Chapter 1

Introduction

1.1 DEFINITIONS OF A SIGNAL

A signal can be defined as a function that conveys *information*. Although signals can be represented in many ways, in all cases the information is contained in some pattern of variations. Signals are represented mathematically as a function of one or more independent variables. If the function depends on a single variable, the signal is said to be *one dimensional*. Eg., Speech signal. A speech signal is represented mathematically as a function of time wherein the amplitude varies with time depending on the spoken word and the person who speaks it. If the function depends on two or more variables, the signal is said to be *multidimensional*. Eg., Image signal. A photographic image is represented as a brightness function of two spatial variables.

Usually the independent variable of the mathematical representation of a signal is taken as *time*. But in some specific cases the independent variable may not be time.

Real-Life examples for signals

In this section, we have discussed few examples for real-life signals.

a) By listening to the heart-beat of a patient, a doctor is able to diagnose the presence or absence of disease. The quantity (heart-beat) represents signal that convey information to the doctor about the state of health of the patient.

b) In listening to a weather forecast over the radio, we get quantities regarding variations in temperature, humidity, the speed of wind etc. The signals represented by these quantities help us to decide whether to go out for a walk or not.

Basically there are two types of signals. (i) Continuous-time signal and (ii) Discrete-time signal.

(i) Continuous-time signal

A signal $x(t)$ is said to be a *continuous-time signal* if it has value of amplitude for all time 't' (i.e., the independent variable 't' is continuous). Fig. 1.1 represents an example of a continuous-time signal whose amplitude varies continuously with time. Continuous-time signals arise naturally when a physical phenomenon (eg., heart-beat, acoustic pressure variation etc.) is converted into an electrical signal using appropriate transducer.

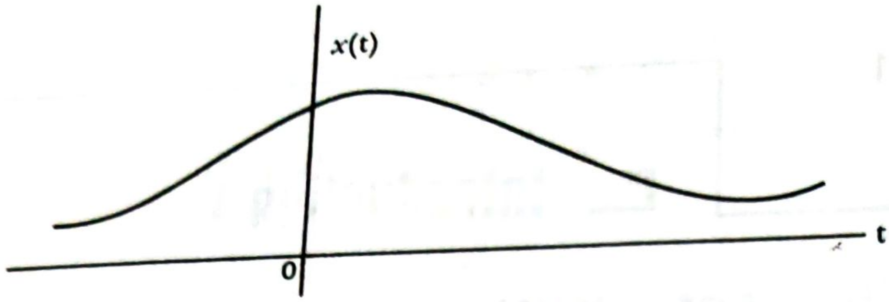


Fig. 1.1 A continuous-time signal

(ii) Discrete-time signal

A *discrete-time signal* is defined only at discrete instants of time. (i.e., the independent variable has discrete values only which are usually uniformly spaced)

A discrete-time signals are represented mathematically as sequence of numbers. A sequence of numbers x in which the n^{th} number in the sequence is denoted by $x(n)$ is written as,

$$x = \{ x(n) \} \qquad ; -\infty < n < \infty$$

where 'n' is an *integer*.

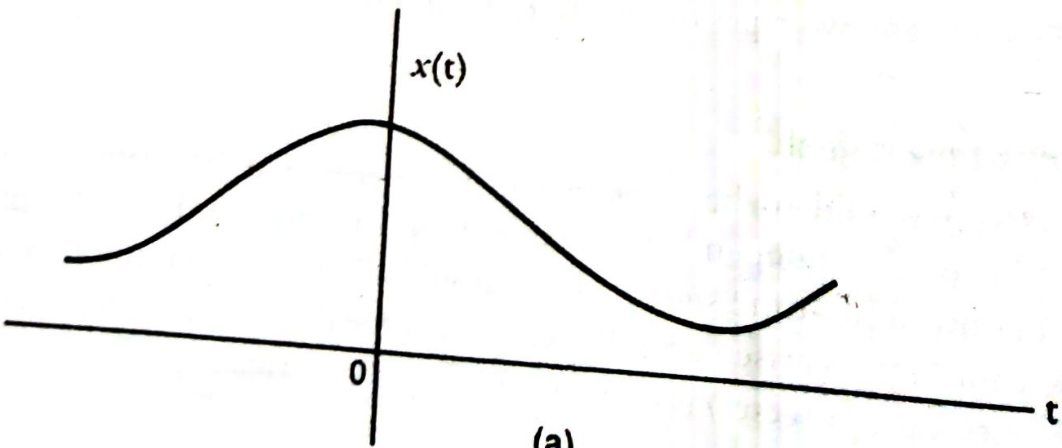
Practically, such sequences are usually obtained from a continuous - time signal by sampling it at a uniform rate.

Consider 'τ' is the sampling period and 'n' denote an integer ($-\infty < n < \infty$). Sampling a continuous-time signal $x(t)$ at time $t=n\tau$ gives a sample value $x(n\tau)$. We write this sampled signal as $x(n)$ such that,

$$x(n) = x(n\tau) \qquad ; \; n = 0, \pm 1, \pm 2, \pm 3, \; \ldots\ldots\ldots$$

Thus a discrete-time signal is represented by a sequence numbers .... $x(-2), x(-1), x(0), x(1), x(2)$ ....

Fig. 1.2 (a) and (b) illustrates the relationship between a continuous-time signal $x(t)$ and discrete-time signal $x(n)$ derived from it.



(a)

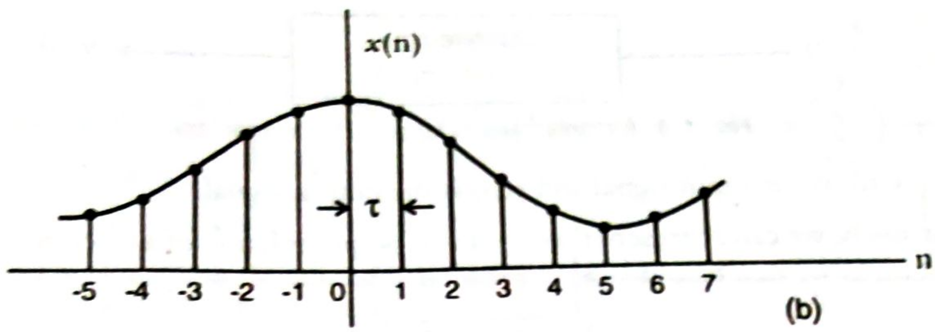


Fig. 1.2 Discrete-time signal $x(n)$ obtained from a continuous-time signal $x(t)$ with sampling period 'τ'

1.2 DEFINITIONS OF A SYSTEM

A *system* is an interacting group of physical objects or physical conditions that are called system *components*. A physical system is an interconnection of components, devices or subsystems.

Signals that enter a system from some external source are referred to as *input signals*. Signals produced by the system by processing the input signals are called the *output signals* or *responses*. Signals that occur within a system and therefore are neither input nor output signals are called *internal signals*. The system responds to one or more input signals to produce one or more output signals. These signals are functions of an independent variable such as time, distance etc.

Many systems are quite complex. They may contain system components and signals of different types. For example, an audio amplifier system contains microphone that convert acoustic signals to electrical signals, amplifier that amplify the electrical signals and speakers that convert electrical signals to acoustic signals.

A *continuous time system* is one where continuous time input signals are applied which results in continuous time output signals. It can be represented pictorially as shown in Fig 1.3.

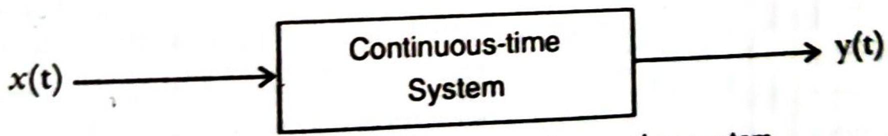


Fig. 1.3 A representation for continuous-time system

where $x(t)$ is the input signal and $y(t)$ is the output signal.

Alternatively, we can represent the input-output relation of a continuous time system by the notation as written below.

$$x(t) \longrightarrow y(t)$$

Similarly, a *discrete-time system* is one where discrete-time input signals are applied which results in discrete-time output signals. It can be represented pictorially as shown in Fig. 1.4.

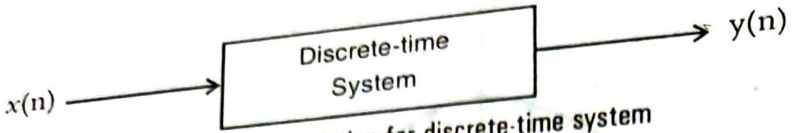


Fig. 1.4 A representation for discrete-time system

where $x(n)$ is the input signal and $y(n)$ is the output signal.

Alternatively, we can represent the input and output relation of a discrete-time system by the notation as written below.

$$x(n) \longrightarrow y(n)$$

Examples for systems :

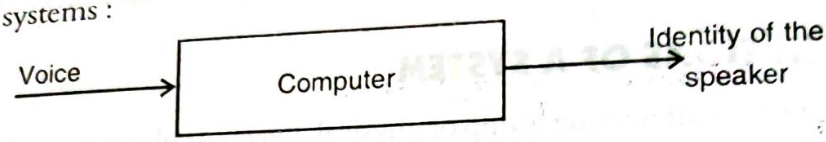


Fig. 1.5 Automatic speaker recognition system

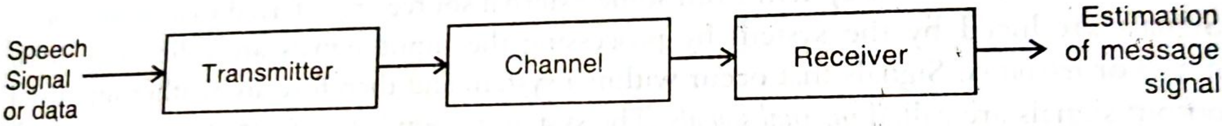


Fig. 1.6 Communication system

Real-Life Examples for Systems

In this section, we will discuss some of the real-life systems in brief.

1. Communication Systems :

The three basic elements of communication system are (i) Transmitter (ii) Channel (iii) Receiver as shown in Fig. 1.6. The *transmitter* and *receiver* are placed at a distance apart which are connected by a physical medium called *channel*. The channel may be free space, optical fiber, co-axial cable etc. The transmitter converts the message signal (eg., speech signal, video signal etc.) produced by a source of information into a form suitable for transmission over the channel. When the transmitted signal travels through the channel it would be distorted due to the physical characteristics of the channel. In addition to this, noise and interfering signals originating from other sources contaminate the message signals. The receiver receives the distorted signal to reconstruct it into a recognizable form or into an estimated form of the original message signal.

Thus, the receiver does the reverse process that of transmitter and also reverses the effect of the channel such as noise elimination, amplification of weak signals etc.

2. Control Systems :

Fig. 1.7 shows the block diagram representation of a closed-loop control system or feedback control system.

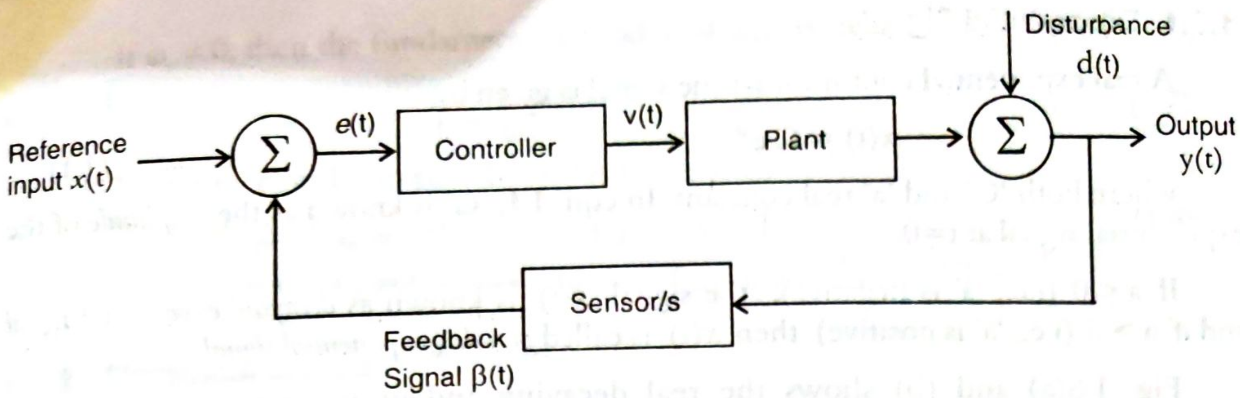


Fig. 1.7 Block diagram of closed loop control system

In any control system, the plant is represented by mathematical operations that generate the output y(t) in response to the plant input v(t) and the external disturbance d(t). The sensors existing in the feedback loop measures the plant output y(t) and converts it into another form β(t) known as *feedback signal*. It is compared against the reference input x(t) to produce an *error signal* e(t). This error signal is applied to a controller which in turn produces the *actuating signal* v(t) that performs the controlling action of the plant.

For example, in an aircraft landing system, the plant is represented by the aircraft body and actuator. The sensors are used by the pilot to determine the lateral position of the aircraft and the controller is a digital computer.

3. Remote Sensing System :

Remote sensing is defined as the process of acquiring informations about an object of interest without being in physical contact with it. The acquisition of informations is accomplished by detecting and measuring the changes that the object creates on the surrounding field. This field may be electromagnetic, acoustic, magnetic etc.

The acquisition of informations can be done in a passive manner by listening to the field that is naturally emitted by the object and processing it or by purposely illuminating the object with a defined field and processing the echo.

1.3 ELEMENTARY CONTINUOUS-TIME SIGNALS

The *elementary* or *basic* signals occur frequently in nature. These signals are the basic building blocks for constructing more complex signals. Many physical signals that occur in nature can be modelled using these elementary signals.

Some of the important elementary continuous-time signals are,

i) Exponential signals
ii) Sinusoidal signals
iii) Exponentially damped sinusoidal signals
iv) Unit step function
v) Unit impulse function
vi) Unit ramp function.

1.3.1 Exponential Signals

A real exponential continuous-time signal is given by,

$$x(t) = C\,e^{at} \qquad \text{.....} \quad (1.1)$$

where both 'C' and 'a' real constant. In eqn. 1.1, 'C' is known as the *amplitude* of the exponential signal at t=0.

If $a < 0$ (i.e., 'a' is negative), the signal $x(t)$ is known as *decaying exponential signal* and if $a > 0$ (i.e., 'a' is positive) then $x(t)$ is called *growing exponential signal*.

Fig. 1.8(a) and (b) shows the real decaying and growing exponential signal respectively.

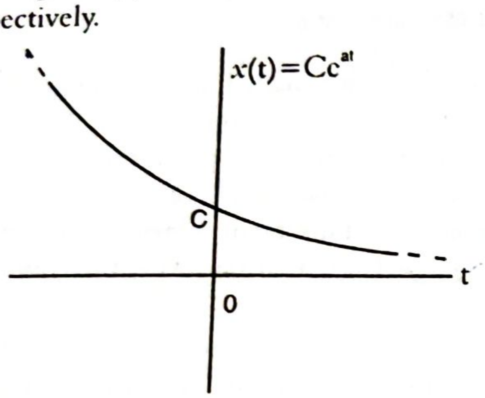


Fig. 1.8(a) : Real decaying exponential signal (i.e., a < o)

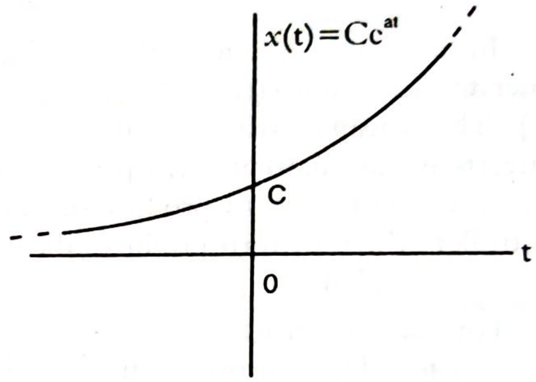


Fig. 1.8(b) : Real growing exponential signal (i.e., a > o)

In eqn. 1.1, if 'C' or 'a' or both are complex numbers, then $x(t)$ is known as *continuous-time complex exponential signal.*

Note : In eqn. 1.1, consider C=1 and 'a' is imaginary.

i.e., $x(t) = e^{j\omega_0 t}$ ..... (1.2)

The signal $x(t)$ given by eqn. 1.2 is periodic with fundamental period $T = \dfrac{2\pi}{\omega_0}$

A continuous-time signal $x(t)$ is periodic with period T,

$$\text{if} \quad x(t) = x(t+T)$$

$$x(t) = e^{j\omega_0 t}$$

$$\therefore \; x(t+T) = e^{j\omega_0 (t+T)} \qquad \text{.....} \quad (1.3)$$

For $x(t)$ to be periodic ..... (1.4)

$$e^{j\omega_0 t} = e^{j\omega_0 (t+T)}$$

$$e^{j\omega_0 t} = e^{j\omega_0 t} . \, e^{j\omega_0 T}$$

Eqn. 1.5 is valid only if ..... (1.5)

$$e^{j\omega_0 T} = 1$$

$\therefore$ If $\omega_0 = 0$ then $e^{j\omega_0 T} = 1$ ..... (1,6)

If $\omega_0 \neq 0$, then the fundamental period T of $x(t)$ is the smallest value of T and is given by,

$$T = \frac{2\pi}{\omega_0} \quad \text{..... (1.7)}$$

Substituting eqn. 1.7 in eqn. 1.6 we get,

$$e^{j\omega_0 \frac{2\pi}{\omega_0}} = e^{j2\pi} = \cos 2\pi + j\sin 2\pi = 1 \quad \text{..... (1.8)}$$

Therefore $e^{j\omega_0 t}$ and $e^{-j\omega_0 t}$ are periodic with fundamental period $T = \frac{2\pi}{\omega_0}$

1.3.2 Sinusoidal signals

A sinusoidal signal is given by,

$$x(t) = A\cos(\omega_0 t + \phi) \quad \text{..... (1.9)}$$

where $\omega_0 = 2\pi f_0$ = angular frequency (rad / sec)

f_0 = linear frequency (Hz)

ϕ = phase shift (radians)

The fundamental period of the signal $x(t)$ in eqn. 1.9 is $T = \frac{2\pi}{\omega_0}$ (sec.)

1.3.3 Exponential damped sinusoidal signals

An exponentially damped sinusoidal signal is given by,

$$x(t) = e^{-at}\sin\omega t \quad ; \text{where } a > 0$$

As 't' increases, the amplitude of sinusoidal oscillation decreases exponentially and approaches zero as $t \to \infty$. An exponentially damped sinusoidal signal is shown in Fig. 1.9.

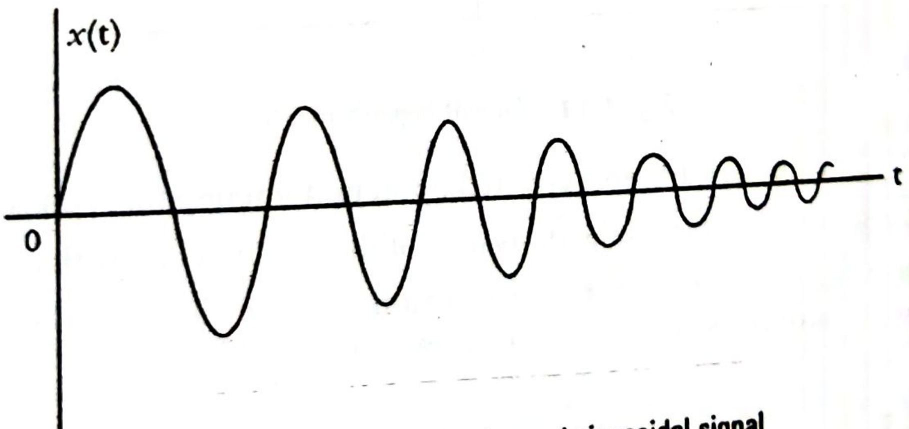


Fig. 1.9 An exponentially damped sinusoidal signal

1.3.4 Unit step function : u(t)

The continuous-time unit step function is defined as,

$$u(t) = \begin{cases} 1 & ; t \geq 0 \\ 0 & ; t < 0 \end{cases}$$

It is shown in Fig. 1.10

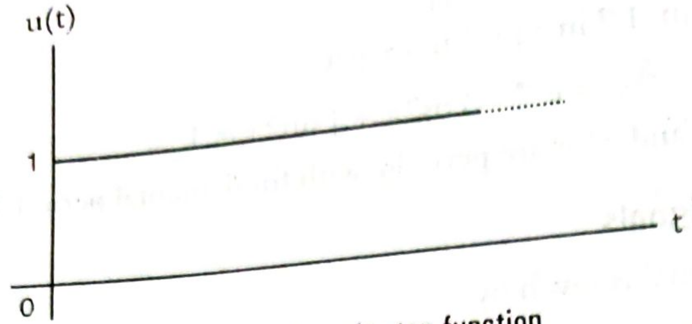


Fig. 1.10 An unit step function

1.3.5 Unit impulse function : $\delta(t)$

The continuous-time unit impulse function $\delta(t)$ is defined as,

$$\delta(t) = 0 \qquad ; t \neq 0$$

$$\text{and} \quad \int_{-\infty}^{\infty} \delta(t)\, dt = 1 \qquad \ldots\ldots \quad (1.10)$$

This function $\delta(t)$ is also known as *Dirac delta function*.

The continuous-time unit impulse function $\delta(t)$ is shown in Fig. 1.11

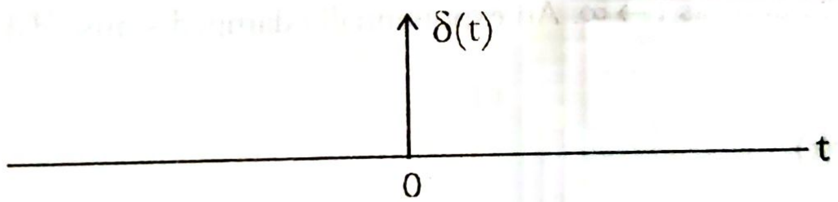


Fig. 1.11 An unit impulse function

Eqn. 1.10 indicates that the area covered by an unit impulse function is *unity*.

The impulse function $\delta(t)$ is the derivative of the step function with respect to time.

Consider a non-idealized unit step function $u_\Delta(t)$ as shown in Fig. 1.12(a). The derivative of $u_\Delta(t)$ with respect to time 't' is shown in Fig. 1.12(b).

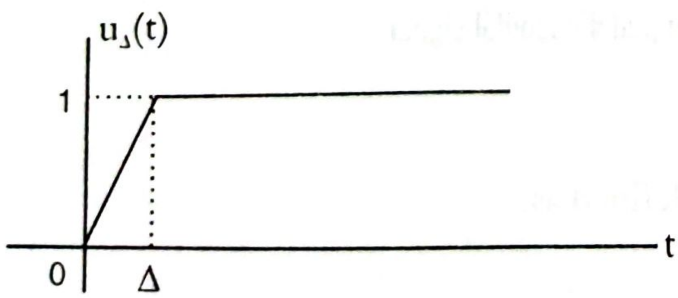


Fig. 1.12(a)

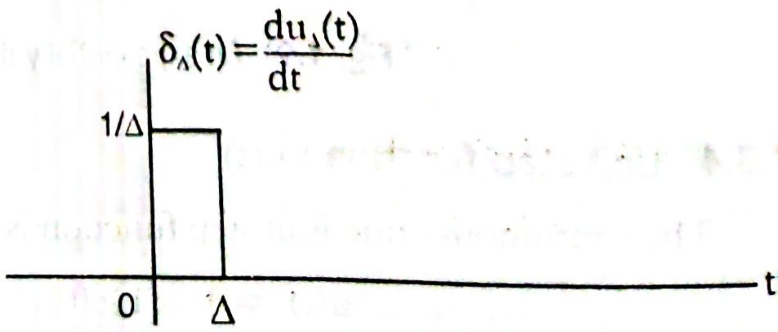


Fig. 1.12(b)

$$\delta_\Delta(t) = \frac{du_\Delta(t)}{dt} \quad \text{.....} \quad (1.11)$$

The area of function $\delta_\Delta(t) = \Delta \cdot \frac{1}{\Delta} = 1$. In Fig. 1.12(b), as $\Delta \to 0$, $\delta_\Delta(t)$ becomes narrower and higher maintaining its unit area.

$$\therefore \quad \delta(t) = \underset{\Delta \to 0}{\text{Lt}} \; \delta_\Delta(t) \quad \text{.....} \quad (1.12)$$

Therefore unit impulse function has,

(i) Zero width
(ii) Infinite height
(iii) Unit area or unit strength

A graphical representation of impulse function with strength 'k' is shown in Fig. 1.13. The strength 'k' indicates the area under the impulse.

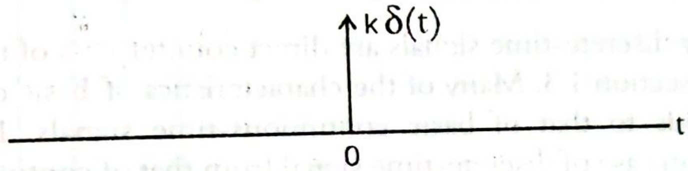


Fig. 1.13 Impulse function with strength 'k'

Properties of continuous-time Impulse function :

(i) $\int_{-\infty}^{\infty} \delta(t)\, dt = 1$

(ii) $\int_{-\infty}^{\infty} k\, \delta(t) = k$

(iii) $x(t)\, \delta(t) = x(0)\, \delta(t)$.

(iv) $\int_{-\infty}^{\infty} x(t)\, \delta(t)\, dt = x(0)$

(v) $x(t).\, \delta(t-t_o) = x(t_o)\, \delta(t-t_o)$ $\Rightarrow$ (sampling property of impulse function)

(vi) $\int_{-\infty}^{\infty} x(t)\, \delta(t-t_o)\, dt = x(t_o)$ $\Rightarrow$ (sifting property of impulse function)

(vii) $\delta(at) = \frac{1}{a}\, \delta(t)$; $a>0$

1.3.6 Unit ramp function $r(t)$

A ramp function is defined as $r(t) = t \quad ; t \geq 0$
$\qquad\qquad\qquad\qquad\qquad\qquad = 0 \quad ; t < 0$

It is the integral of the unit step function $u(t)$. An unit ramp function is shown in Fig. 1.14.

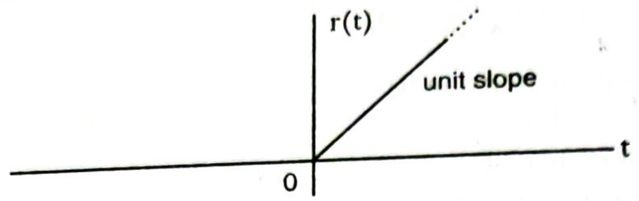


Fig. 1.14 An unit ramp function

1.4 ELEMENTARY DISCRETE-TIME SIGNALS

The elementary discrete-time signals are direct counterparts of the continuous-time signals described in section 1.3. Many of the characteristics of basic discrete-time signals are directly analogous to that of basic continuous-time signals. But, few important characteristics differ in case of discrete-time signal from that of continuous-time signal.

Some of the important basic discrete-time signals are,

(i) Exponential signals.
(ii) Sinusoidal signals.
(iii) Exponentially damped sinusoidal signals.
(iv) Unit step sequence.
(v) Unit impulse sequence.
(vi) Unit ramp sequence.

1.4.1 Discrete-time Exponential Signals

A real exponential discrete-time signal or sequence is given by,

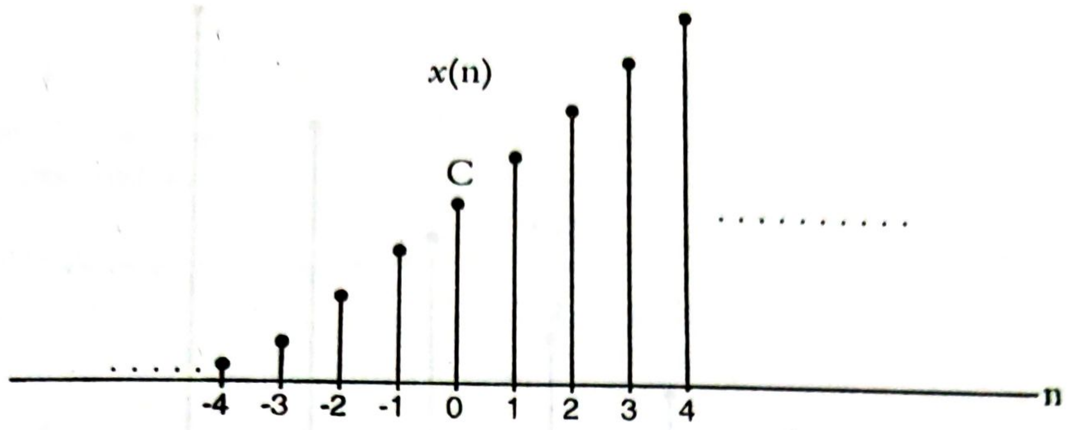
$$x(n) = C\alpha^n \qquad ..... \quad (1.13)$$

where $\qquad \alpha = e^{\beta} \qquad ..... \quad (1.14)$

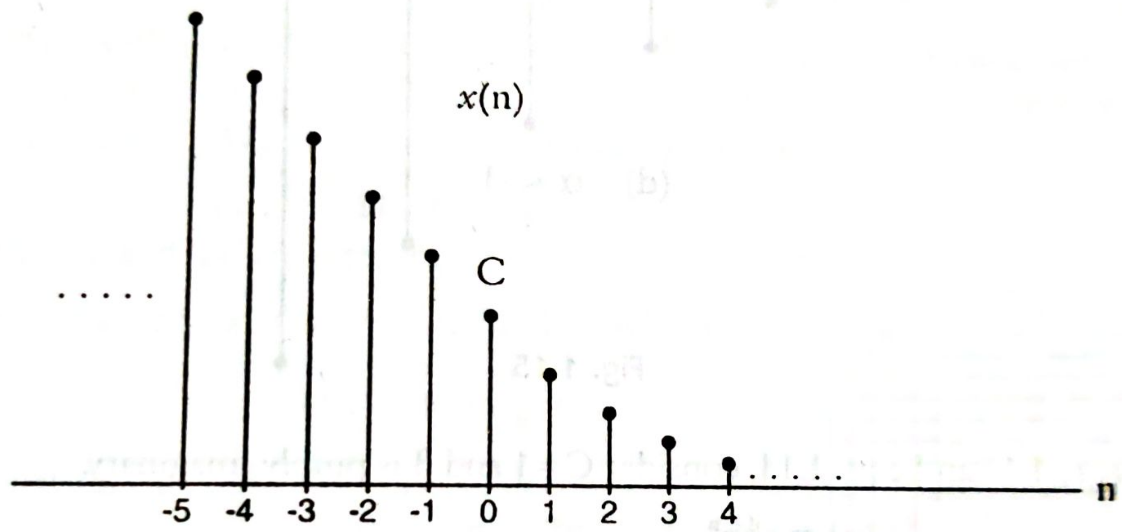
and C, α and β are real constants. In eqn. 1.13, 'C' is known as the *amplitude* of the sequence at n = 0. If $|\alpha| < 1$, the signal decays exponentially. Furthermore, if $\alpha < 0$ (i.e., α is negetive), then the sign of $x(n)$ alternates i.e., when 'n' is positive, $x(n)$ has positive value and when 'n' is negative, $x(n)$ has negative value.

The plots of $x(n) = C\alpha^n$ for $\alpha > 1$; $0 < \alpha < 1$; $-1 < \alpha < 0$ and $\alpha < -1$ are shown in Fig. 1.29 (a), (b), (c) and (d) respectively.

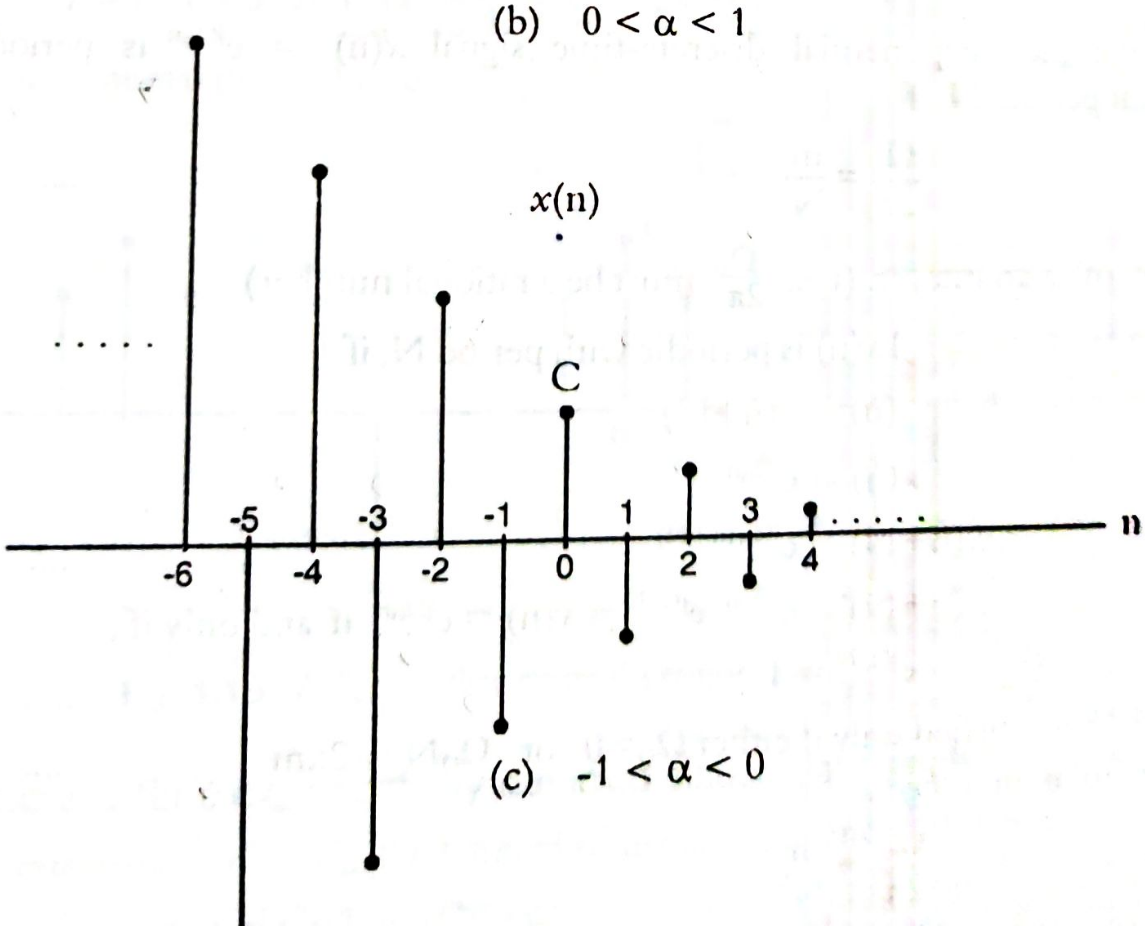
In eqn. 1.13, if 'C' or 'α' or both are complex numbers, then $x(n)$ is known as *discrete-time complex exponential sequence.*



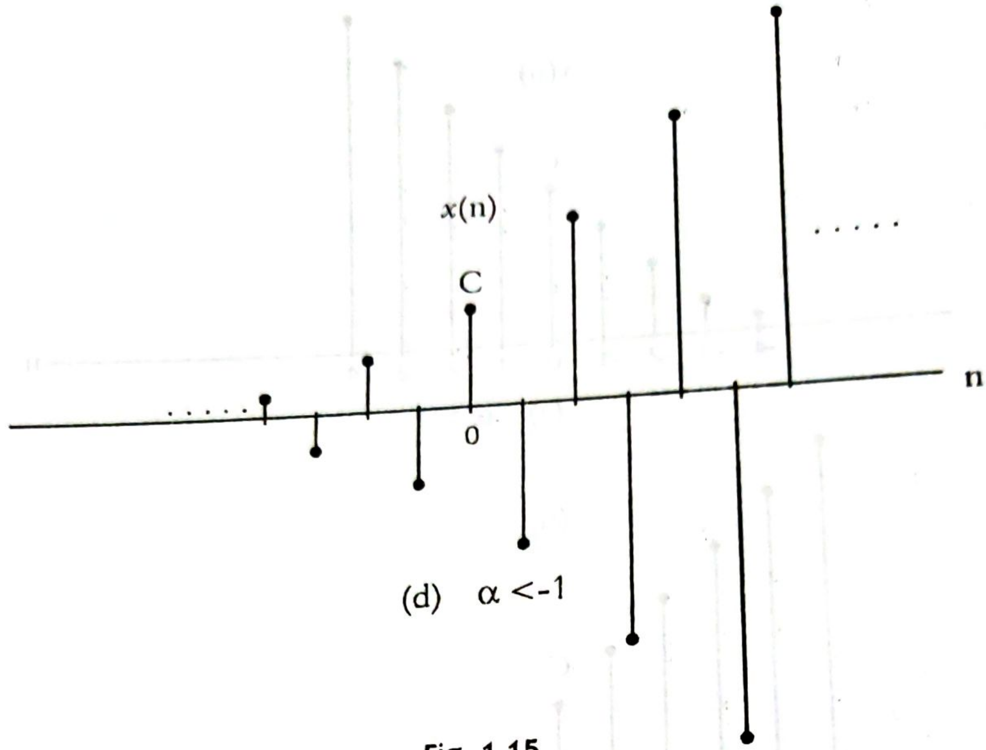
(a) $\alpha > 1$



(b) $0 < \alpha < 1$



(c) $-1 < \alpha < 0$



(d) $\alpha < -1$

Fig. 1.15

Note : In eqn. 1.13 and eqn. 1.14, consider C=1 and β is purely imaginary.

i.e., $\quad x(n) = e^{j\Omega_0 n}$ ..... (1.15)

The complex exponential discrete-time signal $x(n) = e^{j\Omega_0 n}$ is periodic with fundamental period N if,

$$\frac{\Omega_0}{2\pi} = \frac{m}{N} \qquad ..... \quad (1.16)$$

where 'm' is an integer. (i.e., $\frac{\Omega_0}{2\pi}$ must be a rational number)

A discrete-time signal $x(n)$ is periodic with period N, if

$$x(n) = x(n+N)$$

$$x(n) = e^{j\Omega_0 n}$$

$$\therefore x(n+N) = e^{j\Omega_0 (n+N)} \qquad ..... \quad (1.17)$$

$\therefore x(n+N) = e^{j\Omega_0 n} . e^{j\Omega_0 N} = x(n) = e^{j\Omega_0 n}$ **if and only if ,**

$$e^{j\Omega_0 N} = 1 \qquad ..... \quad (1.18)$$

Eqn. 1.18 is satisfied only if either $\Omega_0 = 0$ or $\Omega_0 N = 2\pi m$ ..... (1.19)
where 'm' is an integer.

$$\therefore \quad \frac{\Omega_0}{2\pi} = \frac{m}{N} \qquad ..... \quad (1.20)$$

Therefore discrete-time complex exponential signal **is periodic only if Ω_0 is a rational** multiple of 2π.

i.e., $$\Omega_0 = 2\pi \cdot \frac{m}{N} \quad \text{..... (1.21)}$$

where 'N' is fundamental period, Ω_0 is angular frequency in radians (if 'n' is dimensionless) and 'm' is an integer.

1.4.2 Discrete-time Sinusoidal Signals

A discrete-time version of a sinusoidal signal is given by

$$x(n) = A\cos(\Omega_0 n + \phi) \quad \text{..... (1.22)}$$

where 'A' is maximum value of $x(n)$, Ω_0 is angular frequency and ϕ is phase angle. If 'n' is dimensionless, both Ω_0 and ϕ are measured in radians.

Unlike continuous-time sinusoidal signal, discrete-time sinusoidal signals are not periodic for an arbitrary values of Ω_0. For a discrete-time sinusoidal signal to be periodic, the angular frequency Ω_0 must be a rational multiple of 2π.

i.e., $$\Omega_0 = 2\pi \cdot \frac{m}{N} \quad \text{..... (1.23)}$$

where 'm' and 'N' are integers.

Consider a signal $x(n)$ shown in Fig. 1.16 which is given by,

$$x(n) = \cos\left(\frac{\pi n}{4}\right) \quad \text{..... (1.24)}$$

Comparing eqn. 1.24 with eqn. 1.22, we have

$$A=1,\ \phi=0 \ \text{and}\ \Omega_0 = \frac{\pi}{4} = \frac{2\pi}{8} = 2\pi \cdot \frac{1}{8}$$

$\therefore$ Fundamental period N=8.

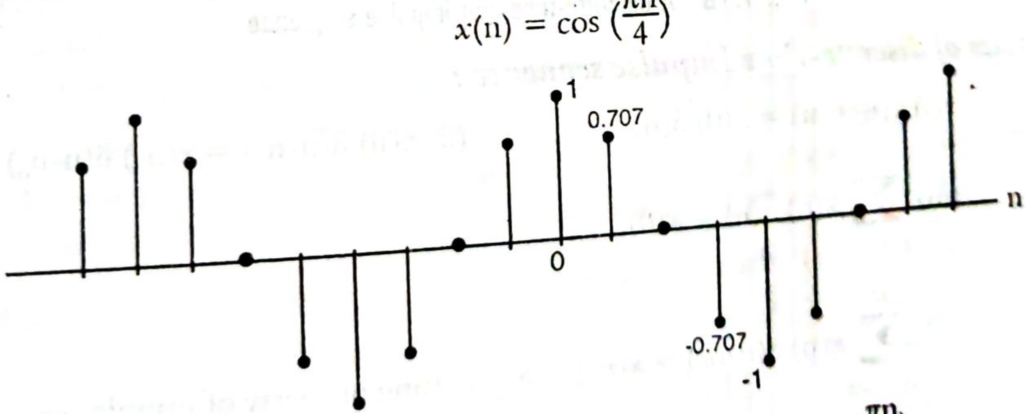


Fig. 1.16 A discrete-time sinusoidal sequence $x(n) = \cos\left(\frac{\pi n}{4}\right)$

1.4.3 Discrete-time exponentially damped sinusoidal signals

A discrete-time exponentially damped sinusoidal sequence is given by,

$$x(n) = C\alpha^n \sin(\Omega n + \phi) \quad ; 0 < |\alpha| < 1 \quad \text{..... (1.25)}$$

The value of $x(n)$ decreases as 'n' increases.

1.4.4 Discrete-time unit step sequence : u(n)

A discrete-time unit step sequence is defined as,

$$u(n) = 1 \quad ; n \geq 0$$
$$= 0 \quad ; n < 0$$

It is shown in Fig. 1.17.

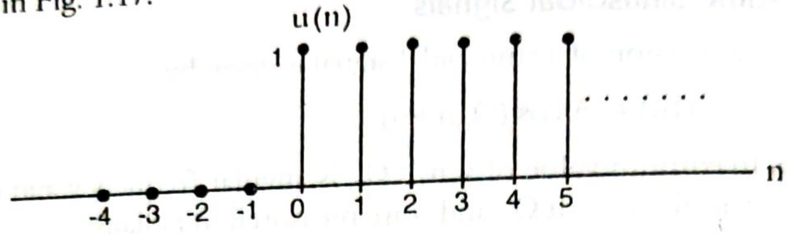


Fig. 1.17 An unit step sequence

1.4.5 Discrete-time unit impulse sequence : δ(n)

A discrete-time impulse sequence is defined as,

$$\delta(n) = 1 \quad ; n=0$$
$$= 0 \quad ; n \neq 0$$

It is shown in Fig. 1.18.

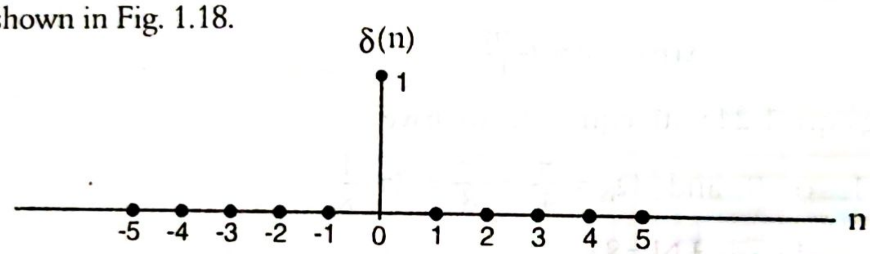


Fig. 1.18 A discrete-time unit impulse sequence

Properties of discrete-time Impulse sequence :

(i) $x(n)\,\delta(n) = x(0)\,\delta(n)$ (ii) $x(n)\,\delta(n-n_o) = x(n_o)\,\delta(n-n_o)$

(iii) $\sum_{n=-\infty}^{\infty} x(n)\,\delta(n) = x(0)$

(iv) $\sum_{n=-\infty}^{\infty} x(n)\,\delta(n-n_o) = x(n_o)$ $\Rightarrow$ (sifting property of impulse sequence)

Note : $u(n) = 1 \quad ; n \geq 0$
$= 0 \quad ; n < 0$

$\therefore\ u(n) = \delta(n) + \delta(n-1) + \delta(n-2) + \ldots.. + \delta(n-\infty)$

$u(n) = \sum_{k=0}^{\infty} \delta(n-k)$ and also $u(n) = \sum_{k=-\infty}^{n} \delta(k)$

1.4.6 Discrete-time unit ramp sequence : r(n)

A discrete-time ramp sequence is defined as,

$$r(n) = n \quad ; n \geq 0$$
$$= 0 \quad ; n < 0$$

It is shown in Fig. 1.19.

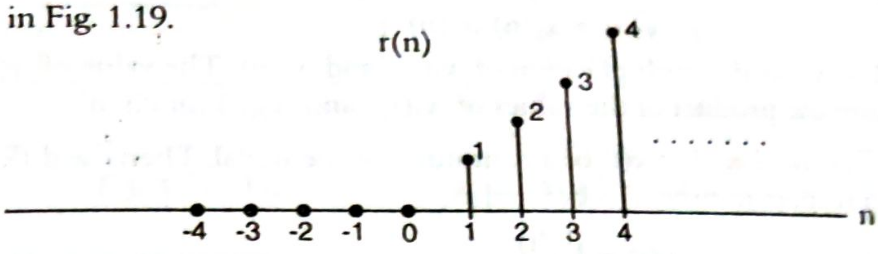


Fig. 1.19 A discrete-time unit ramp sequence

1.5 BASIC OPERATIONS ON SIGNALS

In this section, we describe some of the basic operations that could be performed on signals. One dimensional signal can be defined using two variables.

(i) Dependent variable (ii) Independent variable.

Dependent variable corresponds to the amplitude or value of the signal but the independent variable is time 't' or 'n' for continuous time and discrete-time signal respectively.

1.5.1 Operations performed on the dependent variables

a) ***Amplitude scaling*** : Let $x(t)$ be a continuous time signal. Then the signal

$$y(t) = c\,x(t) \qquad ..... \quad (1.26)$$

is known as amplitude scaled version of $x(t)$ where 'c' is known as *scaling factor*. The signal y(t) is obtained by multiplying the value (amplitude) of $x(t)$ by scalar 'c' at all 't'.

Similarly, let $x(n)$ be a discrete-time signal. Then the signal,

$$y(n) = c\,x(n) \qquad ..... \quad (1.27)$$

is known as amplitude scaled version of $x(n)$ where 'c' is a *scaling factor*. The signal y(n) is obtained by multiplying the value of $x(n)$ by scalar 'c' at all 'n'.

b) ***Addition*** : Let $x_1(t)$ and $x_2(t)$ are continuous-time signals. Then the signal,

$$y(t) = x_1(t) + x_2(t) \qquad ..... \quad (1.28)$$

is known as the addition of $x_1(t)$ and $x_2(t)$. The value of y(t) is obtained by adding the values of $x_1(t)$ and $x_2(t)$ for all 't'.

Similarly, let $x_1(n)$ and $x_2(n)$ are discrete-time signals. Then the signal,

$$y(n) = x_1(n) + x_2(n) \qquad ..... \quad (1.29)$$

is known as the addition of $x_1(n)$ and $x_2(n)$. The value of y(n) is obtained by adding the values of $x_1(n)$ and $x_2(n)$ for all 'n'.

c) ***Multiplication***: Let $x_1(t)$ and $x_2(t)$ are given continuous-time signals. Then the signal,

$$y(t) = x_1(t) \cdot x_2(t) \qquad \ldots\ldots \quad (1.30)$$

is known as the multiplication of $x_1(t)$ and $x_2(t)$. The value or amplitude of $y(t)$ is obtained by taking the product of the values of $x_1(t)$ and $x_2(t)$ for all 't'.

Similarly, let $x_1(n)$ and $x_2(n)$ are discrete-time signals. Then the signal,

$$y(n) = x_1(n) \cdot x_2(n) \qquad \ldots\ldots \quad (1.31)$$

is known as the multiplication of $x_1(n)$ and $x_2(n)$. The value of $y(n)$ is obtained by taking the product of the values of $x_1(n)$ and $x_2(n)$ for all 'n'.

d) ***Differentiation*** : Let $x(t)$ be a continuous-time signal. Then the differentiation of $x(t)$ with respect to time 't' is defined as,

$$y(t) = \frac{dx(t)}{dt} \qquad \ldots\ldots \quad (1.32)$$

e) ***Integration*** : Let $x(t)$ be a continuous-time signal. Then the integration of $x(t)$ with respect to 't' is defined as,

$$y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau \qquad \ldots\ldots \quad (1.33)$$

1.5.2. Operations performed on the independent variables

a) ***Time Scaling*** : Let $x(t)$ be a continuous-time signal. The signal $y(t)$ obtained by scaling the independent variable 't' by a factor 'a' is given by,

$$y(t) = x(at) \qquad \ldots\ldots \quad (1.34)$$

If $a > 1$, the signal $y(t)$ is a compressed version of $x(t)$ and if $0 < a < 1$, the signal $y(t)$ is an expanded version of $x(t)$.

Eg., Let $x(t)$ is as shown in Fig. 1.20.

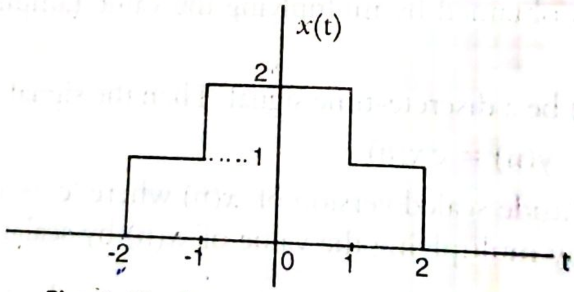


Fig. 1.20 A continuous-time signal $x(t)$

The signals $x(2t)$ and $x(\frac{1}{2}t)$ are shown in Fig. 1.21 and Fig. 1.22 respectively.

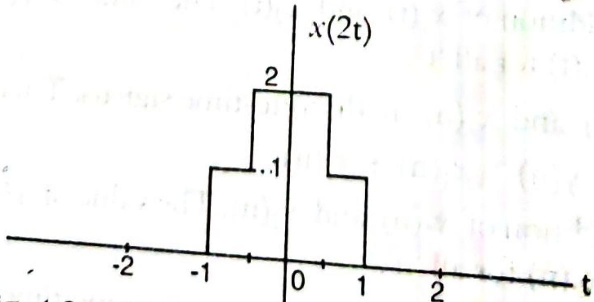


Fig. 1.21 Compressed version of $x(t)$ by a factor 2

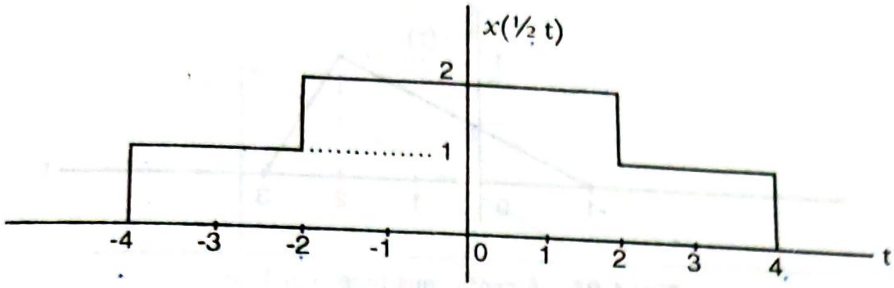


Fig. 1.22 Expanded version of $x(t)$ by a factor 2

In the discrete-time sequence,

$$y(n) = x(kn) \qquad ; k > 0 \qquad \ldots\ldots \quad (1.35)$$

where 'k' is an integer. If $k > 1$, some samples of $x(n)$ would be lost.

Eg., Let $x(n)$ is as shown in Fig. 1.23.

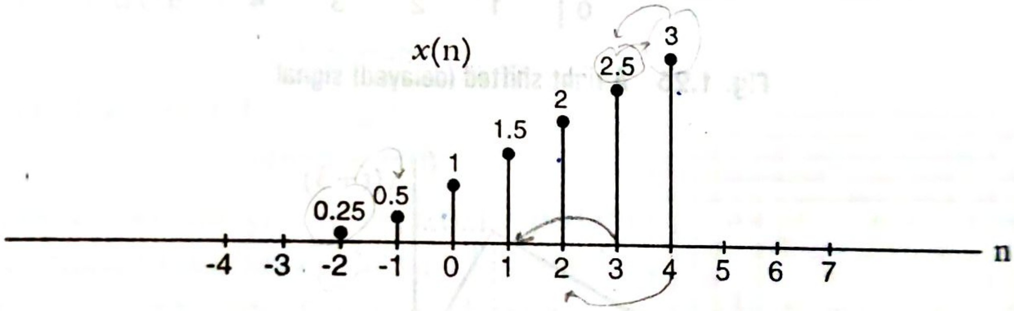


Fig. 1.23 A discrete-time signal $x(n)$

The signal $x(2n)$ is shown below in Fig. 1.24

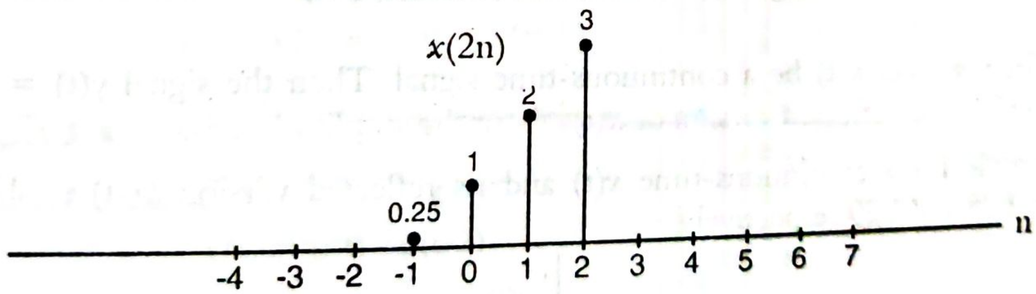


Fig. 1.24 Compressed version of $x(n)$ by a factor 2

b) ***Time Shifting*** : Let $x(t)$ be a continuous-time signal. Then the signal,

$$y(t) = x(t - t_0) \qquad \ldots\ldots \quad (1.36)$$

is known as time shifted version of $x(t)$, where 't_0' is the time shift.

If $t_0 > 0$, the waveform of the signal is shifted to the right. If $t_0 < 0$ the waveform is shifted to the left. Let $x(t)$ is shown in Fig. 1.25.

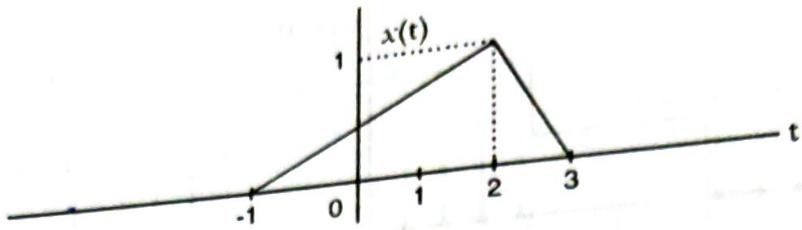


Fig. 1.25 A continuous-time signal

Then, $x(t-2)$ and $x(t+3)$ is shown in Fig. 1.26 and 1.27 respectively.

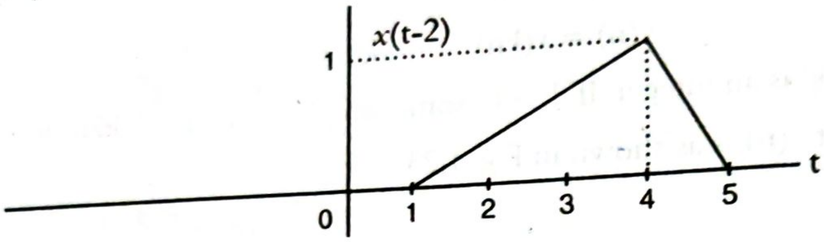


Fig. 1.26 A right shifted (delayed) signal

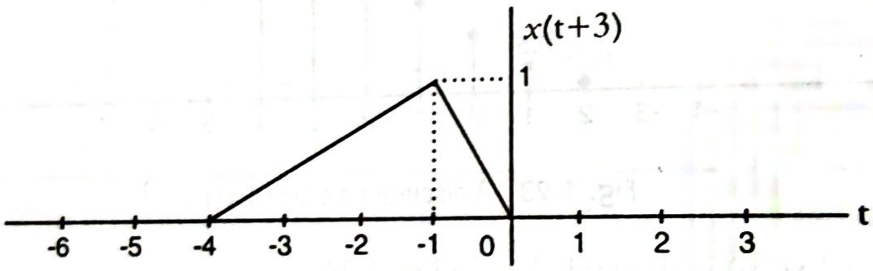


Fig. 1.27 A left shifted (advanced) signal

c) ***Reflection*** : Let $x(t)$ be a continuous-time signal. Then the signal $y(t) = x(-t)$ is known as the reflected version of $x(t)$ about the amplitude axis.

Example for a continuous-time $x(t)$ and its reflected version $x(-t)$ is shown in Fig. 1.28 and 1.29 respectively.

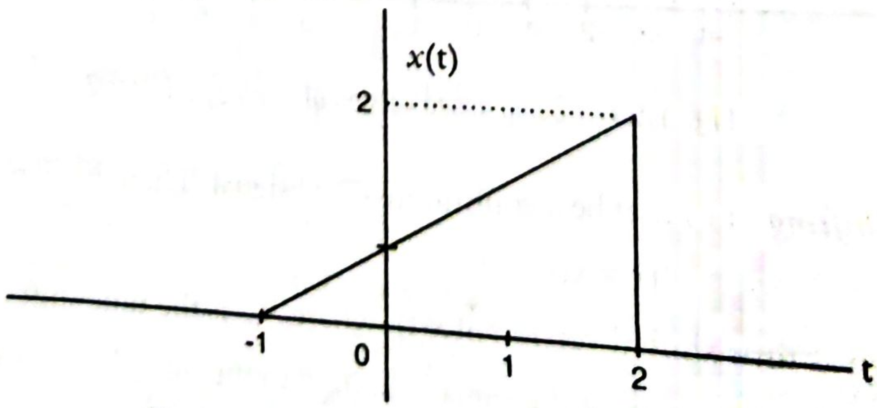


Fig. 1.28 A continuous-time signal $x(t)$

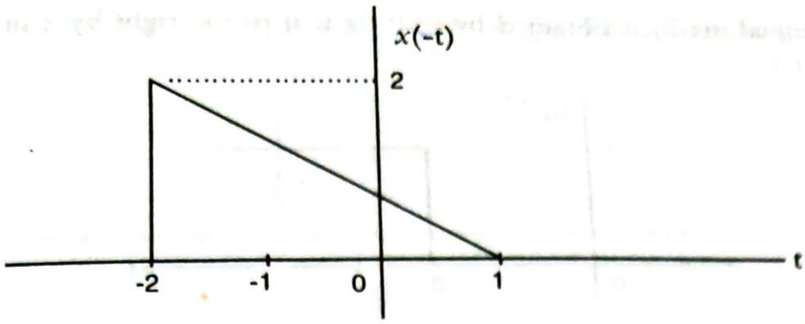


Fig. 1.29 Reflected version of $x(t)$

Precedence Rule : Let the signals $x(t)$ and $y(t)$ are related by the following eqn. 1.37.

$$y(t) = x(at-b) \qquad \text{.....} \quad (1.37)$$

To get $y(t)$ from $x(t)$, we have to perform both time shifting and time scaling operations.

Put $t=0$ in eqn. 1.37 we get,

$$y(0) = x(-b) \qquad \text{.....} \quad (1.38)$$

Put $t=b/a$ in eqn. 1.37 we get,

$$y(b/a) = x(0) \qquad \text{.....} \quad (1.39)$$

Once we obtain $y(t)$ from $x(t)$ by performing time shifting and time scaling operation, it must satisfy the eqn. 1.38 and 1.39.

This is possible only if the time shifting operation is performed first on $x(t)$ which yields an intermediate signal $v(t)$ given by,

$$v(t) = x(t-b) \qquad \text{.....} \quad (1.40)$$

Next, the time scaling operation is performed on $v(t)$ to obtain $y(t)$.

$$\text{i.e.,} \quad y(t) = v(at) = x(at-b) \qquad \text{.....} \quad (1.41)$$

Examples

Example 1.1 Sketch the following signal.

$$x(t) = u(t) - u(t-2)$$

Solution : Given : $x(t) = u(t) - u(t-2)$

The signal unit step function $u(t)$ is shown in Fig. P1.1.1.

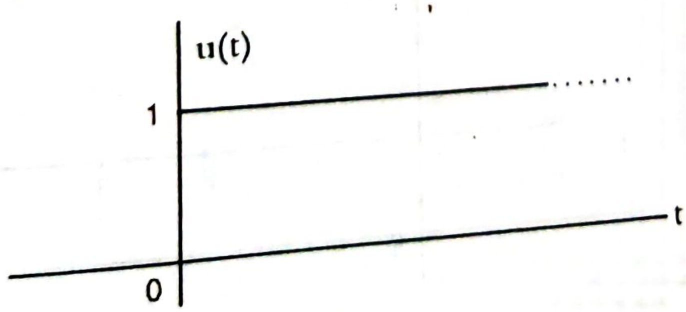


Fig. P1.1.1

The signal u(t-2) is obtained by shifting u(t) to the right by 2 units as shown in Fig. P1.1.2.

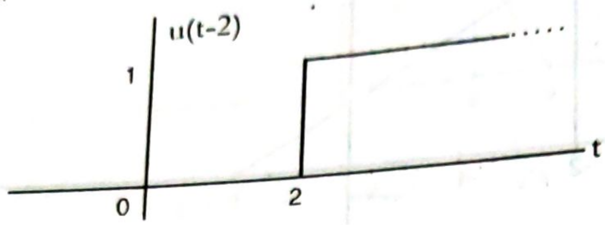


Fig. P1.1.2

i.e., $u(t-2) = 1 \quad ; t \geq 2$

$\qquad\qquad = 0 \quad ; t < 2$

Now $x(t) = u(t) - u(t-2)$ is obtained as follows.

For $t<0$; $u(t) = u(t-2)=0$ $\quad\therefore x(t) = 0 - 0 = 0$

For $0 < t < 2$; $u(t) = 1$ & $u(t-2)=0$ $\quad\therefore x(t) = 1 - 0 = 1$

For $t > 2$; $u(t) = 1$ & $u(t-2)=1$ $\quad\therefore x(t) = 1 - 1 = 0$

The signal $x(t)$ is shown in Fig. P1.1.3.

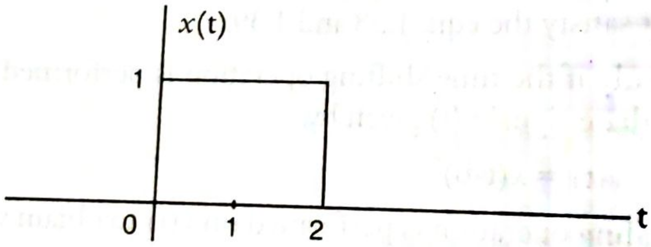
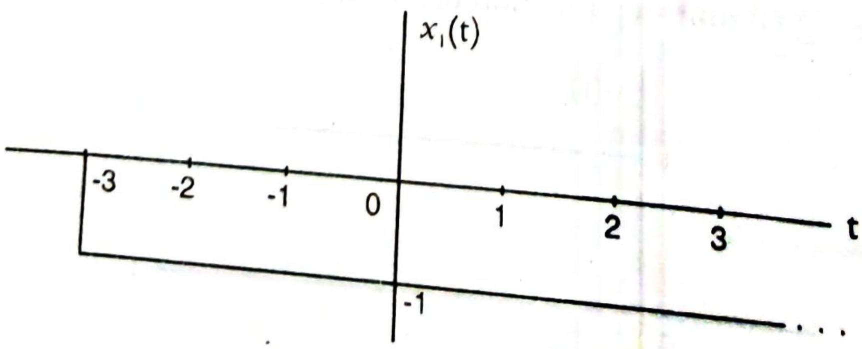


Fig. P1.1.3

Example 1.2 Sketch the signal $x(t) = -u(t+3) + 2u(t+1) - 2u(t-1) + u(t-3)$

Solution : Firstly, we sketch the signals $x_1(t) = -u(t+3)$; $x_2(t) = 2u(t+1)$; $x_3(t) = -2u(t-1)$ and $x_4(t) = u(t-3)$ as shown in Fig. P1.2 (a) to (d)

(a)



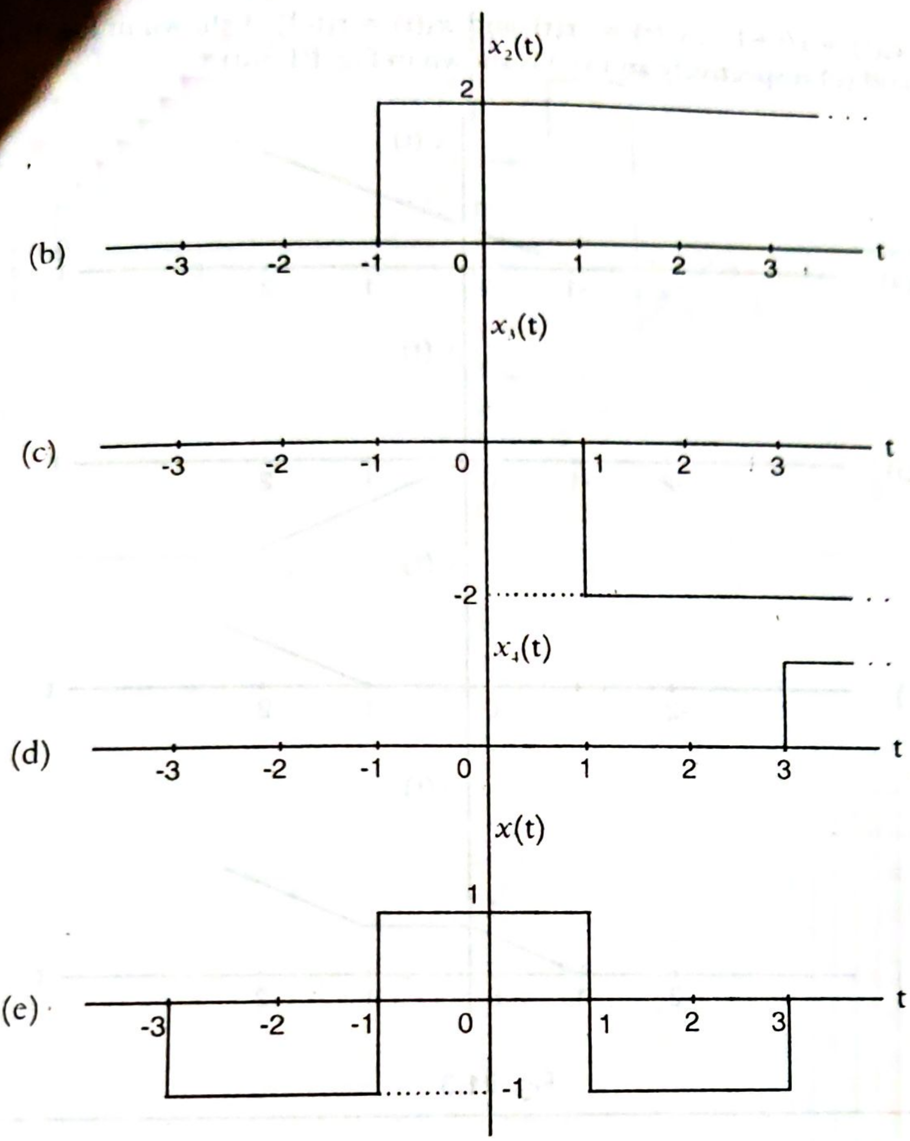


Fig. P1.2

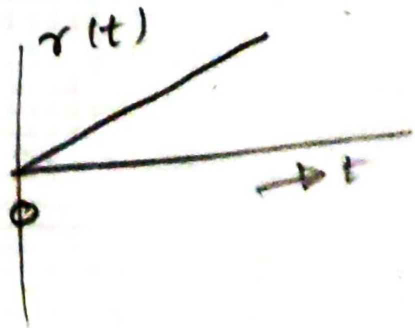
The signal $x(t) = x_1(t)+x_2(t)+x_3(t)+x_4(t)$ is obtained as below and plotted in Fig. P1.2(e).

For $t<-3$	; $x_1(t) = x_2(t) = x_3(t) = x_4(t) = 0$	$\therefore x(t) = 0+0+0+0 = 0$
For $-3<t<-1$	; $x_1(t) = -1$; $x_2(t) = x_3(t) = x_4(t) = 0$	$\therefore x(t) = -1+0+0+0 = -1$
For $-1<t<1$	; $x_1(t) = -1$; $x_2(t) = 2$; $x_3(t) = x_4(t) = 0$	$\therefore x(t) = -1+2+0+0 = 1$
For $1<t<3$	; $x_1(t) = -1$; $x_2(t) = 2$; $x_3(t) = -2$; $x_4(t)=0$	$\therefore x(t) = -1+2-2+0 = -1$
For $t>3$	; $x_1(t) = -1$; $x_2(t) = 2$; $x_3(t) = -2$; $x_4(t) = 1$	$\therefore x(t) = -1+2-2+1 = 0$

Example 1.3 Sketch the signal,

$$x(t) = r(t+1) - r(t) + r(t-1)$$

Solution : Firstly, we draw the signals,



$x_1(t) = r(t+1)$; $x_2(t) = -r(t)$ and $x_3(t) = r(t-1)$ as shown in Fig. P1.3(a), (b) and (c) respectively and $x(t)$ is shown in Fig. P1.3(d).

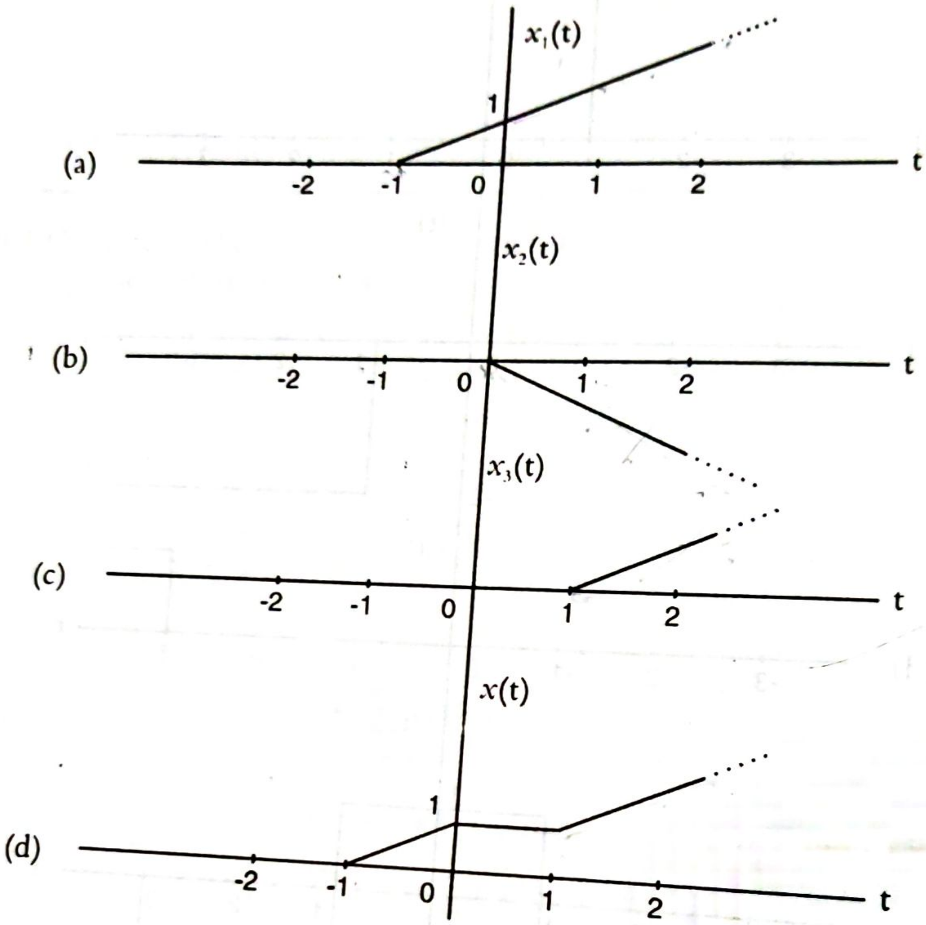


Fig. P1.3

Example 1.4 A continuous-time signal $x(t)$ shown in Fig. P1.4. Draw the signal.

$$y(t) = \{x(t)+x(2-t)\}\, u(1-t)$$

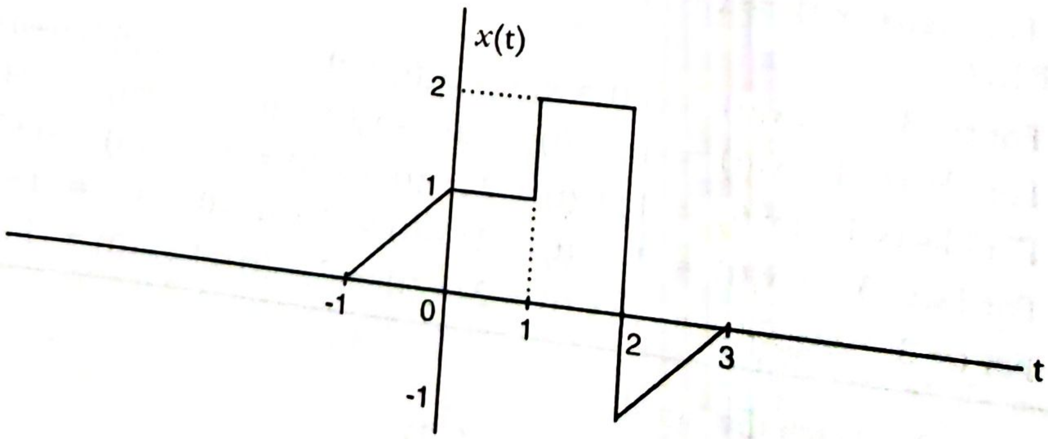


Fig. P1.4

lution :

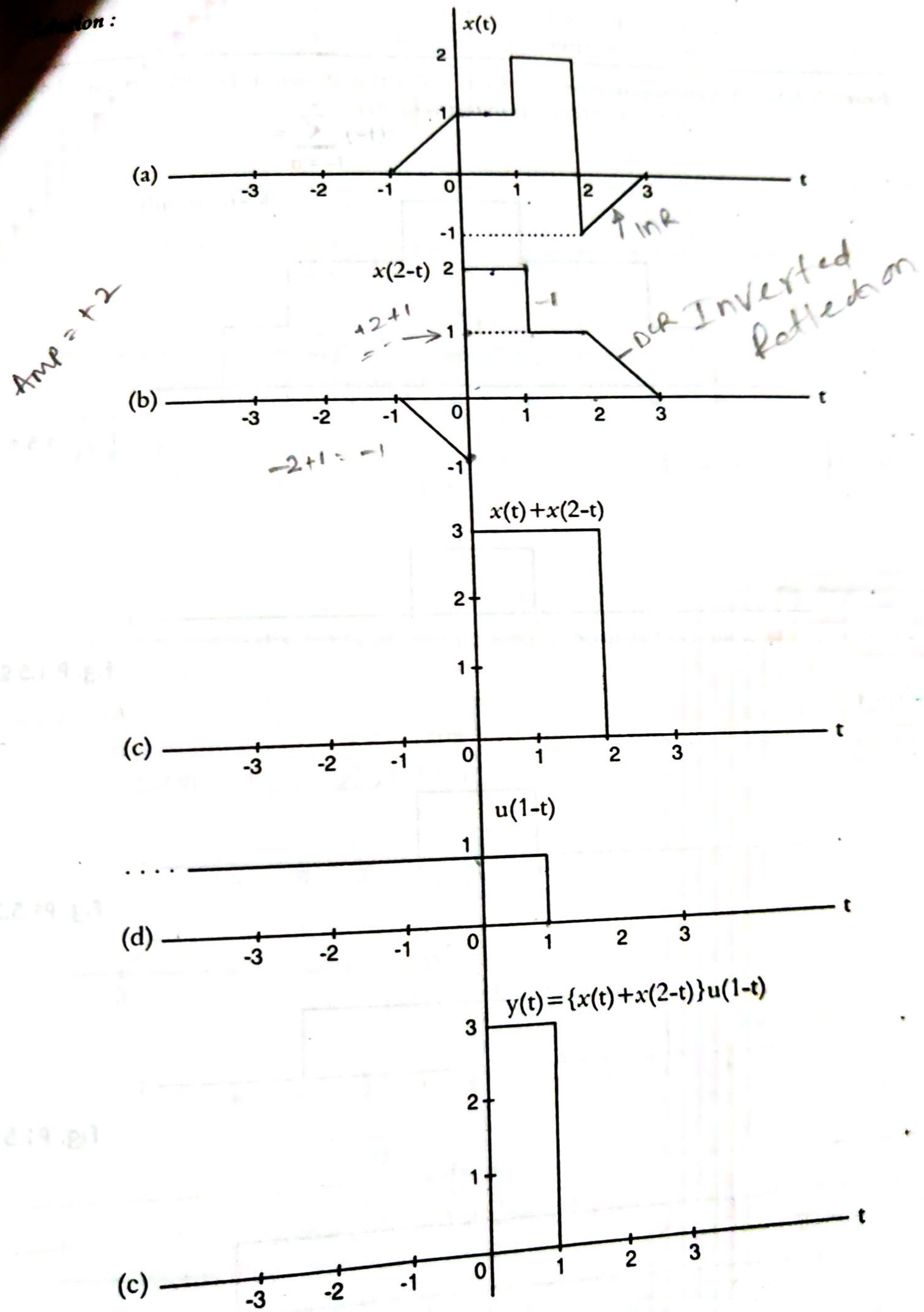


Fig. P1.4.1

$\therefore$ $y(t) = 3 \quad ; 0 \leq t \leq 1$

$\quad\quad = 0 \quad ;$ otherwise

Example 1.5 A continuous-time signal $x(t)$ and $g(t)$ is shown in Fig. P1.5.1 and P1.5.2 respectively. Express $x(t)$ in terms of $g(t)$.

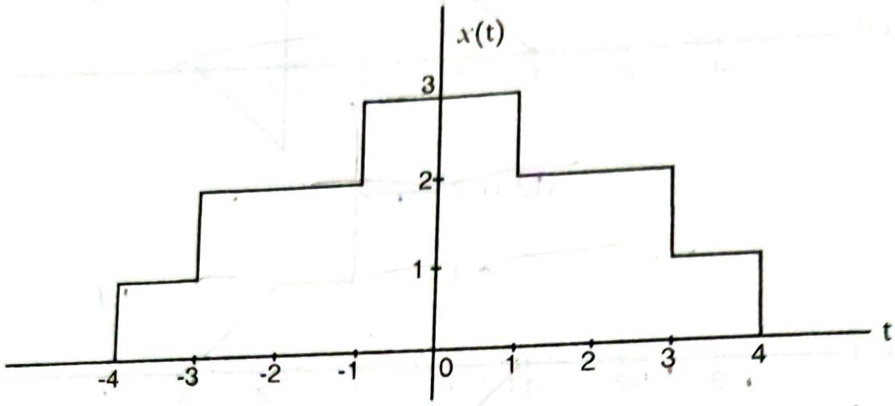


Fig. P 1.5.1

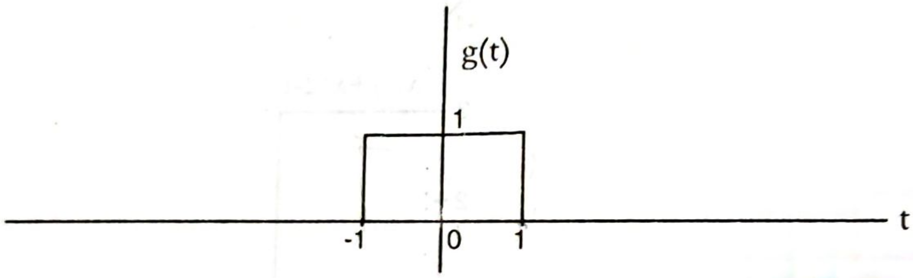


Fig. P 1.5.2

Solution :

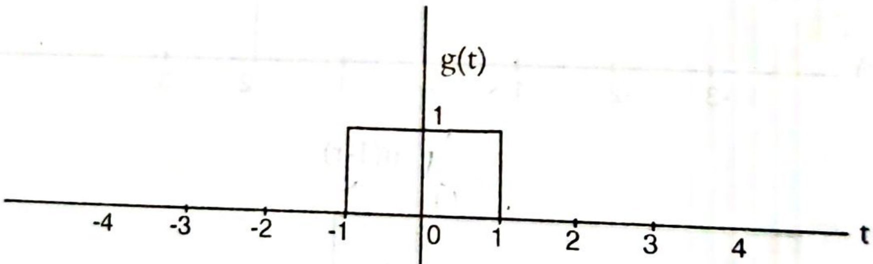


Fig. P1.5.3

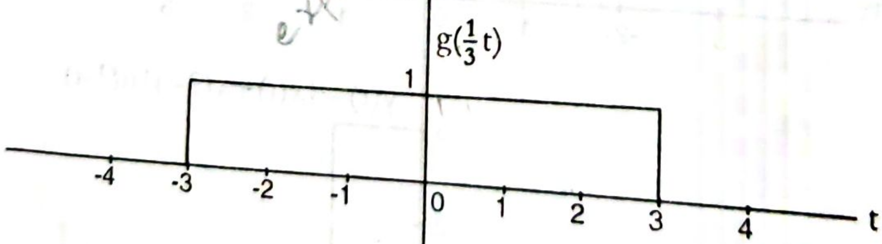


Fig. P1.5.4

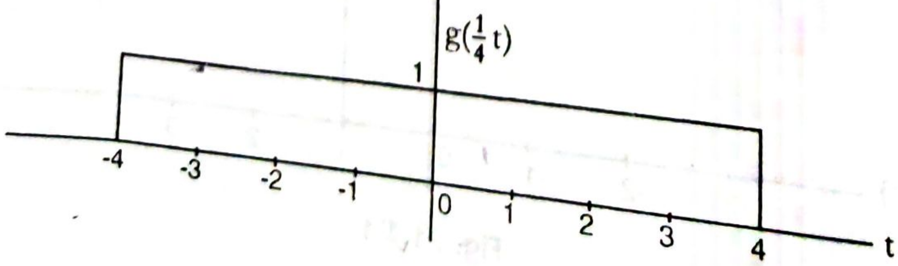


Fig. P1.5.5

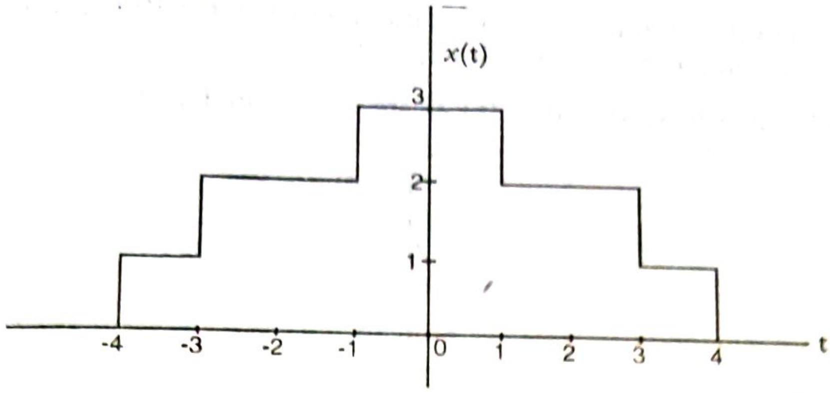


Fig. P1.5.6

Adding g(t), $g(\frac{1}{3}t)$ and $g(\frac{1}{4}t)$ yields the signal $x(t)$ as shown in Fig. P1.5.6.

$$\therefore x(t) = g(t) + g(\tfrac{1}{3}t) + g(\tfrac{1}{4}t)$$

Example 1.6 For the continuous-time signal $x(t)$ shown in Fig. P1.6, sketch the signal $y(t) = x(3t+2)$

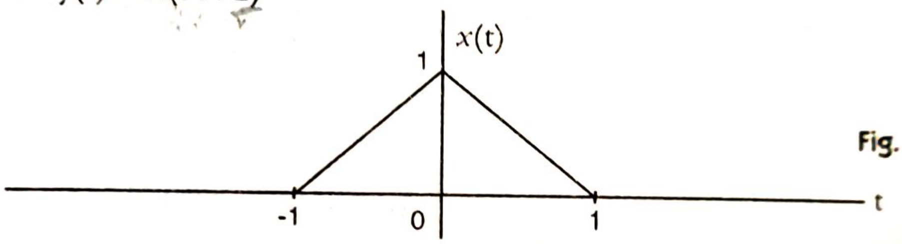


Fig. P1.6

Solution : According to precedence rule, first we obtain an intermediate signal v(t) representing time shifted version of $x(t)$ and then compress it by a factor of 3 to get $y(t) = v(3t) = x(3t+2)$

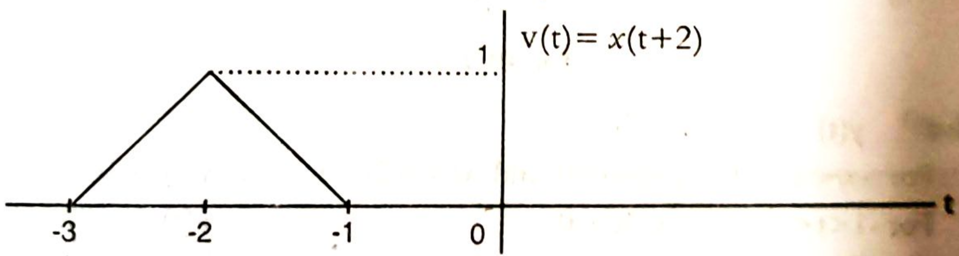


Fig. P1.6.1

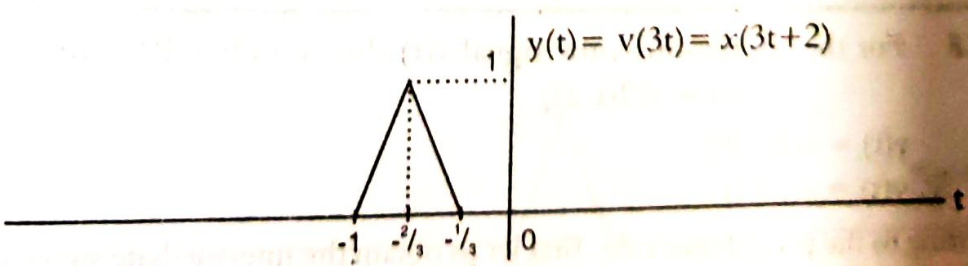


Fig. P1.6.2

Example 1.7 For the continuous-time signal $x(t)$ shown in Fig. P1.6 (previous example), obtain $y(t) = x(3t) + x(3t+2)$.

Solution : The signals $x(3t)$ and $x(3t+2)$ are shown in Fig. P1.7.1 and Fig. P1.7.2 respectively.

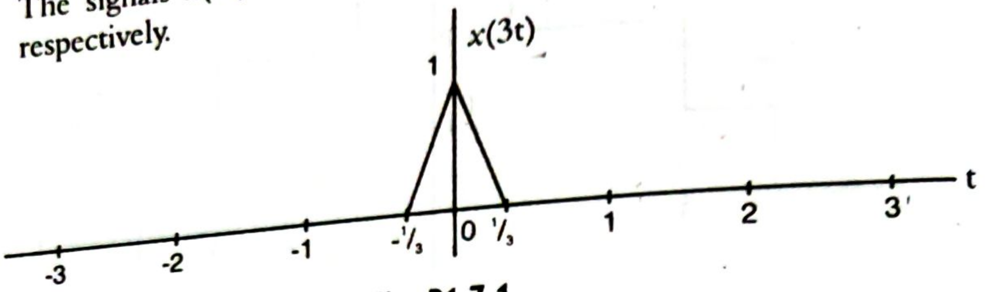


Fig. P1.7.1

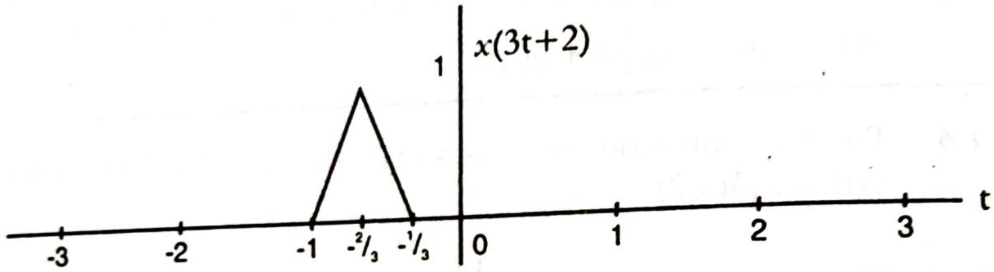


Fig. P1.7.2

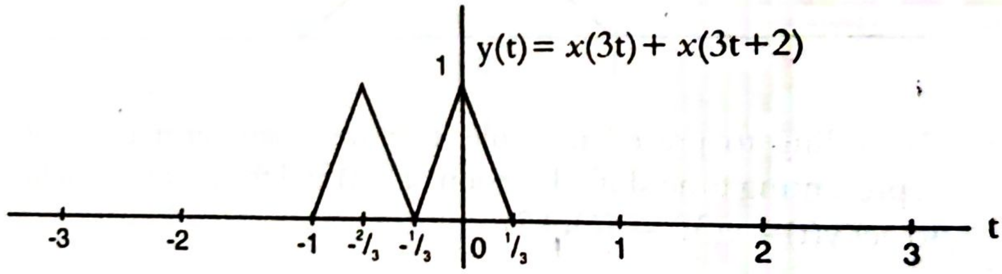


Fig. P1.7.3

We have $y(t) = x(3t) + x(3t+2)$

For $-\infty < t < -1$; $x(3t)=0$ and $x(3t+2)=0$ $\therefore$ $y(t) = 0$

For $-1<t<-1/3$; $x(3t)=0$ $\therefore$ $y(t) = x(3t+2)$

For $-1/3 < t < 1/3$; $x(3t+2)=0$ $\therefore$ $y(t) = x(3t)$

For $t> 1/3$; $x(3t)=0$ and $x(3t+2)=0$ $\therefore$ $y(t) = 0$

Example 1.8 For the continuous-time signal $x(t)$ shown in Fig. P1.6, draw

$$y(t) = x(2(t-2))$$

Solution :

$$y(t) = x(2(t-2))$$

$$\therefore y(t) = x(2t-4)$$

According to the precedence rule, first let us obtain the intermediate signal $v(t)$ which is time shifted version $x(t)$ [i.e., $v(t)=x(t-4)$] as shown in Fig. P1.8.1.

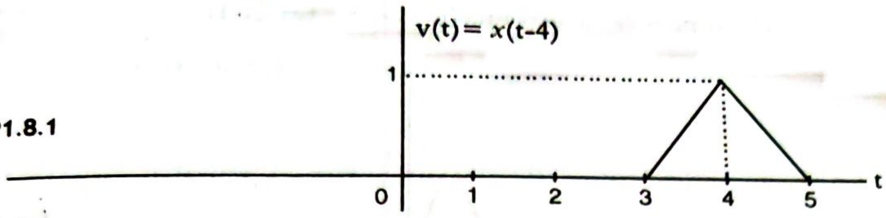


Fig. P1.8.1

Then compress it by a factor of 2 to get $y(t) = v(2t) = x(2t-4)$ as shown in Fig. P1.8.2

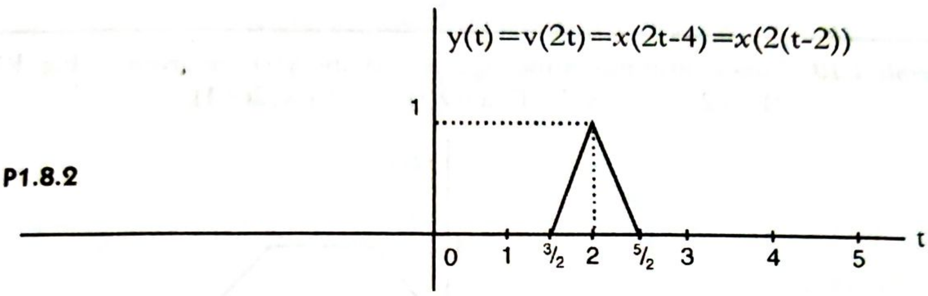


Fig. P1.8.2

Example 1.9 For the continuous time signal $x(t)$ shown in Fig. P1.6, draw

$$y(t) = x(-2t-1)$$

Solution : We have to find $y(t) = x(-2t-1)$.

In this case, we have to perform 3 operations in the following order.

(i) Time shifting (ii) Compression & (iii) Time reversal

Time shifted version $v(t) = x(t-1)$ is shown in Fig. P1.9.1

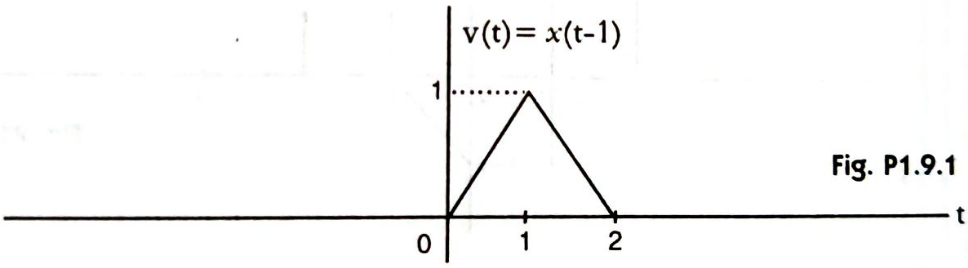


Fig. P1.9.1

The compressed signal $z(t) = v(2t) = x(2t-1)$ is shown in Fig. P 1.9.2.

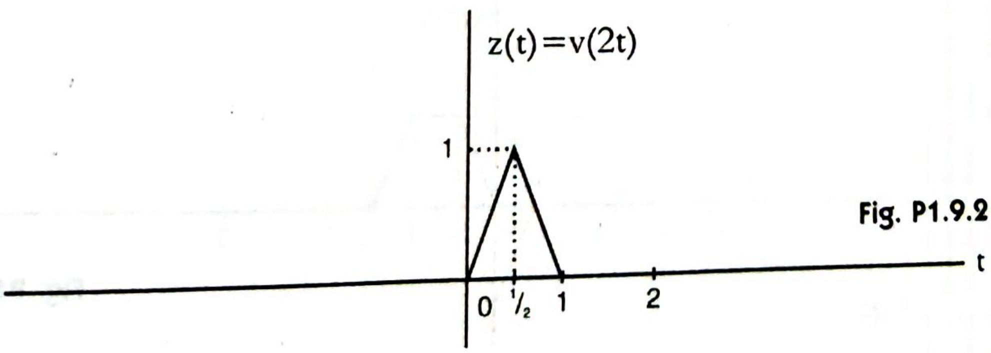


Fig. P1.9.2

The time reversal version $y(t) = z(-t) = x(-2t-1)$

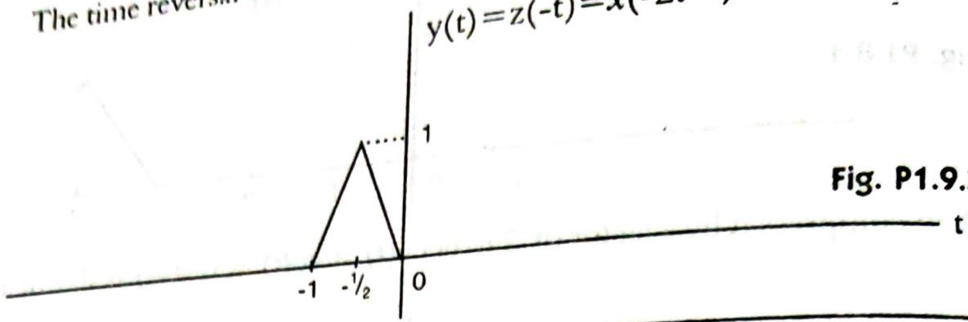


Fig. P1.9.3

Example 1.10 Two continuous time signals $x(t)$ and $y(t)$ are given in Fig. P1.10.1 and P1.10.2 respectively. Draw $z(t) = x(2t) \cdot y(2t+1)$.

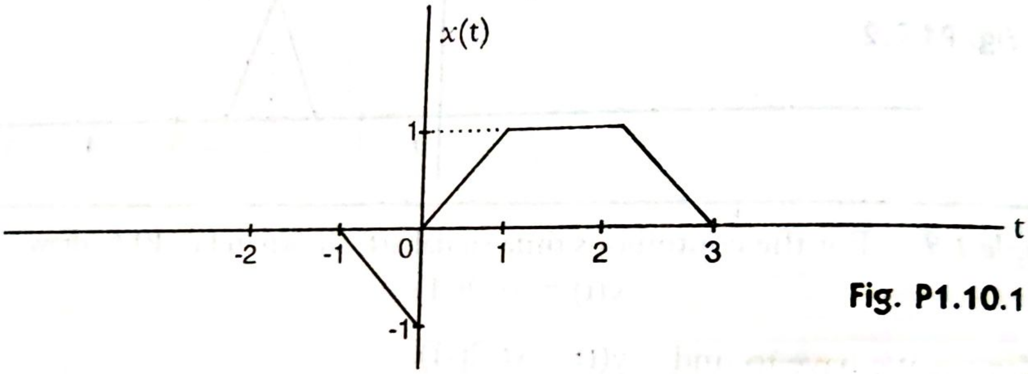


Fig. P1.10.1

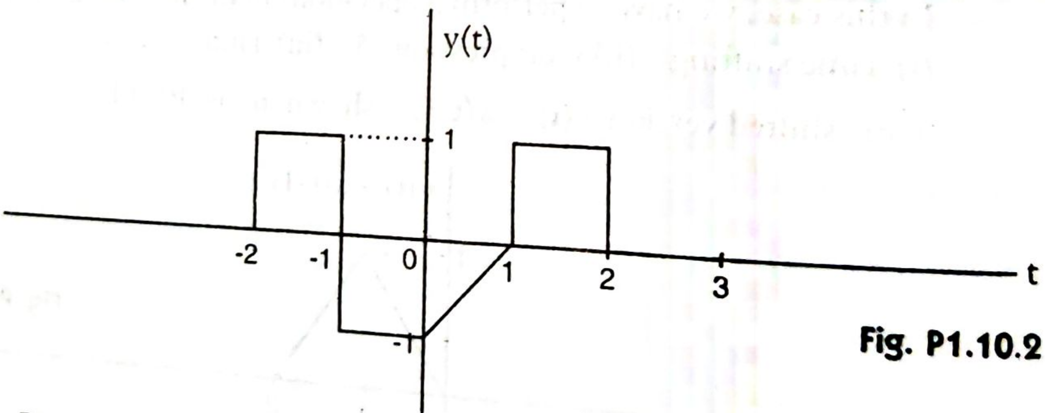


Fig. P1.10.2

Solution : We have to draw $z(t) = x(2t)\, y(2t+1)$ and it is shown in Fig. P1.10.5.

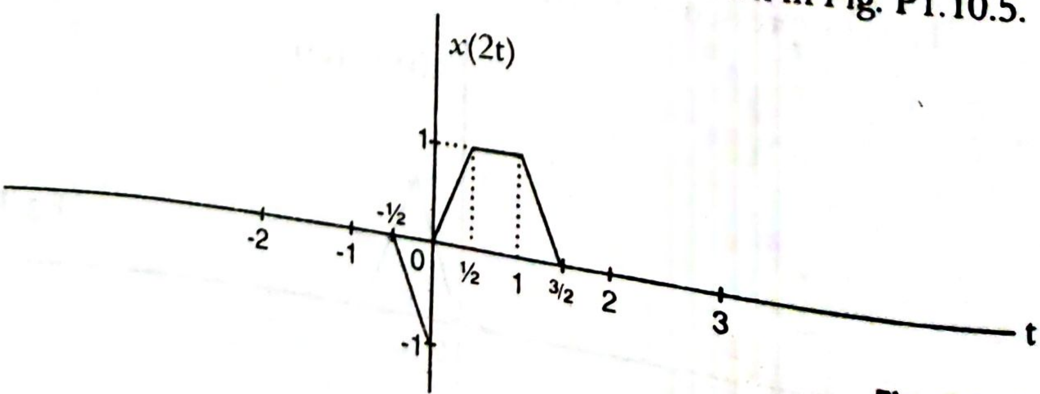


Fig. P1.10.3

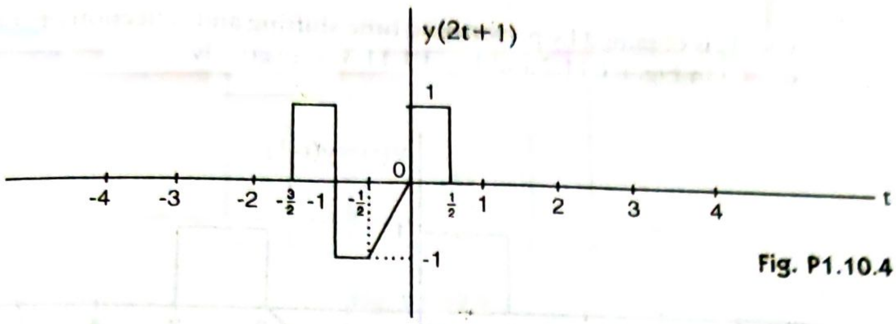


Fig. P1.10.4

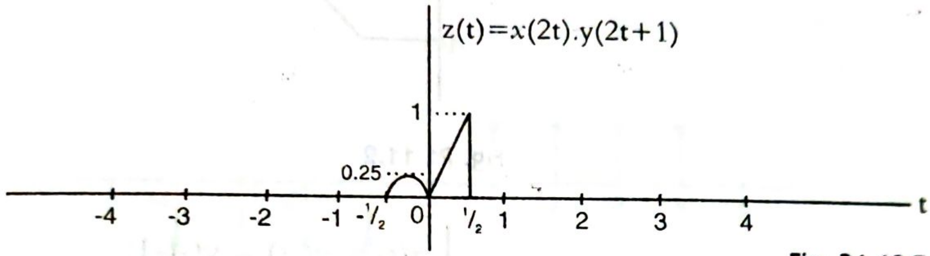


Fig. P1.10.5

For $t < \frac{-3}{2}$; $x(2t)=0$ & $y(2t+1)=0$ $\therefore z(t)=0$

For $\frac{-3}{2} < t < \frac{-1}{2}$; $x(2t)=0$ & $y(2t+1)\neq 0$ $\therefore z(t)=0$

For $\frac{-1}{2} < t < 0$; $x(2t)\neq 0$ & $y(2t+1)\neq 0$ $\therefore z(t)\neq 0$

For $0 < t < \frac{1}{2}$; $x(2t)\neq 0$ & $y(2t+1)=1$ $\therefore z(t)=x(2t)$

For $\frac{1}{2} < t < \frac{3}{2}$; $x(2t)\neq 0$ & $y(2t+1)=0$ $\therefore z(t)=0$

For $t > \frac{3}{2}$; $x(2t)=0$ & $y(2t+1)=0$ $\therefore z(t)=0$

Example 1.11 For the $x(t)$ and $y(t)$ shown in Fig. P1.10.1 & P1.10.2, draw $z(t)=x(t)y(-1-t)$

Solution : We have to draw $z(t) = x(t).\, y(-1-t)$, i.e.,, $z(t) = x(t).\, y(-t-1)$

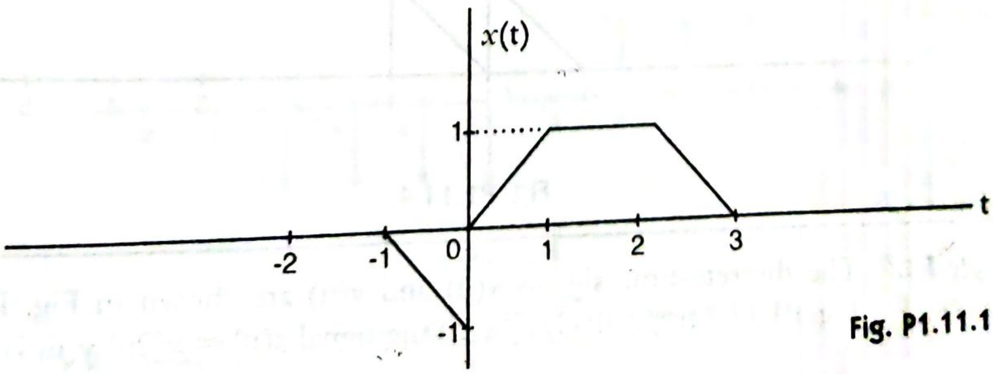


Fig. P1.11.1

y(-t-1) is obtained by performing time shifting and reflection operation y(t) as shown in Fig. P1.11.2 and Fig. P1.11.3 respectively.

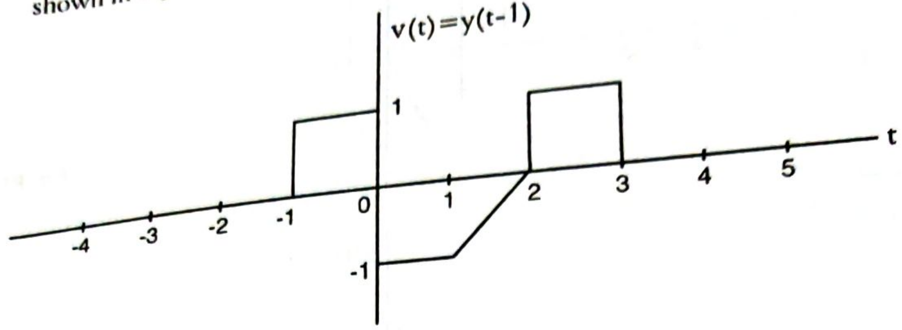


Fig. P1.11.2

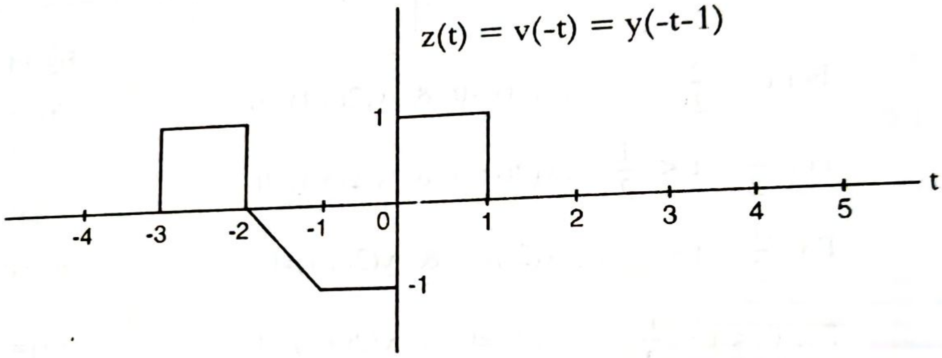


Fig. P1.11.3

The signal z(t) = x(t) y(-t-1)=x(t) y(-1-t) is shown in Fig. P1.11.4

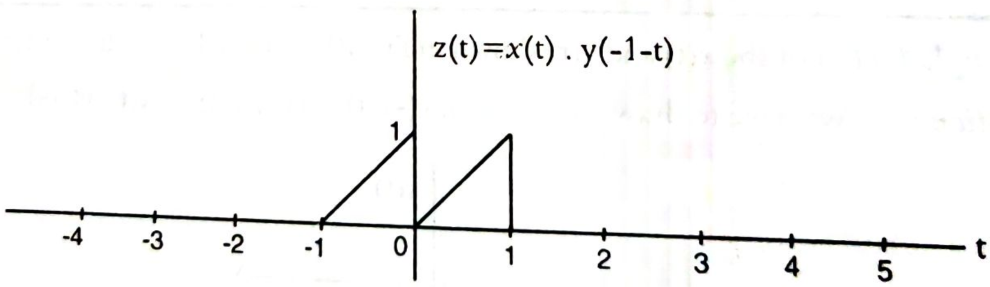


Fig. P1.11.4

Example 1.12 The discrete-time signals $x(n)$ and y(n) are shown in Fig. P1.12.1 and Fig. P1.12.2 respectively. Sketch the signal $z(n) = x(2n)\, y(n-4)$.

$x(n)$

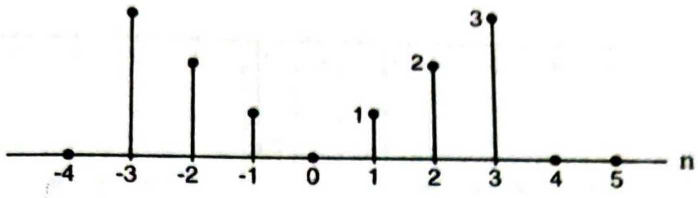


Fig. P1.12.1

$y(n)$

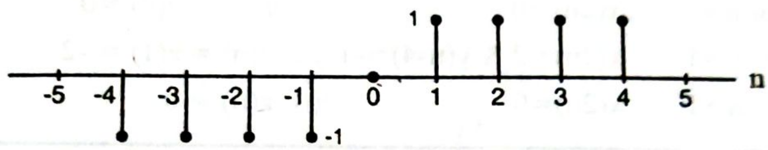


Fig. P1.12.2

Solution : The signal $x(2n)$ and $y(n-4)$ are shown in Fig. P1.12.3 and Fig. P1.12.4 respectively.

$x(2n)$

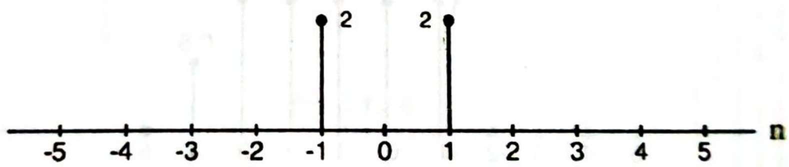


Fig. P1.12.3

$y(n-4)$

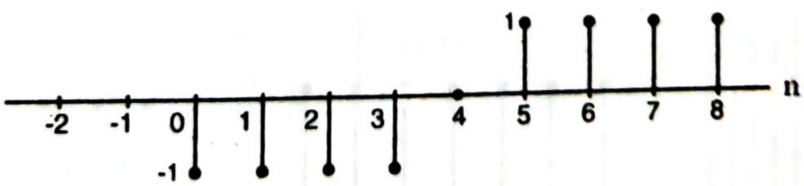


Fig. P1.12.4

The signal $z(n)=x(2n).\ y(n-4)$ is shown in Fig. P1.12.5.

$$z(n) = x(2n) \cdot y(n-4)$$

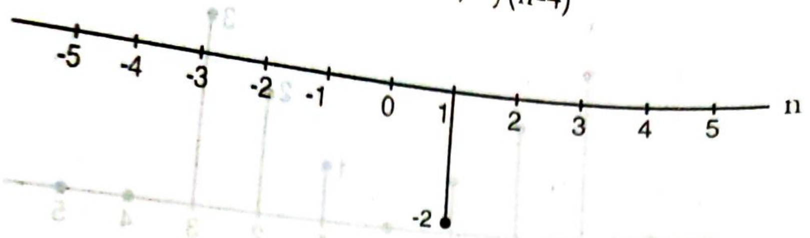


Fig. P1.12.5

From Fig. P1.12.3 and P1.12.4 we have,

For $n<0$; $y(n-4)=0$; $\therefore z(n) = x(2n)y(n-4) = 0$

For $n=0$; $x(2n)=0$; $\therefore z(n) = z(0) = 0$

For $n=1$; $x(2n)=2$ & $y(n-4)=-1$; $\therefore z(n) = z(1) = -2$

For $n>1$; $x(2n)=0$; $\therefore z(n) = 0$

Example 1.13 A discrete-time signal $x(n)$ is shown in Fig. P1.13. Sketch the signal,

$$y(n) = x(n)\, u(2-n)$$

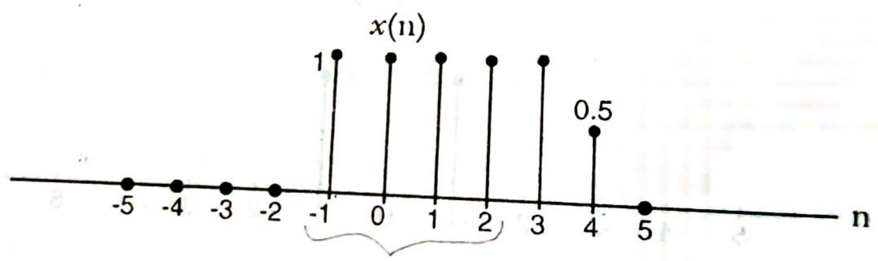


Fig. P1.13

Solution : The signal $u(2-n)$ is shown in Fig. P1.13.1.

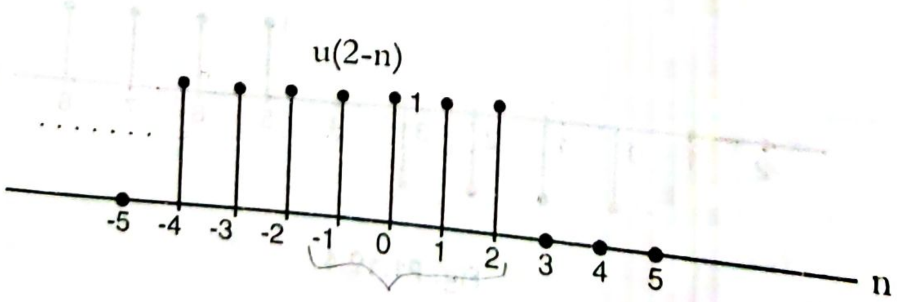


Fig. P1.13.1

Now, the signal $y(n) = x(n).\ u(2-n)$ is shown in Fig. P1.13.2.

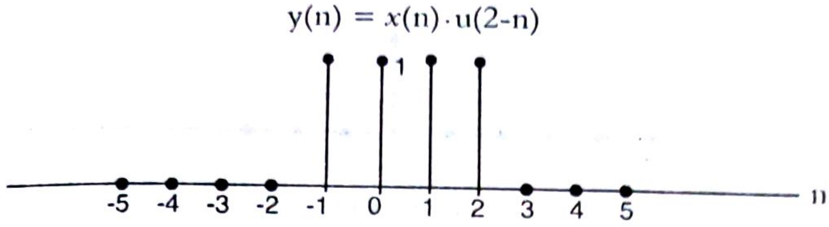


Fig. P1.13.2

Example 1.14 A discrete-time sequence $h(n)$ is shown in Fig. P1.14. Sketch the signal,

$$x(n) = h(3n)\ .\ \delta(n-1)$$

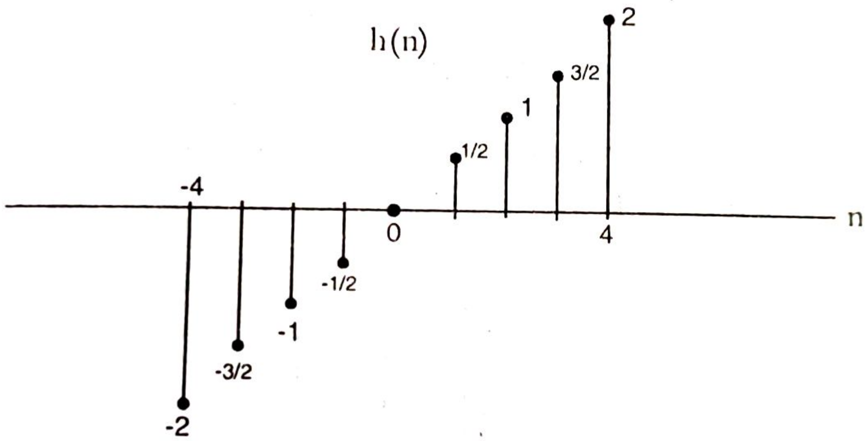


Fig. P1.14

Solution : Given : $x(n) = h(3n)\ \delta(n-1)$

The signal $h(3n)$ is shown in Fig. P1.14.1 below.

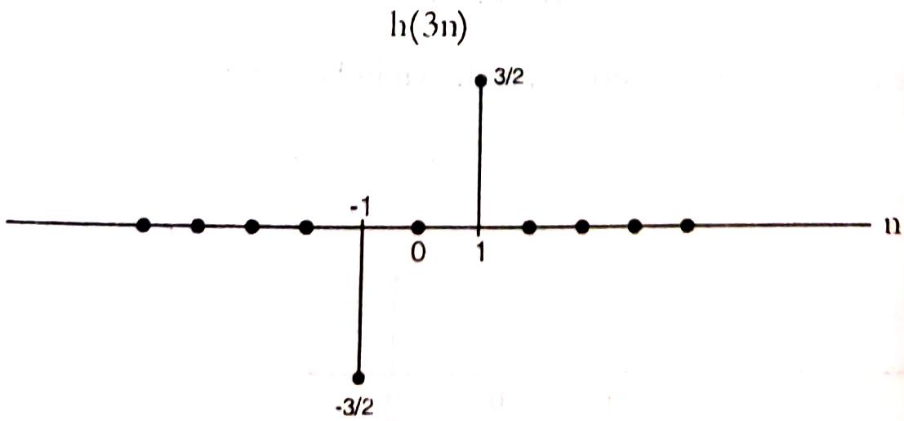


Fig. P1.14.1

Let $x(t) = x_R(t) + j\,x_I(t)$ ..... (1.43)

where $x_R(t)$ is the real part of $x(t)$, $x_I(t)$ is the imaginary part of $x(t)$ and $j=\sqrt{-1}$.

Then complex conjugate of $x(t)$ is,

$$x^*(t) = x_R(t) - j\,x_I(t) \quad \text{..... (1.44)}$$

Decomposition of a signal :

A continuous-time signal $x(t)$ can be decomposed into a sum of two signals, one of which is even $x_e(t)$ the other is odd $x_o(t)$ such that,

$$x(t) = x_e(t) + x_o(t) \quad \text{..... (1.45)}$$

For $x_e(t)$ to be even,

$$x_e(-t) = x_e(t) \quad \text{..... (1.46)}$$

and $x_o(t)$ to be odd,

$$x_o(-t) = -x_o(t) \quad \text{..... (1.47)}$$

Substituting t= -t in eqn. 1.45 we get,

$$x(-t) = x_e(-t) + x_o(-t)$$

$$x(-t) = x_e(t) - x_o(t) \quad \text{..... (1.48)}$$

Solving for $x_e(t)$ and $x_o(t)$ from eqn. 1.45 and 1.48 we get,

$$x_e(t) = \frac{1}{2}[x(t)+x(-t)] \quad \text{..... (1.49)}$$

$$\& \quad x_o(t) = \frac{1}{2}[x(t)-x(-t)] \quad \text{..... (1.50)}$$

Similarly, a discrete-time signal $x(n)$ can be decomposed into a sum of two signals, one of which is even $x_e(n)$ and the other is odd $x_o(n)$ such that,

$$x(n) = x_e(n) + x_o(n) \quad \text{..... (1.51)}$$

where $$x_e(n) = \frac{1}{2}[x(n)+x(-n)] \quad \text{..... (1.52)}$$

and $$x_o(n) = \frac{1}{2}[x(n)-x(-n)] \quad \text{..... (1.53)}$$

Examples

Example 1.16 Find the even and odd components of the signal,

$$x(t) = \cos(t) + \sin(t) + \sin(t)\cos(t)$$

Solution : We have $$x_e(t) = \frac{1}{2}[x(t) + x(-t)] \quad \text{..... P1.16.1}$$

$$\& \quad x_o(t) = \frac{1}{2}[x(t) - x(-t)] \quad \text{..... P1.16.2}$$

Given : $x(t) = \cos(t) + \sin(t) + \sin(t)\cos(t)$ P1.16.3

We can obtain $x(-t)$ from $x(t)$ by replacing t by -t.

$$\therefore \; x(-t) = \cos(-t)+\sin(-t)+\sin(-t)\cos(-t)$$

$$x(-t) = \cos(t) - \sin(t) - \sin(t)\cos(t) \qquad \text{..... P1.16.4}$$

Substituting eqn. P1.16.3 & P1.16.4 in eqn. P1.16.1 we get,

$$x_e(t) = \cos(t)$$

Substituting eqn. P1.16.3 & P1.16.4 in P1.16.2 we get,

$$x_o(t) = \sin(t)\,[1+\cos(t)]$$

Example 1.17 Obtain the even and odd components of the signal $x(t) = (1+t^3)\cos^3(10t)$.

Solution : We have $x_e(t) = \frac{1}{2}[x(t) + x(-t)]$ P1.17.1

& $x_o(t) = \frac{1}{2}[x(t) - x(-t)]$ P1.17.2

Given : $x(t) = (1+t^3)\cos^3(10t)$ P1.17.3

$\therefore \quad x(-t) = [1+(-t)^3]\cos^3[10(-t)]$

$x(-t) = (1-t^3)\cos^3(10t)$ P1.17.4

Substituting eqn. P1.17.3 & P1.17.4 in eqn. P1.17.1 we get,

$$x_e(t) = \cos^3(10t)$$

Substituting eqn. P1.17.3 & P1.17.4 in eqn. P1.17.2 we get,

$$x_o(t) = t^3\cos^3(10t)$$

Example 1.18 Find the even and odd part of the signal $x(t) = 1+t+3t^2+5t^3+9t^4$

Solution : We have $x_e(t) = \frac{1}{2}[x(t) + x(-t)]$ P1.18.1

& $x_o(t) = \frac{1}{2}[x(t) - x(-t)]$ P1.18.2

Given : $x(t) = 1+t+3t^2+5t^3+9t^4$ P1.18.3

$x(-t) = 1+(-t)+3(-t)^2+5(-t)^3+9(-t)^4$

$\therefore x(-t) = 1-t+3t^2-5t^3+9t^4$ P1.18.4

Substituting eqn. P1.18.3 & P1.18.4 in eqn. P1.18.1 we get,

$$x_e(t) = 1+3t^2+9t^4$$

Substituting eqn. P1.18.3 & P1.18.4 in eqn. P1.18.2 we get,

$$x_o(t) = t(1+5t^2)$$

Example 1.19 Determine and sketch the even and odd parts of the signal shown in Fig. P1.19 below.

Solution : We have $x_e(t) = \frac{1}{2}[x(t) + x(-t)]$

$$x_e(t) = \frac{1}{2}x(t) + \frac{1}{2}x(-t)$$

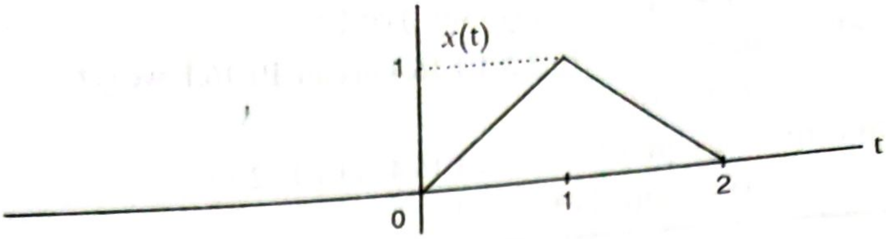


Fig. P1.19

The signal $x_e(t)$ consists 2 terms [i.e., $\frac{1}{2}x(t)$ and $\frac{1}{2}x(-t)$]. The signal $\frac{1}{2}x(t)$ is obtained from $x(t)$ by multiplying its strength (amplitude) by $\frac{1}{2}$ at all 't' as shown below in Fig. P1.19.1.

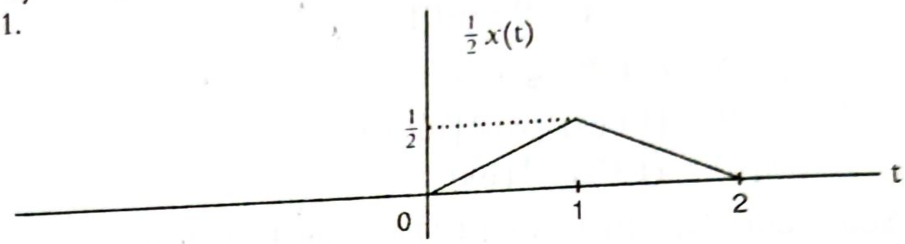


Fig. P1.19.1

Similarly, the signal $\frac{1}{2}x(-t)$ is obtained by taking the mirror image of $x(t)$ to obtain $x(-t)$, then multiplying its strength by $\frac{1}{2}$ at all 't' as shown below in Fig. P1.19.2.

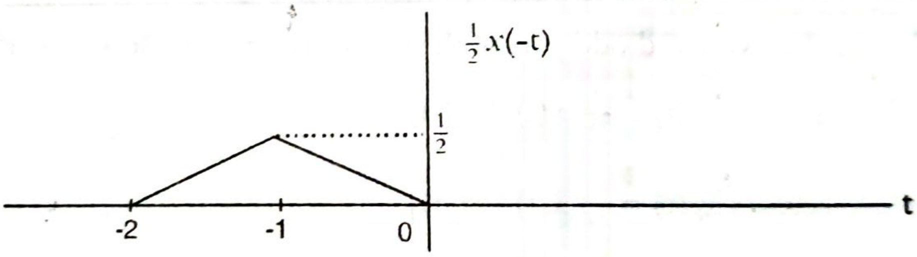


Fig. P1.19.2

Adding $\frac{1}{2}x(t)$ and $\frac{1}{2}x(-t)$, we get $x_e(t)$ as shown in Fig. P1.19.3.

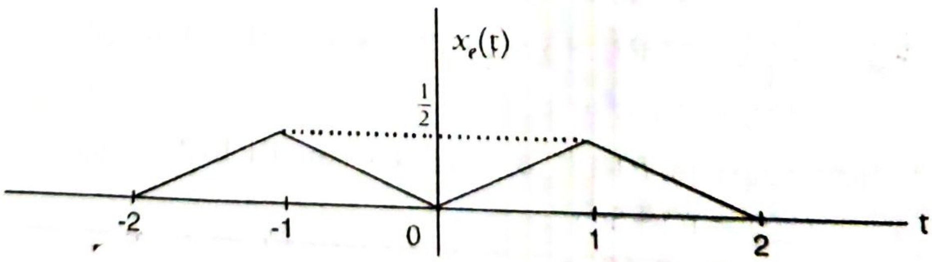


Fig. P1.19.3

Similarly, we have $x_o(t) = \frac{1}{2}[x(t) - x(-t)]$

$$x_o(t) = \frac{1}{2}x(t) - \frac{1}{2}x(-t)$$

Substracting $\frac{1}{2}x(-t)$ from $\frac{1}{2}x(t)$, we get $x_o(t)$ as shown below in Fig. P1.19.4.

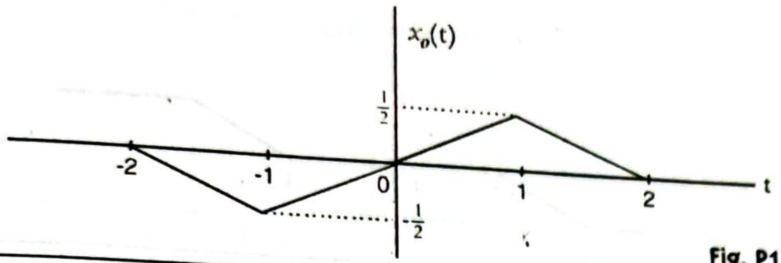


Fig. P1.19.4

Example 1.20 Determine and sketch the even and odd components of the signal $x(t)$ shown in Fig. P1.20.

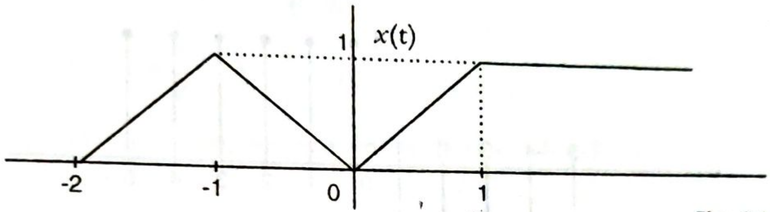


Fig. P1.20

Solution :

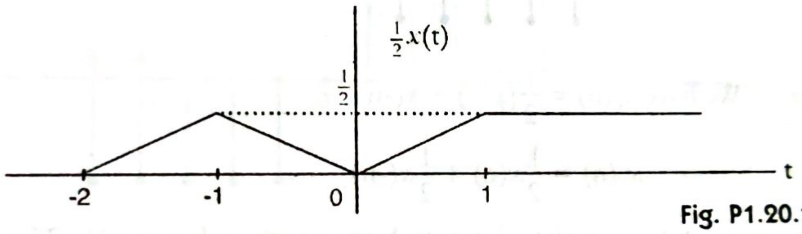


Fig. P1.20.1

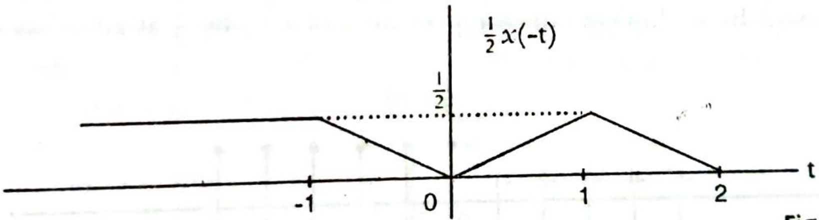


Fig. P1.20.2

Adding $\frac{1}{2}x(t)$ and $\frac{1}{2}x(-t)$, we get $x_e(t)$ as shown below in Fig. P1.20.3.

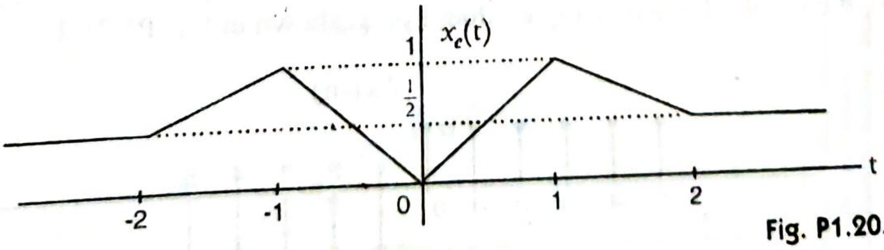


Fig. P1.20.3

Substracting $\frac{1}{2}x(-t)$ from $\frac{1}{2}x(t)$, we get $x_o(t)$ as shown below in Fig. P1.20.4

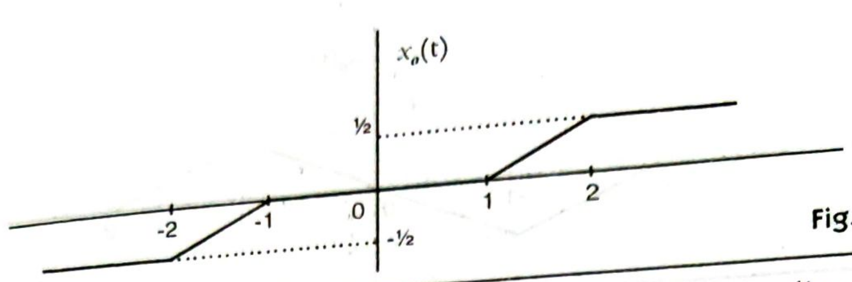


Fig. P1.20.4

Example 1.21 Determine and sketch the even and odd components of the discrete-time signal $x(n)$ shown in Fig. P1.21.

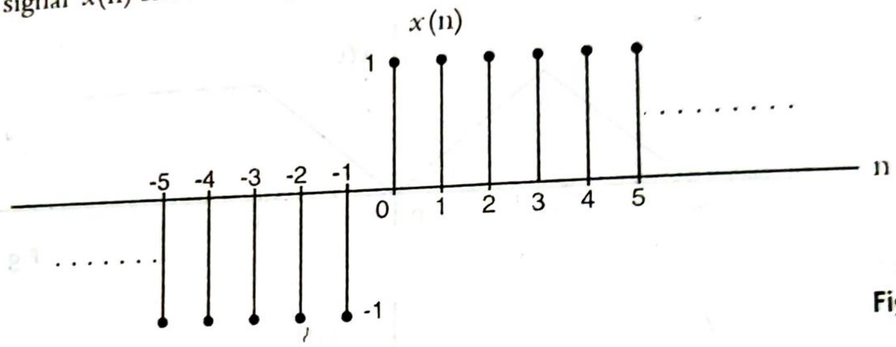


Fig. P1.21

Solution : We have $x_e(n) = \frac{1}{2}[x(n) + x(-n)]$

$$x_e(n) = \frac{1}{2}x(n) + \frac{1}{2}x(-n)$$

The signal $x_e(n)$ consists 2 terms [i.e., $\frac{1}{2}x(n)$ and $\frac{1}{2}x(-n)$]. The signal $\frac{1}{2}x(n)$ is obtained from $x(n)$ by multiplying the sample values of $x(n)$ by $\frac{1}{2}$ at all 'n' as shown in Fig. P1.21.1.

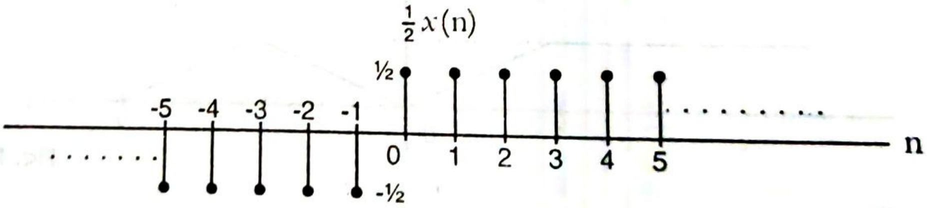


Fig. P1.21.1

Similarly, the signal $\frac{1}{2}x(-n)$ is obtained from $x(n)$ by taking the mirror image of $x(n)$ to get $x(-n)$, then multiplying its sample values by $\frac{1}{2}$ as shown in Fig. P1.21.2.

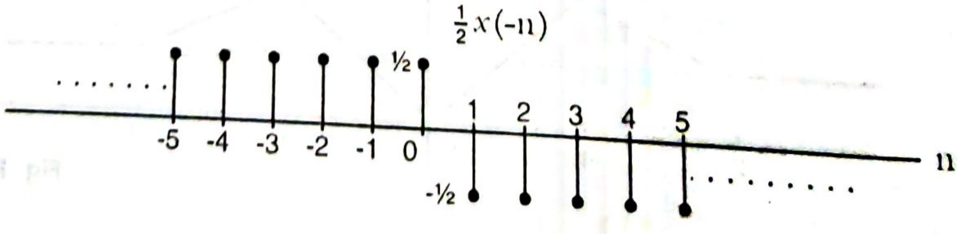


Fig. P1.21.2

Adding $\frac{1}{2}x(n)$ and $\frac{1}{2}x(-n)$, we get $x_e(n)$ as shown in Fig. P1.21.3.

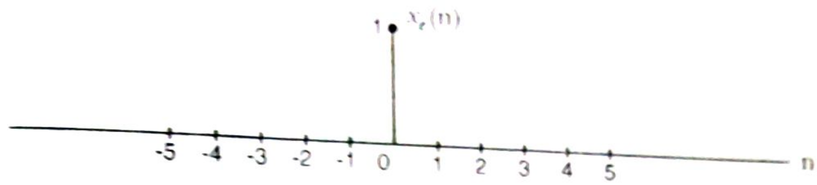


Fig. P1.21.3

Similarly, we have $x_o(n) = \frac{1}{2}[x(n) - x(-n)]$

$$x_o(n) = \frac{1}{2}x(n) - \frac{1}{2}x(-n)$$

Subtracting $\frac{1}{2}x(-n)$ from $\frac{1}{2}x(n)$, we get $x_o(n)$ as shown in Fig. P1.21.4.

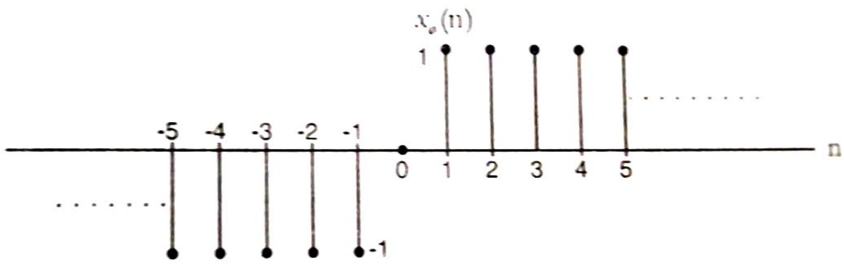


Fig. P1.21.4

Example 1.22 Determine and draw the even and odd parts of the discrete-time signal **x(n)** shown in Fig. P1.22.

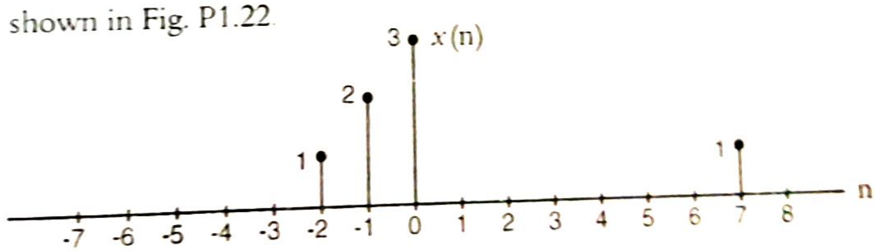


Fig. P1.22

Solution :

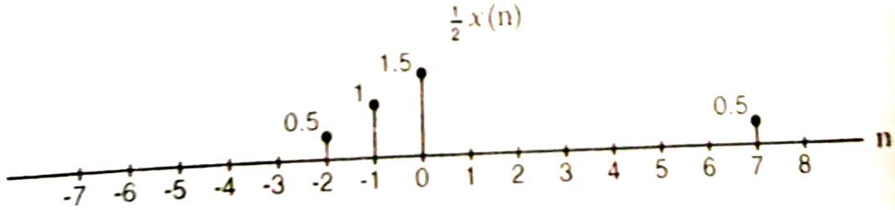


Fig. P1.22.1

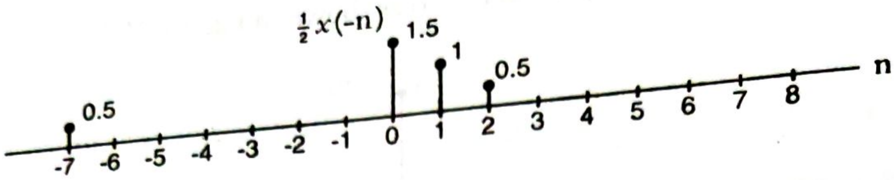


Fig. P1.22.2

Adding $\frac{1}{2}x(n)$ and $\frac{1}{2}x(-n)$, we get $x_e(n)$ as shown in Fig. P1.22.3

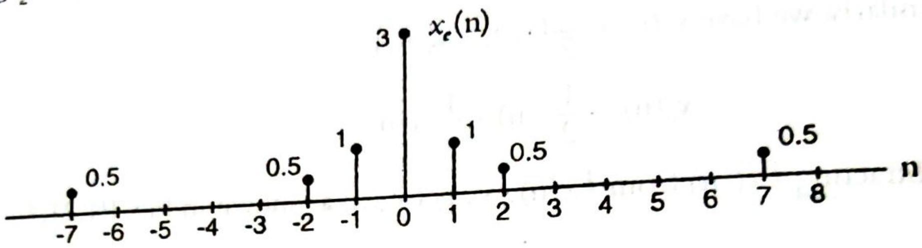


Fig. P1.22.3

Subtracting $\frac{1}{2}x(-n)$ from $\frac{1}{2}x(n)$, we get $x_o(n)$ as shown in Fig. P1.22.4

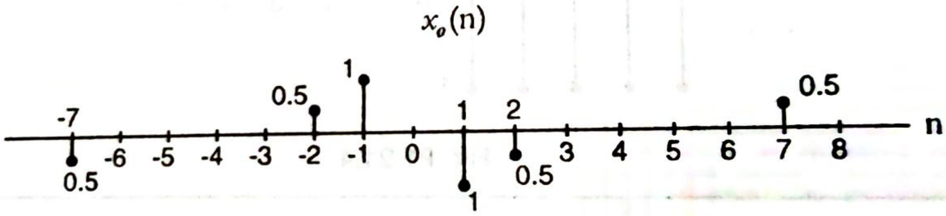


Fig. P1.22.4

Example 1.23 Find the odd and even part of the signal $x(t)$ shown in Fig. P1.23 below.

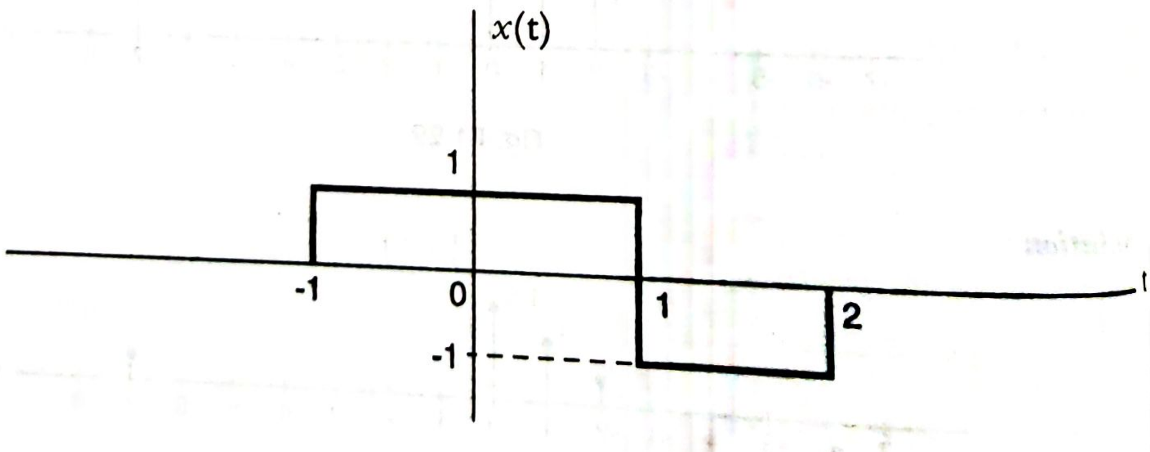


Fig. P1.23

Solution :

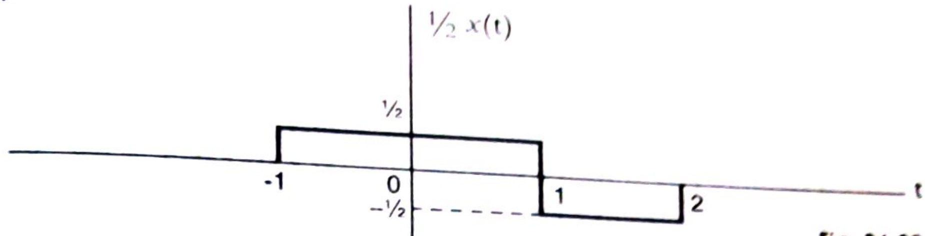


Fig. P1.23.1

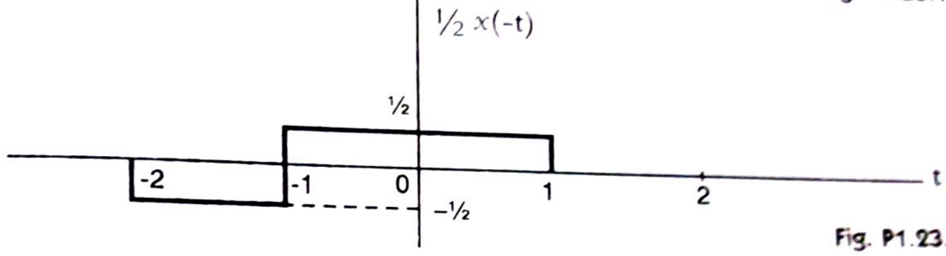


Fig. P1.23.2

Adding signals shown in Fig. P1.23.1 & Fig. P1.23.2 , we get $x_e(t)$ and is shown in Fig. P1.23.3.

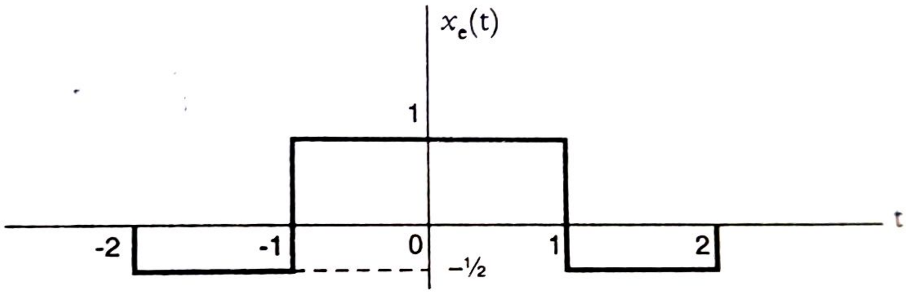


Fig. P1.23.3

Subtracting signal shown in Fig. P1.23.2 from Fig. P1.23.1, we get $x_o(t)$ and is drawn in Fig. P1.23.4.

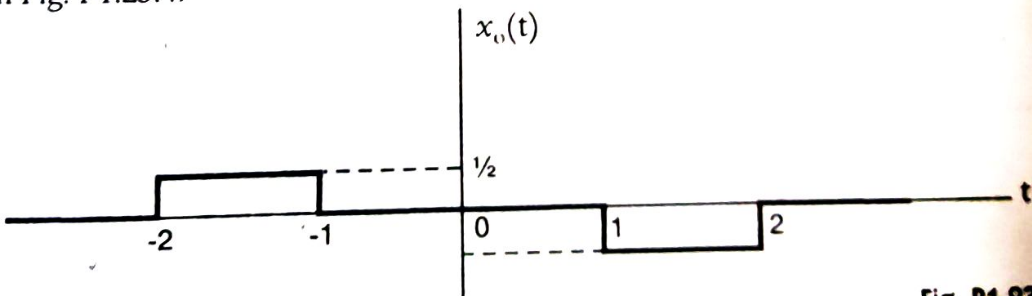
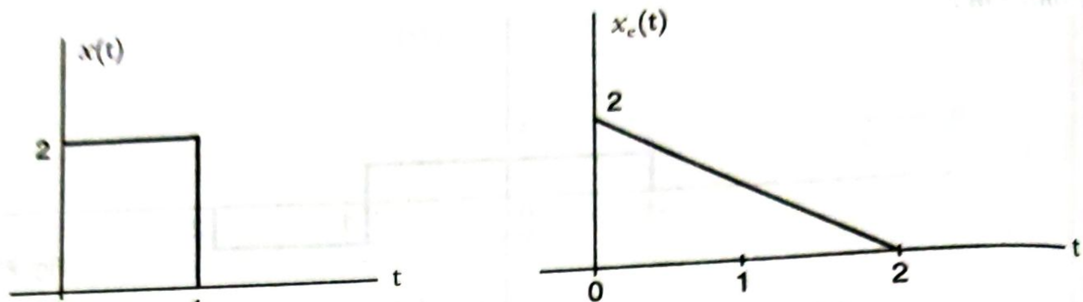


Fig. P1.23.4

Example 1.24 Fig. 1.24(a) and (b) shows part of the signal $x(t)$ and its even part $x_e(t)$ respectively for $t \geq 0$ only. $x(t)$ and even part $x_e(t)$ for $t<0$ is not shown. Complete the plots of $x(t)$ and $x_e(t)$. Also draw the odd part of $x(t)$ [i.e., $x_o(t)$].



(b)

(a) Fig. P1.24

Solution : We know that even part of any signal is symmetric about t=0. Thus, we have the complete even part $x_e(t)$ as shown in Fig. P1.24.1 below.

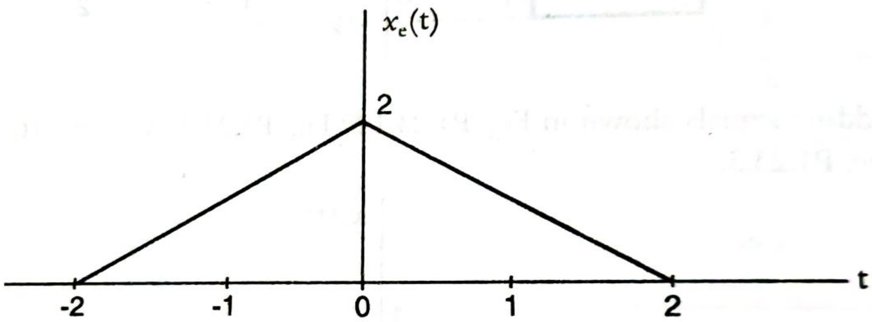


Fig. P1.24.1

We know that $\quad x(t) = x_e(t) + x_o(t)$

Now, $x_o(t)$ for $t \geq 0$ is obtained in such away that if we add $x_e(t)$ and $x_o(t)$ for $t \geq 0$, we should get the given $x(t)$ for $t \geq 0$ shown in Fig. P1.24(a).

$\therefore$ $x_o(t)$ for $t \geq 0$ is shown in Fig. P1.24.2.

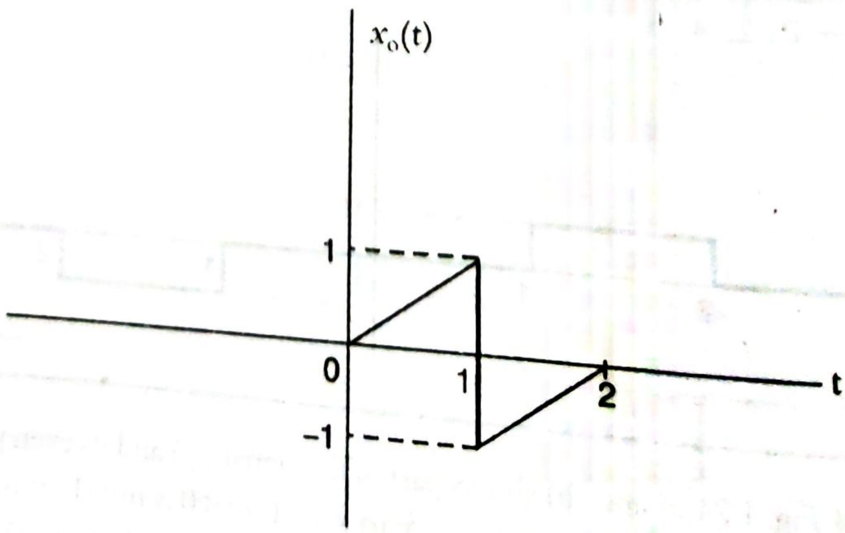


Fig. P1.24.2

If we add the signal shown in Fig. P1.24(b) and P1.24.2, we get $x(t)$ for $t \geq 0$ as shown in Fig. P1.24(a).

Now, we know that odd part of any real signal is antisymmetric about $t=0$. Therefore, the complete $x_o(t)$ is shown in Fig. P1.24.3 below.

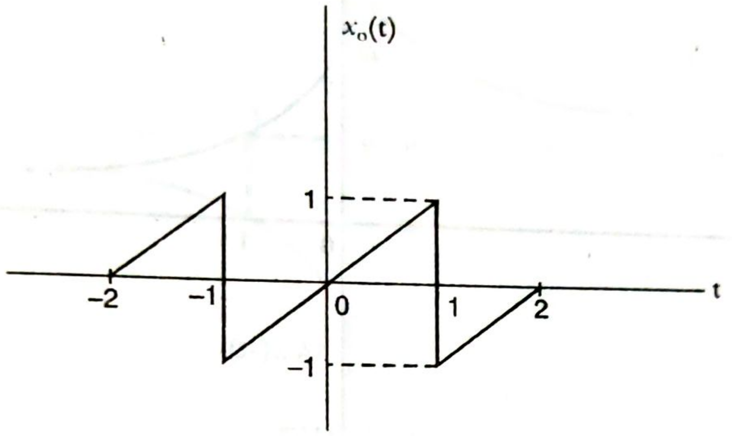


Fig. P1.24.3

Now the complete $x(t)$ is obtained by adding complete $x_e(t)$ and $x_o(t)$ as shown in Fig. P1.24.1 and P1.24.3. It is shown in Fig. P1.24.4 below.

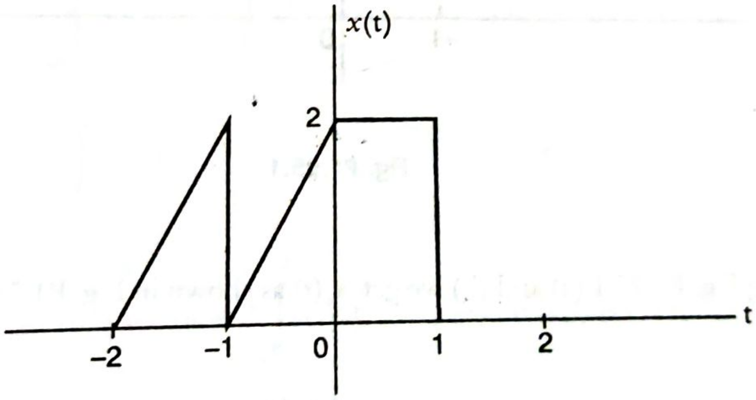


Fig. P1.24.4

Example 1.25 Find and sketch the even and odd components of the signal,

$$x(t) = e^{-\left(\frac{t}{4}\right)} u(t)$$

Solution : Given :

$$x(t) = e^{-\left(\frac{t}{4}\right)} u(t)$$

$$= (0.779)^t u(t)$$

$$\frac{1}{2} x(t) = \frac{1}{2} (0.779)^t u(t)$$

$$\frac{1}{2} x(-t) = \frac{1}{2} (0.779)^{-t} u(-t)$$

The sketch of $\frac{1}{2}x(t)$ and $\frac{1}{2}x(-t)$ are shown in Fig. P1.25.1(a) & (b) below.

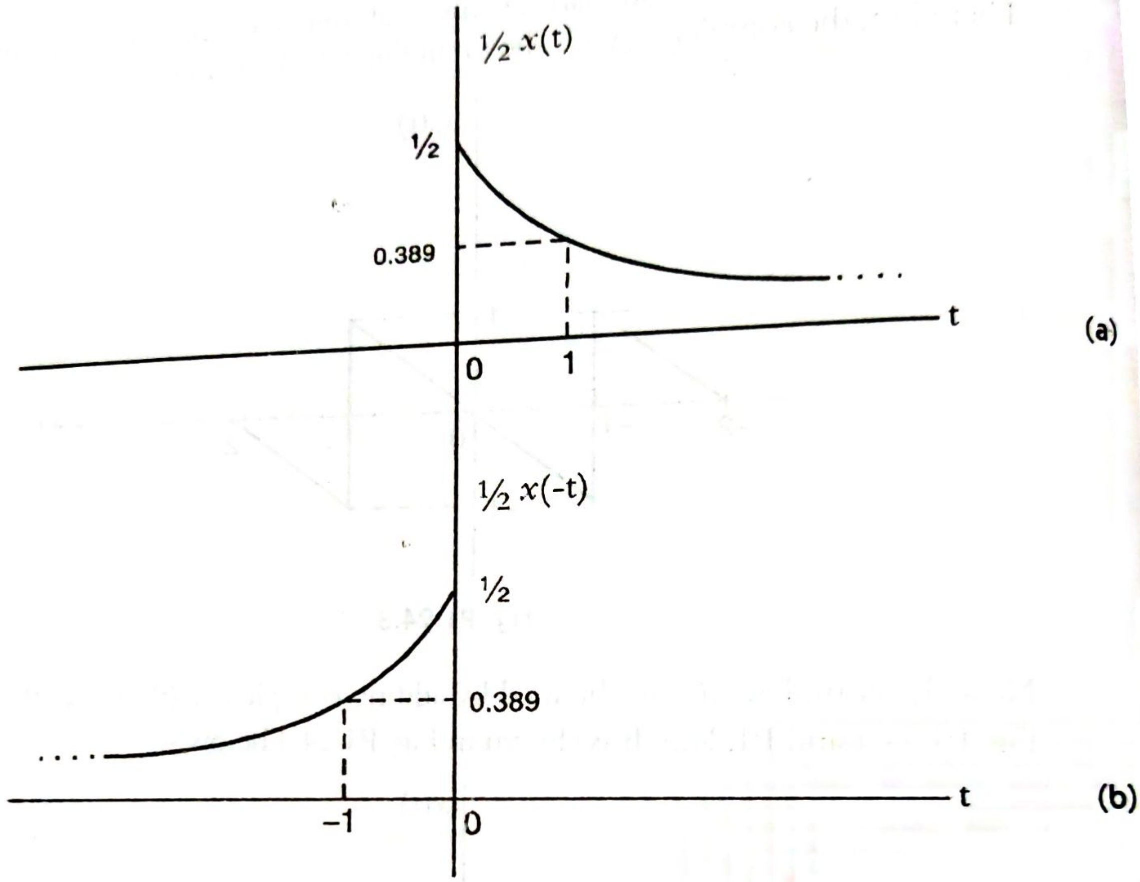


Fig. P1.25.1

Adding Fig. P1.25.1 (a) and (b) we get $x_e(t)$ as shown in Fig. P1.25.2 below.

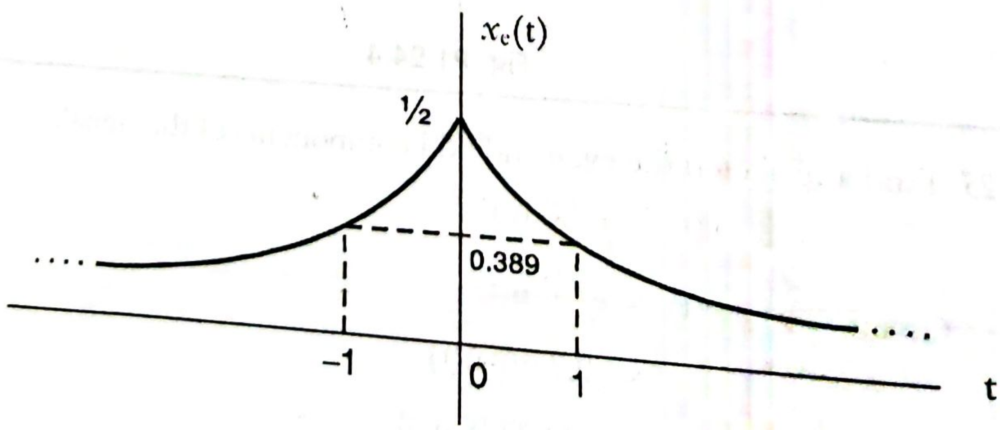


Fig. P1.25.2

Subtracting Fig. P1.25.1 (b) from (a), we get $x_o(t)$ as shown in Fig. P1.25.3 below.

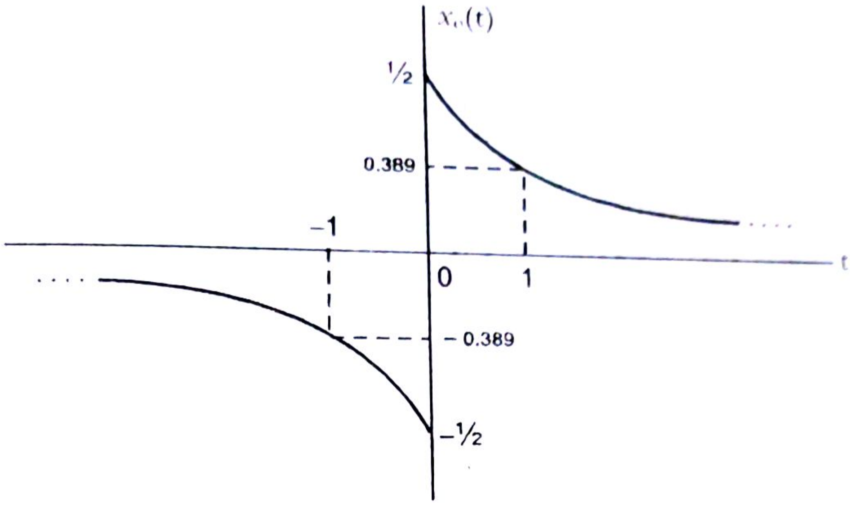


Fig. P1.25.3

Example 1.26 Prove that,

(i) $$\int_{-a}^{a} x(t)\,dt = 2\int_{0}^{a} x(t)dt \quad ;\ \text{if } x(t) \text{ is even}$$

(ii) $$\int_{-a}^{a} x(t)\,dt = 0 \quad ;\ \text{if } x(t) \text{ is odd}$$

Solution : (i) To prove $$\int_{-a}^{a} x(t)\,dt = 2\int_{0}^{a} x(t)dt \quad ;\ \text{if } x(t) \text{ is even}$$

$$\text{LHS} = \int_{-a}^{a} x(t)\,dt$$

$$= \int_{-a}^{0} x(t)dt + \int_{0}^{a} x(t)dt$$

$$= -\int_{0}^{-a} x(t)\,dt + \int_{0}^{a} x(t)dt$$

Put $t=-t$ in first term, $[dt = -dt]$

$$= \int_0^a x(-t)dt + \int_0^a x(t)dt$$

$$= \int_0^a x(t)dt + \int_0^a x(t)dt \qquad \left[\begin{array}{l} \because x(t) \text{ is even} \\ \therefore x(-t) = x(t) \end{array}\right]$$

$$= 2\int_0^a x(t)dt$$

$$= \text{RHS}$$

(ii) To prove $\int_{-a}^{a} x(t)dt = 0$; if $x(t)$ is odd,

$$\text{LHS} = \int_{-a}^{a} x(t)dt$$

$$= \int_{-a}^{0} x(t)dt + \int_0^a x(t)dt$$

$$= -\int_0^{-a} x(t)dt + \int_0^a x(t)dt$$

Put $t=-t$ in the first term, $[dt=-dt]$

$$= \int_0^a x(-t)dt + \int_0^a x(t)dt$$

$$= -\int_0^a x(t)dt + \int_0^a x(t)dt \qquad \left[\begin{array}{l} \because x(t) \text{ is odd} \\ \therefore x(-t) = -x(t) \end{array}\right]$$

$$= 0$$

$$= \text{RHS}$$

Example 1.27 Show that if $x(n)$ is an odd signal then,

$$\sum_{n=-\infty}^{\infty} x(n) = 0$$

Solution : A discrete-time sequence $x(n)$ is said to be odd signal if $x(-n) = -x(n)$.

$$\text{LHS} = \sum_{n=-\infty}^{\infty} x(n)$$

$$= \sum_{n=-\infty}^{-1} x(n) + \cancel{x(0)}^{0} + \sum_{n=1}^{\infty} x(n)$$

$$= \sum_{n=1}^{\infty} x(-n) + \cancel{x(0)}^{0} + \sum_{n=1}^{\infty} x(n)$$

$$= x(0) + \sum_{n=1}^{\infty} \{x(n) + x(-n)\} \qquad \text{..... P1.27.1}$$

For odd signal $x(0) = 0$ & $x(-n) = -x(n)$.

Substituting these in eqn. P1.27.1 we get,

$$\therefore \sum_{n=-\infty}^{\infty} x(n) = 0 + \sum_{n=1}^{\infty} \{x(n) - x(n)\}$$

$$= 0$$

Hence the proof.

Example 1.28 Show that if $x_1(n)$ is an odd signal and $x_2(n)$ is an even signal, then $x_1(n)\, x_2(n)$ is an odd signal.

Solution :

Consider $y(n) = x_1(n) \,.\, x_2(n)$ P1.28.1

$\therefore$ $y(-n) = x_1(-n) \,.\, x_2(-n)$ P1.28.2

Given : $x_1(n)$ is odd. $\therefore x_1(-n) = -x_1(n)$ P1.28.3

& $x_2(n)$ is even. $\therefore x_2(-n) = x_2(n)$ P1.28.4

Substituting eqn. P1.28.3 & P1.28.4 in eqn. P1.28.2, we get

$$y(-n) = -x_1(n) \,.\, x_2(n) = -y(n)$$

i.e., $y(-n) = -y(n)$

i.e., $y(n)$ satisfies the condition of odd signal.

$\therefore$ $y(n) = x_1(n)\, x_2(n)$ is an odd signal.

Example 1.29 Let $x(n)$ be an arbitrary signal with even and odd parts denoted by $x_e(n)$ and $x_o(n)$ respectively. Show that.

$$\sum_{n=-\infty}^{\infty} x^2(n) = \sum_{n=-\infty}^{\infty} x_e^2(n) + \sum_{n=-\infty}^{\infty} x_o^2(n)$$

Solution : $$\text{LHS} = \sum_{n=-\infty}^{\infty} x^2(n)$$

$$= \sum_{n=-\infty}^{\infty} \{x_e(n) + x_o(n)\}^2$$

$$= \sum_{n=-\infty}^{\infty} \{x_e^2(n) + x_o^2(n) + 2x_e(n) \cdot x_o(n)\}$$

$$= \sum_{n=-\infty}^{\infty} x_e^2(n) + \sum_{n=-\infty}^{\infty} x_o^2(n) + 2\sum_{n=-\infty}^{\infty} x_e(n) \cdot x_o(n) \qquad \text{..... P1.29.1}$$

In eqn. P1.29.1, the last term contains the product of even and odd signal respectively. In example 1.28 we proved that it results in an odd signal. By applying the equation proved in example 1.27, this term is equal to zero.

$$\therefore \quad \sum_{n=-\infty}^{\infty} x^2(n) = \sum_{n=-\infty}^{\infty} x_e^2(n) + \sum_{n=-\infty}^{\infty} x_o^2(n)$$

Hence the proof.

1.6.3 Periodic and Non-periodic signals

A continuous-time signal $x(t)$ is said to be *periodic* if it satisfies the condition,

$$x(t) = x(t+T) \quad ; \text{for all 't'} \qquad \text{..... (1.54)}$$

where 'T' is a positive constant.

If the condition in eqn. 1.54 is satisfied for $T=T_o$, then it is also satisfied for any $T=nT_o$ where $n=1, 2, 3.......$

The smallest value of 'T' that satisfies eqn. 1.54 is called the *fundamental period* of $x(t)$. This fundamental period is the time taken by the signal $x(t)$ to complete its one cycle. The reciprocal of the fundamental period 'T' is known as the *fundamental frequency* of the signal.

Fundamental frequency $f = \dfrac{1}{T}$ (Hertz)

The fundamental angular frequency 'ω' is given by

$$\omega = 2\pi f = \frac{2\pi}{T} \qquad \text{(rad/sec)}$$

Any continuous-time signal $x(t)$ which does not satisfy eqn. 1.54 is called *non-periodic* or *aperiodic signal*.

Examples for periodic and non-periodic continuous-time signal is shown in Fig. 1.30 and 1.31 respectively.

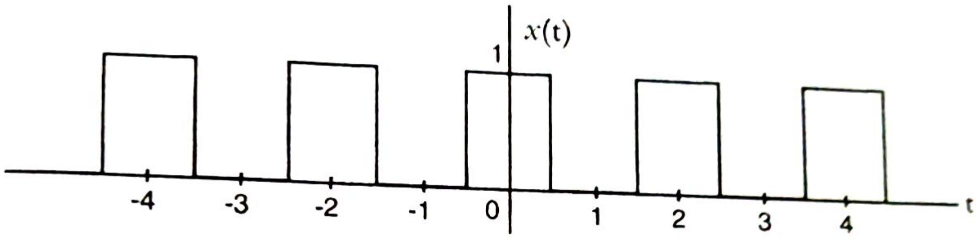


Fig. 1.30 A continuous-time periodic signal with the fundamental period T = 2

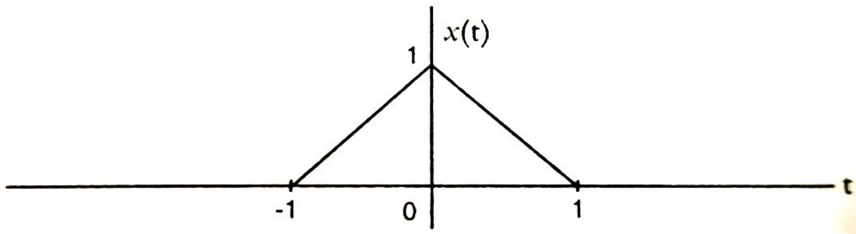


Fig. 1.31 A continuous-time non-periodic signal

Similarly, a discrete-time signal $x(n)$ is said to be *periodic* if it satisfies the condition,

$$x(n) = x(n+N) \quad ; \text{for all 'n'} \qquad \text{.....} \quad (1.55)$$

where N is a positive integer. The smallest value of N which satisfies eqn. 1.55 is called the *fundamental period* of the signal $x(n)$.

The fundamental angular frequency of $x(n)$ is given by,

$$\Omega = \frac{2\pi}{N} \qquad \text{(radians)}$$

Any discrete-time signal $x(n)$ which doesnot satisfy eqn. 1.55 is called *non-periodic* or *aperiodic signal*.

Examples for periodic and non-periodic discrete-time signal is shown in Fig. 1.32 and Fig. 1.33 respectively.

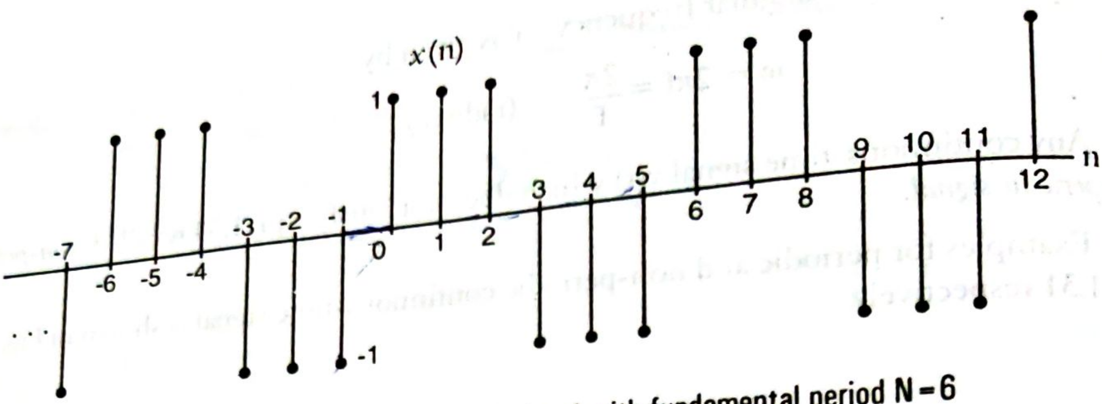


Fig. 1.32 A discrete-time periodic signal with fundamental period N=6

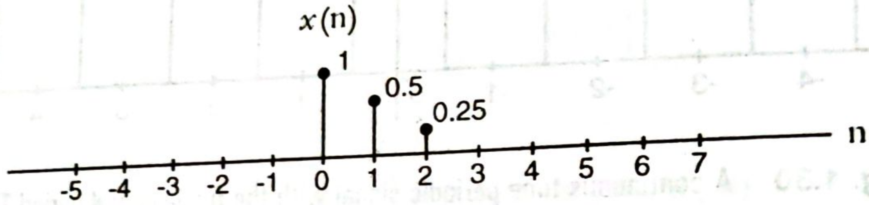


Fig. 1.33 A discrete-time non-periodic signal

Examples

Example 1.30 Determine whether the continuous time signal $x(t) = [\cos(2\pi t)]^2$ is periodic or not. If periodic, find the fundamental period T.

Solution : Given :

$$x(t) = [\cos(2\pi t)]^2$$

$$= \frac{1}{2}[1+\cos 4\pi t]$$

$$x(t) = \frac{1}{2}+\frac{1}{2}\cos 4\pi t$$

In the above expression for $x(t)$, the first term is constant (i.e., average value). The second term is a cosine signal with maximum amplitude of $\frac{1}{2}$. Comparing with $\cos \omega_0 t$, we have $\omega_0 = 4\pi$.

The signal $\cos\omega_0 t$ is periodic with period $T = \frac{2\pi}{\omega_0}$

$$\therefore \omega_0 = 2\pi f = 4\pi = 2\pi(2) = \frac{2\pi}{0.5} \quad \text{..... P1.30.1}$$

We have $\omega_0 = \frac{2\pi}{T}$ P1.30.2

Comparing P1.30.1 and P1.30.2 we have,

Fundamental period T=0.5 sec.

Verification $\Rightarrow$ $x(t+T) = x(t)$

$$x(t+0.5) = [\cos(2\pi(t+0.5))]^2$$

$$= \frac{1}{2} + \frac{1}{2}\cos 4\pi(t+0.5)$$

Using $\cos(A+B) = \cos A \cos B - \sin A . \sin B.$

$$\therefore x(t+0.5) = \frac{1}{2}(1+\cos 4\pi t . \overset{1}{\cancel{\cos 2\pi}} - \sin 4\pi t \sin \overset{0}{\cancel{2\pi}})$$

$$= \frac{1}{2}(1+\cos 4\pi t)$$

$$= [\cos(2\pi t)]^2 = x(t)$$

$\therefore x(t+0.5) = x(t)$. Therefore $x(t)$ is periodic with fundamental period T=0.5 sec.

Example 1.31 Determine whether the signal $x(t) = \sum_{k=-2}^{2} y(t-2k)$ for $y(t)$ shown in Fig. P1.31 is periodic or not. If periodic, find its fundamental period.

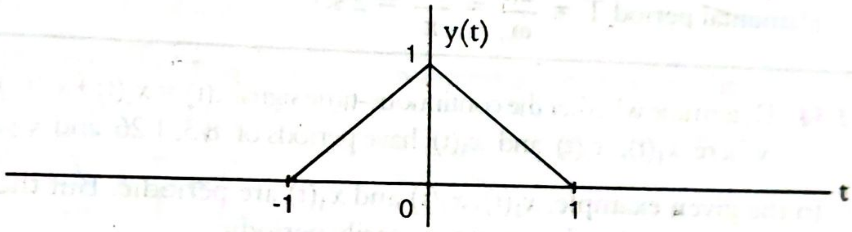


Fig. P1.31

Solution :

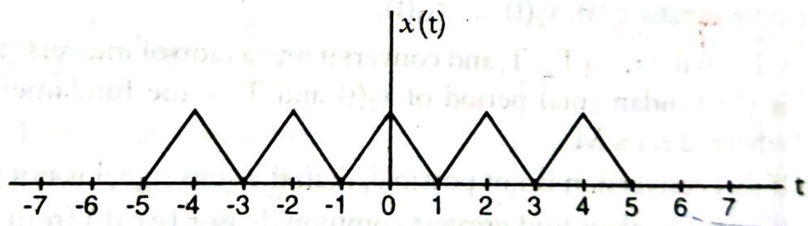


Fig. P1.31.1

The signal $x(t) = \sum_{k=-2}^{2} y(t-2k) = y(t+2) + y(t+1) + y(t) + y(t-1) + y(t-2)$

The signal $x(t)$ is plotted in Fig. P1.31.1.

Here the cycle repeats only between t = -5 and t = 5. For the signal to be periodic, the cycle must repeat between t = -∞ and t = ∞. ∴ the given $x(t)$ is non-periodic.

Note : The signal $x(t) = \sum_{k=-\infty}^{\infty} y(t-2k)$ is periodic with fundamental period T=2 sec.

Example 1.32 Check for the periodicity of $x(t) = 2\cos\left(3t+\frac{\pi}{4}\right)$

Solution : Comparing with $x(t) = A\cos(\omega_o t+\phi)$, we have fundamental angular frequency $\omega_o = 3 = 2\pi f_o$ (rad/sec).

$$\therefore f_o = \frac{3}{2\pi}$$

$$\therefore \text{Fundamental period } T = \frac{1}{f_o} = \frac{2\pi}{3} \text{ sec.}$$

Example 1.33 Check for periodicity of the signal $x(t) = e^{j\pi t}$

Solution : Note : A complex exponential signal $x(t) = e^{j\omega_o t}$ is periodic with fundamental period $T = 2\pi/\omega_o$. It is explained in section 1.3.

Given : $x(t) = e^{j\pi t}$

Comparing this signal with a complex exponential signal $x(t) = e^{j\omega_o t}$, we have,

$$\omega_o = \pi \quad \text{rad/sec.}$$

$$\therefore \text{Fundamental period } T = \frac{2\pi}{\omega_o} = \frac{2\pi}{\pi} = 2 \text{ sec.}$$

Example 1.34 Determine whether the continuous-time signal $x(t) = x_1(t)+x_2(t)+x_3(t)$ is periodic, where $x_1(t)$, $x_2(t)$ and $x_3(t)$ have periods of 8/3, 1.26 and $\sqrt{2}$ sec. respectively.

Solution : In the given example, $x_1(t)$, $x_2(t)$ and $x_3(t)$ are periodic. But the summation of these periodic signals is not necessarily periodic.

Note : The following steps can be used to determine the period of the summation of N periodic signals $x_1(t), x_2(t) \ldots\ldots x_M(t)$.

1. Obtain the ratio T_1/T_i and convert it into a ratio of integers (rational), where T_1 is the fundamental period of $x_1(t)$ and T_i is the fundamental period of $x_i(t)$ where $2 \le i \le M$.
2. If this conversion is not possible, then the sum signal is not periodic.
3. If possible, then find greatest common divisor (g.c.d.) from the numerator and denominator of each individual ratio.
4. Then find least common multiple (l.c.m) of the denominators of the resulting ratios, say it is 'l'.
5. Then the period of the sum signal is given by $T = T_1 . l$

Given : $x(t) = x_1(t) + x_2(t) + x_3(t)$

Period of $x_1(t)$: $T_1 = 8/3$ sec.

Period of $x_2(t)$: $T_2 = 1.26$ sec.

Period of $x_3(t)$: $T_3 = \sqrt{2}$ sec.

Step 1 : $\frac{T_1}{T_2} = \frac{8/3}{1.26} = \frac{8}{3.78} = \frac{800}{378} = \frac{400}{189}$ = rational

$$\frac{T_1}{T_3} = \frac{8/3}{\sqrt{2}} = \frac{8}{3\sqrt{2}} = \text{not rational}$$

T_1/T_3 cannot be brought to the form of ratio of integers.

Step 2 : Therefore $x(t) = x_1(t)+x_2(t)+x_3(t)$ is not periodic.

Example 1.35 Repeat example 1.34 for the signal $y(t) = y_1(t)+y_2(t)+y_3(t)$ where $y_1(t)$, $y_2(t)$ and $y_3(t)$ have periods of 1.08, 3.6 and 2.025 sec. respectively.

Solution : Given : $y(t) = y_1(t)+y_2(t)+y_3(t)$

Period of $y_1(t)$: $T_1 = 1.08$ sec.

Period of $y_2(t)$: $T_2 = 3.6$ sec.

Period of $y_3(t)$: $T_3 = 2.025$ sec.

Step 1 : $\dfrac{T_1}{T_2} = \dfrac{1.08}{3.6} = \dfrac{108}{360} = \dfrac{3}{10}$ = rational

$$\frac{T_1}{T_3} = \frac{1.08}{2.025} = \frac{1080}{2025} = \frac{8}{15} = \text{rational}$$

Step 2 : The ratio T_1/T_i is converted into ratio of integers i.e., they are rational. Therefore $y(t)$ is periodic.

Step 3 : $\dfrac{T_1}{T_2} = \dfrac{3}{10}$ & $\dfrac{T_1}{T_3} = \dfrac{8}{15}$

g.c.d of (3, 8) = 1

g.c.d of (10, 15) = 5

$$\therefore \frac{T_1}{T_2} = \frac{3(1)}{2(5)} \ \& \ \frac{T_1}{T_3} = \frac{8(1)}{3(5)}$$

$\dfrac{3}{10}$, $\dfrac{8}{15}$ $\dfrac{3(1)}{2(5)}$ $\dfrac{8(1)}{3(5)}$

Step 4 : L.C.M. of the denominators i.e., l.c.m of (2), (3), (5) = 30 = l

LCM of 3, 2, 5

30

Step 5 : Period of the sum signal y(t) is,

$$T = T_1 . l = 1.08(30) = 32.4 \text{ sec.}$$

$T = T_1 . l =$

Example 1.36 Determine whether the continuous time signal $x(t) = [\sin(t-\frac{\pi}{6})]^2$ is periodic. If periodic, find its fundamental period.

Solution : Given : $x(t) = [\sin(t-\dfrac{\pi}{6})]^2$

$$\therefore x(t) = \frac{1}{2}[1 - \cos 2(t-\frac{\pi}{6})] \qquad [\because \sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)]$$

$$x(t) = \frac{1}{2} - \frac{1}{2}[\cos(2t-\frac{\pi}{3})]$$

$$= \frac{1}{2} - \frac{1}{2}\left(\cos 2t \cos\frac{\pi}{3} + \sin 2t \sin\frac{\pi}{3}\right) \quad [\because \cos(A-B) = \cos A . \cos B + \sin A . \sin B]$$

$$= \frac{1}{2} - \frac{1}{4} \cdot \cos 2t - \frac{\sqrt{3}}{4} \cdot \sin 2t$$

$$x(t) = \frac{1}{2} - \frac{1}{4}\cos 2t - \frac{\sqrt{3}}{4}\sin 2t$$

$$\therefore T = 2\pi/\omega_0 = 2\pi/2 = \pi \text{ sec.}$$

Example 1.37 Repeat example 1.36 for $x(t) = (\cos 2\pi t)\, u(t)$

Solution : Given : $x(t) = (\cos 2\pi t)\, u(t)$

Even though $\cos 2\pi t$ is a periodic signal with fundamental period $T = 2\pi/\omega_0 = 2\pi/2\pi = 1$ sec, the signal $x(t) = (\cos 2\pi t)\, u(t)$ is non-periodic because it appears only for $t \geq 0$. If any signal is multiplied by $u(t)$ (i.e., unit step function), the resultant signal appears only for $t \geq 0$. The difference between $\cos 2\pi t$ and $(\cos 2\pi t)u(t)$ is shown in Fig. P1.37.1 and Fig. P1.37.2 below.

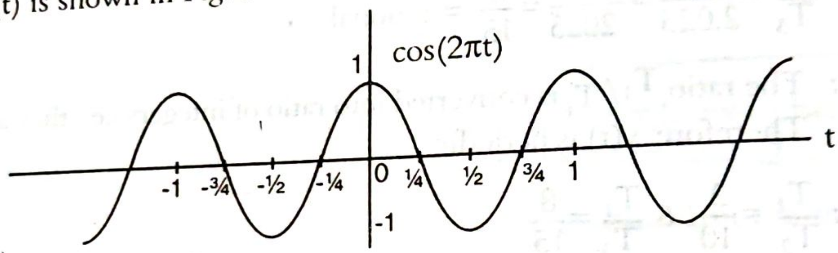


Fig. P1.37.1

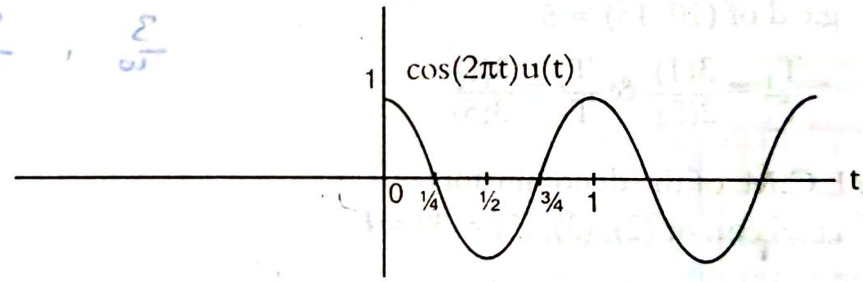


Fig. P1.37.2

A signal is periodic, if the cycle repeats along the entire time axis. [i.e., $-\infty < t < \infty$]

$\therefore x(t) = (\cos 2\pi t)\, u(t)$ is non-periodic.

Example 1.38 Determine whether the discrete-time signal $x(n) = (-1)^n$ is periodic. If periodic, find the fundamental period.

Solution : Given : $x(n) = (-1)^n$

i.e., $x(n) = 1 \quad ; n = \text{even}$

$\quad\quad\; = -1 \quad ; n = \text{odd}$

The signal $x(n)$ is plotted in Fig. P1.38.1.

$$\Omega_1 = 2\pi \cdot \frac{1}{5}$$

$\therefore$ Fundamental period $N_1 = 5$

Similarly, fundamental period of $\cos\left(\frac{2\pi n}{7}\right) = N_2 = 7$

$$\therefore \frac{N_1}{N_2} = \frac{5}{7} = \text{rational}$$

$x(n)$ is periodic.

$$\frac{N_1}{N_2} = \frac{5}{7(1)} \rightarrow \text{g.c.d of denominator}$$

of the denominator is $l = 7$

ndamental period $N = N_1 \cdot l = 5(7) = 35$.

erministic and Random Signals

ninistic signal behaves in a fixed known way with respect to time. It can be a function of time 't' (i.e., continuous-time signal) or a function of a sample e., discrete time signal).

deterministic signal mathematically, the range of values for 't' or 'n' must be wise it valids for all values of 't' or 'n'.

ignal takes on one of several possible values at each time for which a signal That is, it is a signal about which there is uncertainty with respect to its The existence of a random signals in a system is a random process which istic models. Examples of random signals are the noise generated in the receiver, the ECG signal etc.

d Power Signals

l energy and signal power describe signal characteristics. They are not energy and power because the energy or power absorbed by a system ion of the component and the signal that passes through (current) or ıge).

m, a signal may be in the form of voltage or current. Consider a s a resistor resulting in a current $i(t)$. Then the instantaneous power

$$p(t) = \frac{\upsilon^2(t)}{R} = R.i^2(t) \quad \ldots\ldots (1.56)$$

ended over the time interval $t_1 \le t \le t_2$ is given by,

$$= \int_{t_1}^{t_2} \frac{\upsilon^2(t)}{R}\,dt = \int_{t_1}^{t_2} R.i^2(t)\,dt$$

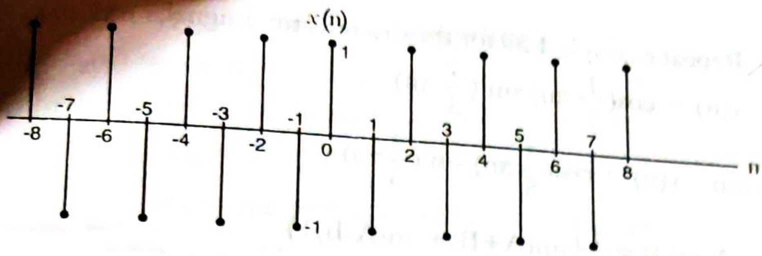


Fig. P1.38

By observation, it is periodic with fundamental period N=2.

Example 1.39 Repeat example 1.38 for the signal $x(n)$ shown in Fig. P1.39.

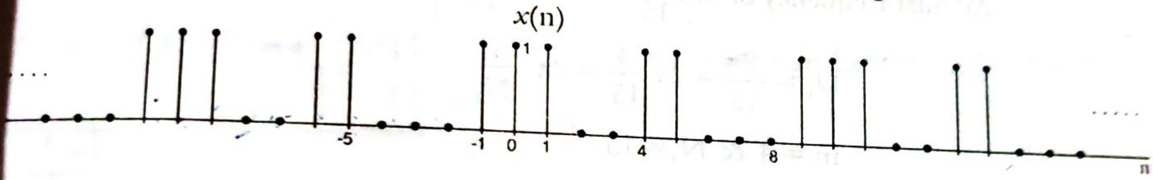


Fig. P1.39

Solution : By observation, the given $x(n)$ is periodic with fundamental period N=10

Example 1.40 Determine whether the discrete-time signal z(n) is periodic, where $z(n) = z_1(n) + z_2(n)$ where $z_1(n)$ and $z_2(n)$ are periodic with period of 90 and 54 respectively.

Solution : (Note : The period of discrete-time signal is integer. To determine the period of the summation of M discrete-time periodic signals follow the steps given in example 1.34)

Given : $z(n) = z_1(n) + z_2(n)$

Period of $z_1(n) : N_1 = 90$

Period of $z_2(n) : N_2 = 54$

Step 1 :

$$\frac{N_1}{N_2} = \frac{90}{54} = \frac{5}{3} = \text{rational}$$

Step 2 :

$\therefore$ z(n) is periodic

Step 3 :

$$\frac{N_1}{N_2} = \frac{5}{3(1)} \rightarrow \text{g.c.d of denominater.}$$

Step 4 : L.C.M. of the denominator is 'l' = 3

Step 5 : Fundamental period $N = N_1 . l = 90(3) = 270$

Example 1.41 Repeat example 1.39 for the discrete-time signal,

$$x(n) = \cos(\frac{1}{5}\pi n)\sin(\frac{1}{3}\pi n)$$

Solution : Given : $x(n) = \cos(\frac{1}{5}\pi n)\sin(\frac{1}{3}\pi n)$

$$(\sin A\cos B = \frac{1}{2}\{\sin(A+B)+\sin(A-B)\})$$

$$\therefore x(n) = \frac{1}{2}\left\{\sin\frac{8\pi}{15}n + \sin\frac{2\pi}{15}n\right\}$$

Angular frequency of $\sin\frac{8\pi}{15}n = \Omega_1 = \frac{8\pi}{15}$

$$\Omega_1 = \frac{8\pi}{15} = 2\pi\cdot\frac{4}{15} = 2\pi\cdot\frac{m}{N_1}$$

$$m = 4 \ \&\ N_1 = 15$$

$\therefore$ Fundamental period $N_1 = 15$.

Similarly, fundamental period of $\sin\frac{2\pi}{15}n$ is $N_2 = 15$

$$\frac{N_1}{N_2} = \frac{15}{15} = \frac{1}{1}$$

l.c.m. of the denominator $l = 1$

$\therefore$ Period of $x(n)$ $N = N_1 l$

$$= 15\,(1) = 15$$

Example 1.42 Check whether the following signals are periodic or not. If periodic, find the fundamental period.

(i) $x_1(n) = \cos 2\pi n$, (ii) $x_2(n) = \cos 2n$

Solution : Remember that the signal $\cos\Omega_o n$ is periodic only if Ω_o is rational (ratio of integers) multiple of 2π.

i.e.,, $\Omega_o = 2\pi\cdot\frac{m}{N}$ where 'm' and 'N' are integers and the fundamental period is N.

Given : (i) $x_1(n) = \cos 2\pi n$

Comparing with $\cos\Omega_o n$ we have,

$$\Omega_o = 2\pi = 2\pi\cdot\underbrace{\frac{1}{1}}_{\text{rational}}$$

$\therefore$ cos $2\pi n$ is periodic with fundamental period N=1

(ii) $x_2(n) = \cos 2n$

Comparing with $\cos \Omega_o n$ we have,

$$\Omega_0 = 2$$

Here 2 cannot be expressed as rational multiple of 2π.

i.e., $2 \neq 2\pi \cdot \frac{m}{N}$

$\therefore x_2(n) = \cos 2n$ is non-periodic.

Example 1.43 Repeat example 1.39 for the discrete-time signal $x(n)=\cos\left(\frac{8\pi n}{7}+2\right)$

Solution : Given : $x(n)=\cos\left(\frac{8\pi n}{7}+2\right)$

Comparing with $x(n) = \cos(\Omega n + \phi)$

Angular frequency : $\Omega = \frac{8\pi}{7} = 2\pi \cdot \frac{4}{7} = 2\pi \frac{m}{N}$

$$m=4 \;\&\; N=7 \;\&\; \phi=2$$

$\therefore$ Fundamental period N=7.

Example 1.44 Determine whether the following signal is periodic or not.

$$x(n) = (\cos \frac{1}{3}\pi n)\,(\sin 2n)$$

Solution : Here 2 signals are multiplied.

First signal ($\cos \frac{1}{3}\pi n$) is periodic with period N = 6. But the second signal ($\sin 2n$) is non-periodic because 2 cannot be expressed in terms of rational multiple of 2π. Multiplying any periodic signal with non-periodic signal results in non-periodic signal. Therefore the given signal $x(n)$ is non-periodic.

Example 1.45 Determine whether the following signal is periodic or not. If periodic, find its fundamental period.

$x(t) = \upsilon(t)$ and $\upsilon(-t)$ where $\upsilon(t) = \sin(t)\,u(t)$

Solution : The plots of $\upsilon(t)$ and $\upsilon(-t)$ are shown in Fig. 1.45 (a) & (b) respectively.

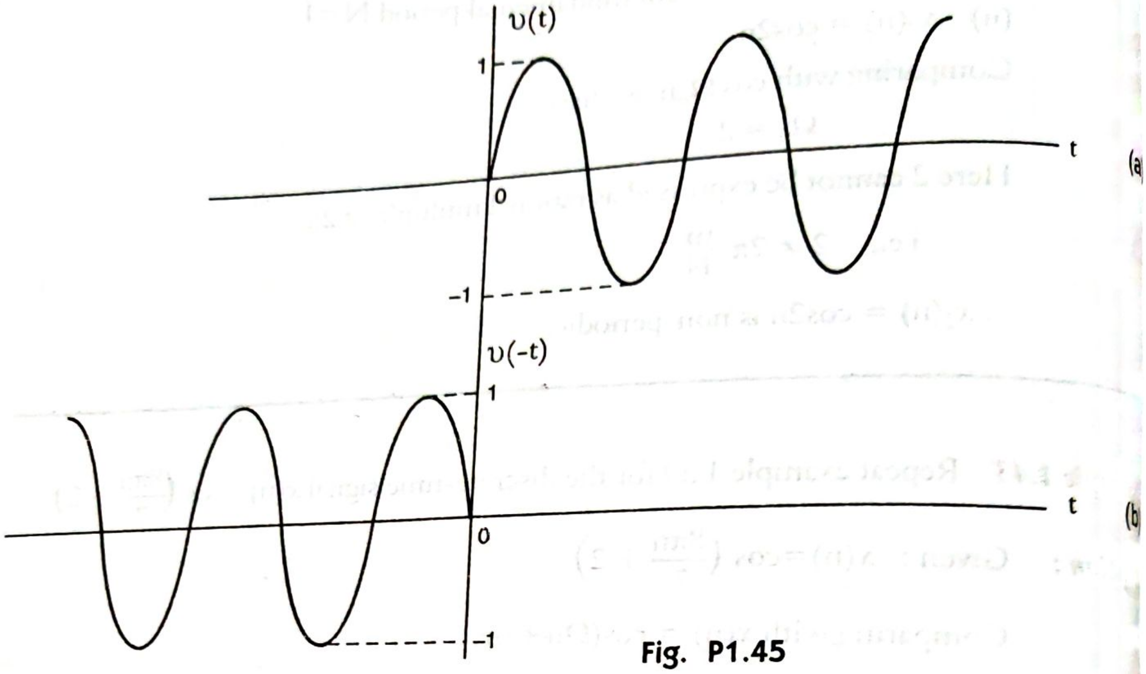


Fig. P1.45

By adding signals shown in Fig. P1.45 (a) & (b), we get the signal $x(t)$ shown in Fig. P1.45.1.

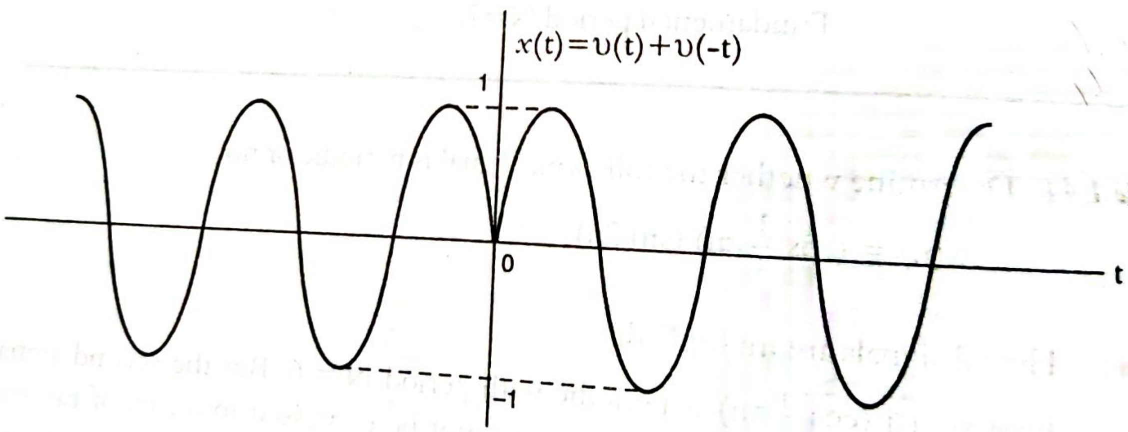


Fig. P1.45.1

By observation, $x(t)$ is non-periodic.

Example 1.46 Find the periodicity of the signal,

$$x(n) = \cos\left(\frac{2\pi n}{5}\right) + \cos\left(\frac{2\pi n}{7}\right)$$

Solution : Given : $x(n) = \cos\left(\frac{2\pi n}{5}\right) + \cos\left(\frac{2\pi n}{7}\right)$

Angular frequency of $\cos\frac{2\pi n}{5}$: $\Omega_1 = \frac{2\pi}{5}$

$$\Omega_1 = 2\pi \cdot \frac{1}{5}$$

$\therefore$ Fundamental period $N_1 = 5$

Similarly, fundamental period of $\cos\left(\frac{2\pi n}{7}\right) = N_2 = 7$

$$\therefore \frac{N_1}{N_2} = \frac{5}{7} = \text{rational}$$

$\therefore x(n)$ is periodic.

$$\frac{N_1}{N_2} = \frac{5}{7(1)} \rightarrow \text{g.c.d of denominator}$$

l.c.m of the denominator is $l = 7$

$\therefore$ Fundamental period $N = N_1 \cdot l = 5(7) = 35.$

1.6.4 Deterministic and Random Signals

A *deterministic* signal behaves in a fixed known way with respect to time. It can be modelled as a function of time 't' (i.e., continuous-time signal) or a function of a sample number 'n' (i.e., discrete time signal).

To model deterministic signal mathematically, the range of values for 't' or 'n' must be specified, otherwise it valids for all values of 't' or 'n'.

A *random* signal takes on one of several possible values at each time for which a signal value is defined. That is, it is a signal about which there is uncertainty with respect to its value at any time. The existence of a random signals in a system is a random process which requires probabilistic models. Examples of random signals are the noise generated in the amplifier of a radio receiver, the ECG signal etc.

1.6.5 Energy and Power Signals

The term signal energy and signal power describe signal characteristics. They are not actually measures of energy and power because the energy or power absorbed by a system component is a function of the component and the signal that passes through (current) or existing across it (voltage).

In electrical system, a signal may be in the form of voltage or current. Consider a voltage $\upsilon(t)$ exists across a resistor resulting in a current $i(t)$. Then the instantaneous power $p(t)$ is given by

$$p(t) = \frac{\upsilon^2(t)}{R} = R.i^2(t) \quad \ldots\ldots \quad (1.56)$$

The total energy expended over the time interval $t_1 \leq t \leq t_2$ is given by,

$$\int_{t_1}^{t_2} p(t)\, dt = \int_{t_1}^{t_2} \frac{\upsilon^2(t)}{R}\, dt = \int_{t_1}^{t_2} R.i^2(t)\, dt$$

and the average power over the time interval is,

$$\frac{1}{t_2-t_1}\int_{t_1}^{t_2} p(t)dt = \frac{1}{t_2-t_1}\int_{t_1}^{t_2} \frac{v^2(t)}{R}\, dt$$

In general, in many systems we are interested in finding power and energy in signals over an infinite time interval [i.e., for $-\infty < t < \infty$ (CT signal) or $-\infty < n < +\infty$ (DT signal)]. The total energy of a continuous time signal $x(t)$ is,

$$E = \underset{T\to\infty}{Lt} \int_{-T/2}^{T/2} x^2(t)\, dt$$

$$= \int_{-\infty}^{\infty} x^2(t)\, dt$$

If $x(t)$ is complex then,

$$E = \underset{T\to\infty}{Lt} \int_{-T/2}^{T/2} |x(t)|^2\, dt$$

$$E = \int_{-\infty}^{\infty} |x(t)|^2\, dt \quad \text{..... (1.57)}$$

But for some signal eqn. 1.57 might not converge. Such signals have infinite energy. The average power of a continuous-time signal $x(t)$ is given by,

$$P = \underset{T\to\infty}{Lt} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2\, dt \quad \text{..... (1.58)}$$

The average power of a periodic continuous-time signal $x(t)$ of fundamental period T is given by,

$$P = \frac{1}{T}\int_{-T/2}^{T/2} x^2(t)\, dt \quad \text{..... (1.59)}$$

Similarly, the total energy of a discrete time signal $x(n)$ is given by,

$$E = \underset{N\to\infty}{Lt} \sum_{n=-N}^{N} |x(n)|^2$$

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2 \quad \text{..... (1.60)}$$

But for some signal eqn.1.60 might not converge. Such signal have infinite energy.

The average power of a discrete-time sequence $x(n)$ is given by,

$$P = \underset{N\to\infty}{Lt} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2 \qquad \ldots\ldots \quad (1.61)$$

The average power of a periodic discrete-time signal $x(n)$ of fundamental period N is given by,

$$P = \frac{1}{N} \sum_{n=0}^{N-1} x^2(n) \qquad \ldots\ldots \quad (1.62)$$

The signal is referred to as an *energy signal* if the total energy E of the signal satisfies the condition,

$0 < E < \infty$ [i.e., E must be finite]

whereas it is referred to as a *power signal* if the average power P of the signal satisfies the condition,

$0 < P < \infty$ [i.e., P must be finite]

Examples for power signals are,

(i) All periodic signals

(ii) Random signals.

All signals which are both deterministic and non-periodic are examples for energy signals.

Examples

Example 1.47 What is the total energy of the rectangular pulse shown in Fig. P1.47.

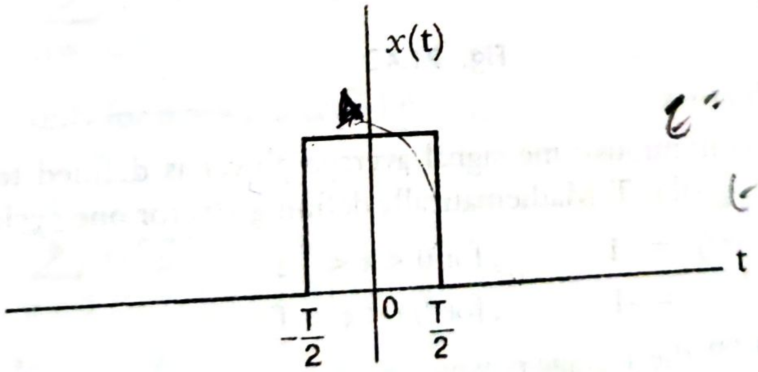


Fig. P1.47

$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$

$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$

$P = \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\, dt$

$P = \sum_{n=0}^{N-1} x^2(k)$

Solution : We have $x(t) = A \quad ; \frac{-T}{2} < t < \frac{T}{2}$

$= 0 \quad$; elsewhere

The total energy of a continuous-time signal is given by (from eqn. 1.57),

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

Since the $x(t)$ has non-zero value in the range $\frac{-T}{2} < t < \frac{T}{2}$, we can write,

$$E = \int_{-T/2}^{T/2} |x(t)|^2 \, dt$$

$$= \int_{-T/2}^{T/2} |A|^2 \, dt$$

$$= A^2 . t \Big|_{-T/2}^{+T/2}$$

$$= A^2 . T$$

Example 1.48 What is the average power of the square wave shown in Fig. P1.48.

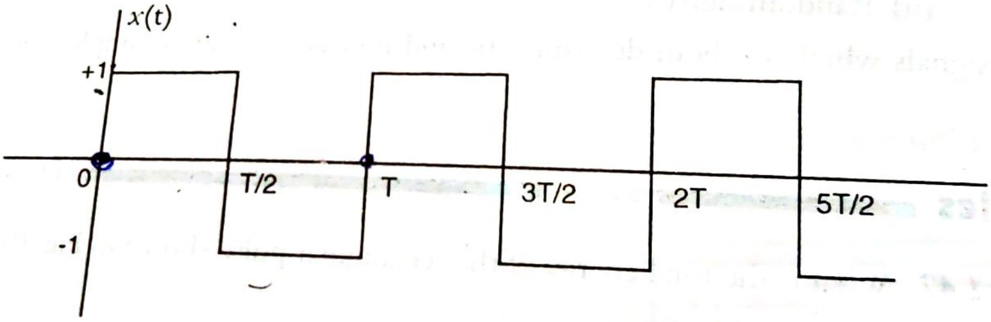


Fig. P1.48

Solution :

In a periodic continuous-time signal average power is defined for one cycle. The period of the given signal is T. Mathematically defining $x(t)$ for one cycle,

$$x(t) = 1 \quad ; \text{for } 0 < t < {}^{T}/_{2}$$

$$= -1 \quad ; \text{for } {}^{T}/_{2} < t < T$$

∴ From eqn. 1.59, the average power is given by,

$$P = \frac{1}{T} \int_{-T/2}^{T/2} x^2(t) \, dt$$

Integration must be carried out for one complete cycle.

$$\therefore P = \frac{1}{T}\int_0^T x^2(t)\,dt$$

$$= \frac{1}{T}\left(\int_0^{T/2}(1)^2\,dt + \int_{T/2}^{T}(-1)^2\,dt\right)$$

$$= \frac{1}{T}\left(t\Big|_0^{T/2} + t\Big|_{T/2}^{T}\right)$$

$$= \frac{1}{T}\left((T/2 - 0) + (T - T/2)\right)$$

$$= 1$$

Example 1.49 What is the total energy of the discrete-time signal $x(n)$ shown in Fig. P1.49.

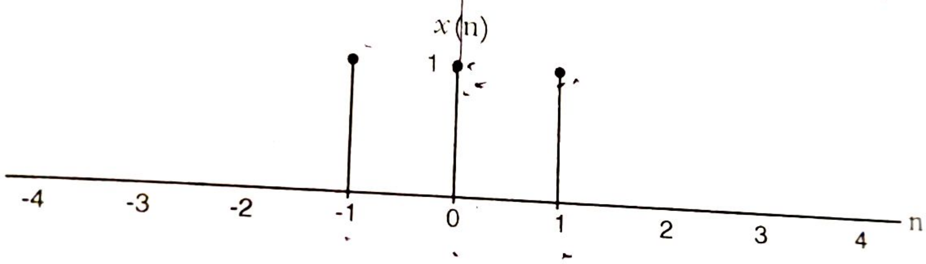


Fig. P1.49

Solution : From eqn. 1.60, we have the total energy of the discrete-time signal is given by,

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

In the given $x(n)$, only for $n = -1, 0$ & $+1$ the signal has non-zero value and its value equal to 1.

$$\therefore E = \sum_{n=-1}^{1} |x(n)|^2$$

$$= |x(-1)|^2 + |x(0)|^2 + |x(1)|^2$$

$$= 1^2 + 1^2 + 1^2$$

$$= 3$$

Example 1.50 What is the average power of the periodic discrete-time signal shown in Fig. P1.50.

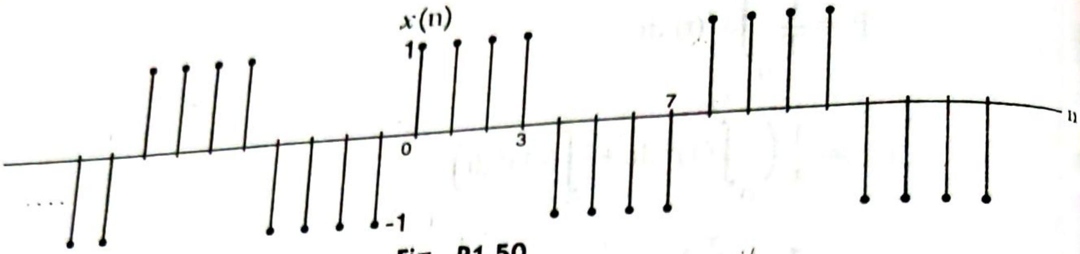


Fig. P1.50

Solution : The given discrete-time signal $x(n)$ is periodic with period $N=8$. From eqn. 1.62, the average power of a periodic discrete-time signal $x(n)$ of fundamental period N is given by,

$$P = \frac{1}{N}\sum_{n=0}^{N-1} x^2(n)$$

$$= \frac{1}{8}\sum_{n=0}^{7} x^2(n)$$

$$= \frac{1}{8}\{x^2(0) + x^2(1) + x^2(2) + \ldots\ldots + x^2(7)\}$$

$$= \frac{1}{8}\{1^2+1^2+1^2+\ldots\ldots\ldots +1^2\}$$

$$= 1$$

Example 1.51 What is the average power of the triangular wave shown in Fig. P1.51.

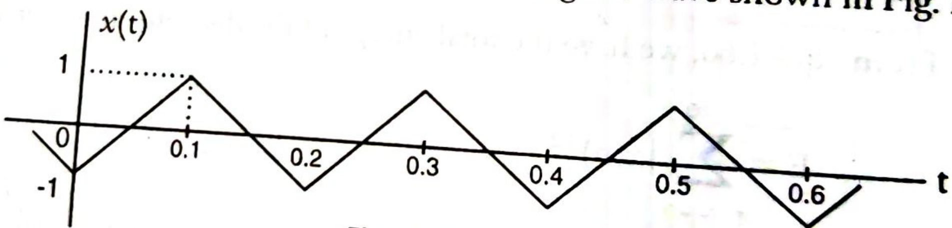


Fig. P1.51

Solution : It is a periodic signal with period T=0.2. From eqn. 1.59 we have,

$$\text{Average power } P = \frac{1}{T}\int_{-T/2}^{T/2} x^2(t)\, dt$$

Integration must be carried out for one complete cycle.

We have $x(t) = 20t-1 \quad ; 0 < t < 0.1$

$= -20t+3 \quad ; 0.1 < t < 0.2$

$$\therefore P = \frac{1}{0.2}\left(\int_{0}^{0.1}(20t-1)^2\, dt + \int_{0.1}^{0.2}(-20t+3)^2\, dt\right) = \frac{1}{3}$$

Example 1.52 For the trapezoidal pulse $x(t)$ shown in Fig. P1.52, find the total energy.

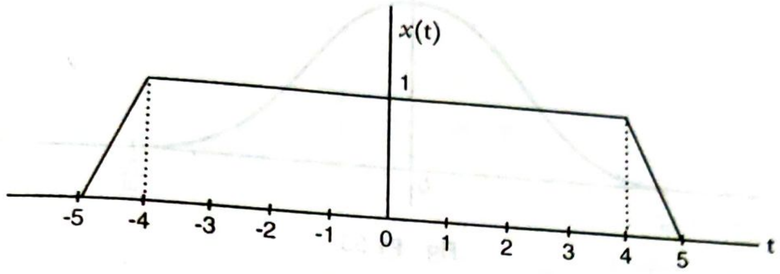


Fig. P1.52

Solution : From the Fig. P1.52 we have,

$$\begin{aligned} x(t) &= 5-t && ; 4 \le t \le 5 \\ &= 1 && ; -4 \le t \le 4 \\ &= t+5 && ; -5 \le t \le -4 \\ &= 0 && ; \text{otherwise} \end{aligned}$$

From eqn. 1.57 we have,

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

But the signal has non-zero value only in the range $-5 \le t \le 5$

$$\therefore E = \int_{-5}^{5} |x(t)|^2 \, dt$$

$$= \int_{-5}^{-4} (t+5)^2 \, dt + \int_{-4}^{4} dt + \int_{4}^{5} (5-t)^2 \, dt = \frac{26}{3}$$

Example 1.53 The raised-cosine pulse $x(t)$ shown in Fig. P1.53 is given by,

$$\begin{aligned} x(t) &= \frac{1}{2}[\cos(\omega t)+1] && ; \frac{-\pi}{\omega} \le t \le \frac{\pi}{\omega} \\ &= 0 && ; \text{otherwise} \end{aligned}$$

Find the total energy of $x(t)$.

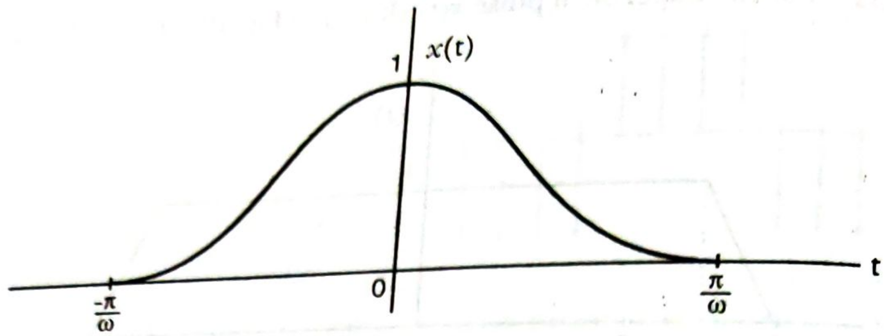


Fig. P1.53

Solution : From eqn. 1.57 we have,

$$\text{Energy} \quad E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

The given signal $x(t)$ appears only in the range $\frac{-\pi}{\omega} \leq t \leq \frac{\pi}{\omega}$

$$\therefore E = \int_{-\pi/\omega}^{\pi/\omega} \left| \left\{ \frac{1}{2}[\cos(\omega t)+1] \right\} \right|^2 dt$$

$$= \frac{1}{4} \int_{-\pi/\omega}^{\pi/\omega} (\cos^2 \omega t + 1 + 2\cos \omega t) \, dt$$

$$= \frac{1}{4} \int_{-\pi/\omega}^{\pi/\omega} \left(\frac{1}{2} + \frac{1}{2}\cos 2\omega t + 1 + 2\cos\omega t \right) dt$$

$$= \frac{3\pi}{4\omega}$$

Example 1.54 Consider the sinusoidal signal given by,

$$x(t) = A\cos(\omega t + \phi)$$

Determine the average power of it.

Solution : The given signal $x(t)$ is periodic with angular frequency ω. The fundamental period is given by,

$$\therefore T = \frac{2\pi}{\omega}$$

$$\therefore \omega T = 2\pi$$

$\therefore$ From eqn. 1.59 we have,

$$\text{Average Power } P = \frac{1}{T}\int_{-T/2}^{T/2} x^2(t)\,dt$$

$$= \frac{1}{T}\int_{-T/2}^{T/2} A^2\cos^2(\omega t+\phi)\,dt$$

$$= \frac{A^2}{2T}\int_{-T/2}^{T/2}(1+\cos 2\phi\cos 2\omega t - \sin 2\phi \sin 2\omega t)\,dt$$

$$= \frac{A^2}{2}$$

Example 1.55 The angular frequency Ω of the sinusoidal signal $x(n) = A\cos(\Omega n+\phi)$ satisfies the condition of periodicity. Determine the average power of $x(n)$.

Solution : The given signal $x(n)$ is periodic with fundamental period $N = 2\pi/\Omega$. From eqn. 1.62, the average power is given by,

$$P = \frac{1}{N}\sum_{n=0}^{N-1} x^2(n)$$

$$= \frac{1}{N}\sum_{n=0}^{N-1} A^2\cos^2(\Omega n+\phi)$$

$$P = \frac{A^2}{N}\sum_{n=0}^{N-1}\cos^2\left(\frac{2\pi n}{N}+\phi\right)$$

Example 1.56 Check whether the following signal $x(t)$ is energy or power signal and find the corresponding value.

$$x(t) = \begin{cases} t & ; 0 \le t \le 1 \\ 2-t & ; 1 \le t \le 2 \\ 0 & ; \text{otherwise.} \end{cases}$$

Solution : From eqn. 1.57,

$$E = \int_{-\infty}^{\infty} |x(t)|^2\,dt$$

$$E = \int_0^1 t^2\,dt + \int_1^2 (2-t)^2\,dt$$

$$= \frac{2}{3} < \infty$$

Since E is finite, $x(t)$ is energy signal.

Example 1.57 Check whether the following signal $x(n)$ is energy or power signal and find the corresponding value.

$$x(n) = \begin{cases} n & ; 0 \le n \le 5 \\ 10-n & ; 5 < n \le 10 \\ 0 & ; \text{otherwise} \end{cases}$$

Solution : From eqn. 1.60,

$$E = \sum_{n=-\infty}^{\infty} x^2(n)$$

$$= \sum_{n=0}^{5} n^2 + \sum_{n=6}^{10} (10-n)^2$$

$$= [0+1^2+2^2+3^2+4^2+5^2] + [(10-6)^2+(10-7)^2+(10-8)^2+(10-9)^2$$

$$= (1+4+9+16+25) + (16+9+4+1)$$

$$= 85 < \infty \quad \therefore x(n) \text{ is energy signal.}$$

Example 1.58 Check whether the following are energy or power signals ? Also find the corresponding value.

(i) $x_1(n) = \cos(\pi n)$; $-4 \le n \le 4$
$= 0$; otherwise

(ii) $x_2(n) = \cos(\pi n)$; $n \ge 0$
$= 0$; otherwise

Solution : (i) Given : $x_1(n) = \cos\pi n$; $-n \le n \le 4$
$= 0$; otherwise

The given signal is non-periodic and exist between $-4 \le n \le 4$. Therefore it is energy signal.

$$\text{Energy} = E_1 = \sum_{n=-\infty}^{\infty} = |x_1(n)|^2$$

$$= \sum_{n=-4}^{4} (\cos \pi n)^2$$

$$= \sum_{n=-4}^{4} (-1)^{2n}$$

Put $m=n+4$:

$$\therefore\ E_1 = \sum_{m=0}^{8} (-1)^{2(m-4)}$$

$$= (-1)^{-8} \sum_{m=0}^{8} (-1)^{2m}$$

$$= 1 \sum_{m=0}^{8} 1 \qquad \left[\because \sum_{n=0}^{N-1} 1 = N\right]$$

$$= 9$$

(ii) Given : $x_2(n) = \cos(\pi n)$; $n \geq 0$

$= 0$; otherwise

The given signal $x_2(n)$ is of infinite length. So if we calculate energy, it will be infinite. Therefore it is not an energy signal. It is power signal. It repeats for every 2 samples. $\therefore$ N=2.

Power $$P = \frac{1}{N} \sum_{n=0}^{N-1} x_2^2(n)$$

$$= \frac{1}{2} \sum_{n=0}^{1} (\cos \pi n)^2$$

$$= \frac{1}{2} \sum_{n=0}^{1} (-1)^{2n}$$

$$= \frac{1}{2} \sum_{n=0}^{1} 1 = 1$$

Example 1.59 The trapezoidal pulse $x(t)$ shown in Fig. P1.59 is applied to a differentiator defined by,

$$y(t) = \frac{d}{dt} x(t)$$

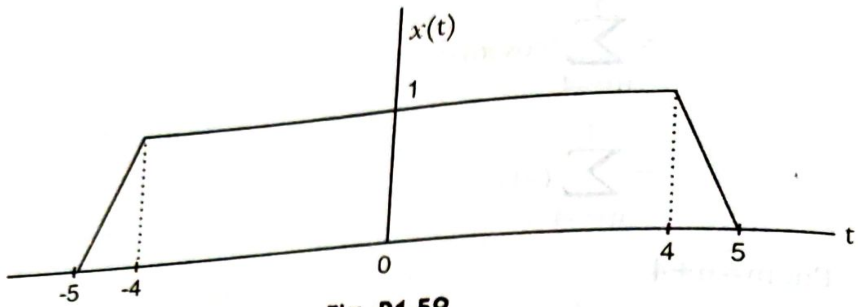


Fig. P1.59

(i) Find the resulting output y(t) of the differentiator.
(ii) Find the total energy of y(t).

Solution : The signal $x(t)$ is given by,

$$x(t) = \begin{cases} 5-t & ; 4 \le t \le 5 \\ 1 & ; -4 \le t \le 4 \\ t+5 & ; -5 \le t \le -4 \\ 0 & ; \text{otherwise.} \end{cases}$$

$$\therefore\ y(t) = \frac{dx(t)}{dt} = \begin{cases} -1 & ; 4 \le t \le 5 \\ 1 & ; -5 \le t \le -4 \\ 0 & ; \text{otherwise.} \end{cases}$$

The plot of y(t) is shown in Fig. P1.59.1

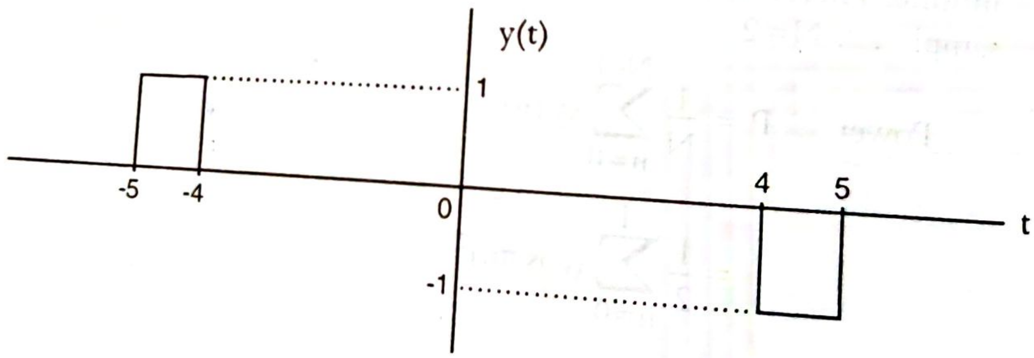


Fig. P1.59.1

∴ From eqn. 1.57 we have,

$$\text{Energy } E = \int_{-\infty}^{\infty} |x(t)|^2\, dt$$

The signal $x(t)$ has non-zero value only in the range $-5 < t < -4$ and $4 < t < 5$.

$$\therefore\ E = \int_{-5}^{-4} 1^2\, dt + \int_{4}^{5} (-1)^2\, dt = 2$$

Example 1.60 A rectangular pulse $x(t)$ is defined by,

$$x(t) = \begin{cases} A & ; 0 \le t \le T \\ 0 & ; \text{otherwise} \end{cases}$$

is applied to an integrator defined by,

$$y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$$

Find the total energy of the output y(t).

Solution : The signal $x(t)$ is drawn in Fig. P1.60.1.

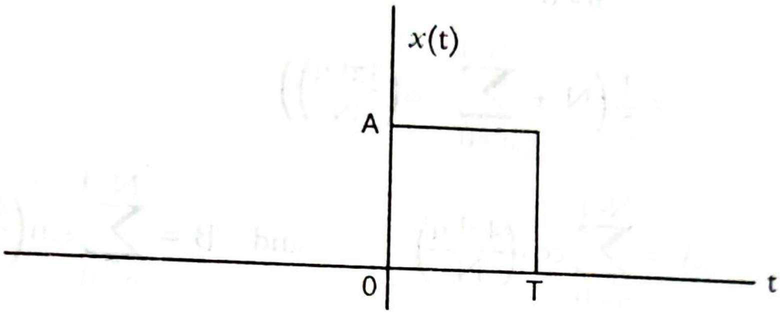


Fig. P1.60.1

We have $$y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$$

$$= \int_{0}^{t} A\, d\tau \qquad ; \text{for } 0 \le \tau \le T$$

$$= A \cdot t$$

$\therefore$ From eqn. 1.57 we have,

$$\text{Energy} = \int_{-\infty}^{\infty} |y(t)|^2\, dt$$

$$= \int_{0}^{T} A^2.t^2.\, dt$$

$$= \frac{A^2T^3}{3}$$

Example 1.61 Compute the energy of the length - N sequence.

$$x(n) = \cos\left(\frac{2\pi kn}{N}\right) \qquad ; 0 \le n \le N-1$$

Solution : We have

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$= \sum_{n=0}^{N-1} \cos^2\left(\frac{2\pi kn}{N}\right)$$

$$= \frac{1}{2}\sum_{n=0}^{N-1}\left(1+\cos\left(\frac{4\pi kn}{N}\right)\right)$$

$$= \frac{1}{2}\left(N + \sum_{n=0}^{N-1}\cos\left(\frac{4\pi kn}{N}\right)\right) \qquad \text{..... (P1.61.1)}$$

Let $A = \sum_{n=0}^{N-1}\cos\left(\frac{4\pi kn}{N}\right)$ and $B = \sum_{n=0}^{N-1}\sin\left(\frac{4\pi kn}{N}\right)$

$$A = \sum_{n=0}^{N-1}\frac{e^{j\frac{4\pi kn}{N}} + e^{-j\frac{4\pi kn}{N}}}{2} \quad \text{and} \quad B = \sum_{n=0}^{N-1}\frac{e^{j\frac{4\pi kn}{N}} - e^{-j\frac{4\pi kn}{N}}}{2j}$$

$$\therefore A + jB = \sum_{n=0}^{N-1} e^{j\frac{4\pi kn}{N}}$$

$$= \frac{1-e^{j4\pi k}}{1-e^{j\frac{4\pi k}{N}}} \qquad \left(\sum_{n=0}^{N-1}\alpha^n = \frac{1-\alpha^N}{1-\alpha} \; ;\alpha \neq 1 \right.$$

$$\qquad\qquad = N \; ;\alpha = 1$$

$A+jB = 0 \qquad (\because e^{j4\pi k} = 1)$

$\therefore\; A = 0$ and $B = 0$

$\therefore$ From eqn. P1.61.1 we get,

$$E = \frac{N}{2}$$

Example 1.62 Determine the average power and the energy of the following sequences.

(i) $x_1(n) = u(n)$

(ii) $x_2(n) = nu(n)$

(iii) $x_3(n) = A_0 e^{j\Omega_0 n}$

Solution: (i) Given : $x_1(n) = u(n)$

Energy $$E = \sum_{n=-\infty}^{\infty} |x_1(n)|^2 = \sum_{n=0}^{\infty} 1^2 = \infty$$

Average Power $$P = \underset{N\to\infty}{Lt} \frac{1}{2N+1} \sum_{n=-N}^{N} |x_1(n)|^2$$

$$= \underset{N\to\infty}{Lt} \frac{1}{2N+1} \sum_{n=0}^{N} 1^2$$

$$= \underset{N\to\infty}{Lt} \frac{1}{2N+1} (N+1) \qquad \left(\because \sum_{n=0}^{N-1} 1 = N\right)$$

$$= \frac{1}{2}$$

(ii) Given : $x_2(n) = nu(n)$

$$E = \sum_{n=-\infty}^{\infty} |(x_2(n))|^2 = \sum_{n=0}^{\infty} n^2 = \infty$$

Average Power $$P = \underset{N\to\infty}{Lt} \frac{1}{2N+1} \sum_{n=-N}^{N} |x_2(n)|^2$$

$$= \underset{N\to\infty}{Lt} \frac{1}{2N+1} \sum_{n=0}^{\infty} n^2$$

$$= \infty$$

(iii) Given : $x_3(n) = A_0 e^{j\Omega_0 n}$

Energy $$E = \sum_{n=-\infty}^{\infty} |x_3(n)|^2 = \sum_{n=-\infty}^{\infty} |A_0 e^{j\Omega_0 n}|^2$$

$$= A_0^2 \sum_{n=-\infty}^{\infty} |e^{j2\Omega_0 n}| \qquad [\because |e^{j\phi}| = 1]$$

$$= A_0^2 \cdot \sum_{n=-\infty}^{\infty} \cdot 1 = \infty$$

$$\text{Average Power} \quad P = \underset{N\to\infty}{Lt} \frac{1}{2N+1} \sum_{n=-N}^{N} |x_3(n)|^2$$

$$= \underset{N\to\infty}{Lt} \frac{1}{2N+1} \sum_{n=-N}^{N} |A_o e^{j\Omega_o n}|^2$$

$$= \underset{N\to\infty}{Lt} \frac{1}{2N+1} \sum_{n=-N}^{N} |A_o|^2 \qquad [\because |e^{j\Omega_o n}| = 1]$$

$$= \underset{N\to\infty}{Lt} \frac{2N+1}{2N+1} \cdot |A_o|^2$$

$$= A_o^2$$

1.7 SYSTEMS VIEWED AS INTERCONNECTION OF OPERATIONS

A system is an interconnection of operations that transforms an input signal into an output signal. The properties of these output signals are entirely different from that of the input signal.

Consider a continuous-time system represented by an operator 'H'. An input signal $x(t)$ applied to this system results in an output signal $y(t)$ is described as,

$$y(t) = H\{x(t)\} \qquad \ldots\ldots (1.63)$$

The block diagram representation of eqn. 1.63 is shown in Fig. 1.34.

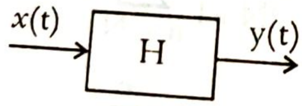


Fig. 1.34

Similarly, the block diagram representation for a discrete-time system described by

$$y(n) = H\{x(n)\}$$

is as shown in Fig. 1.35 below

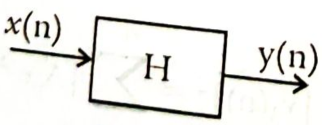


Fig. 1.35

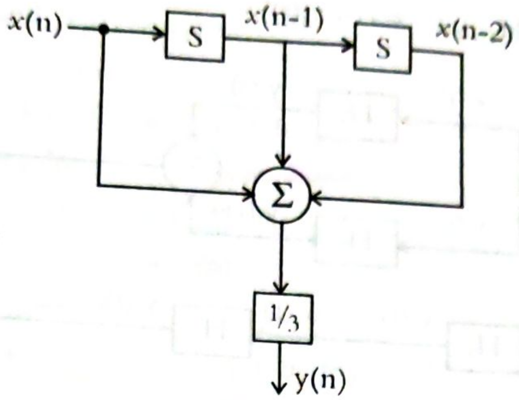
Examples

Example 1.63 Find the overall operator of a system whose output signal $y(n)$ is given by,

$$y(n) = \frac{1}{3}\left[x(n) + x(n-1) + x(n-2)\right]$$

Also draw the block diagram representation.

Solution : The block diagram representation is shown in Fig. P1.63.1.



S → The operator which delays the signal by one unit.

Fig. P1.63.1

where H = S is the shift operator.

$$H = \frac{1}{3}(1+S+S^2)$$

Example 1.64 Find the overall operator of a system whose output signal y(n) is given by,

$$y(n) = \frac{1}{3}\left[x(n+1) + x(n) + x(n-1)\right]$$

Also draw the block diagram representation.

Solution : The block diagram representation is shown in Fig. P1.64.1

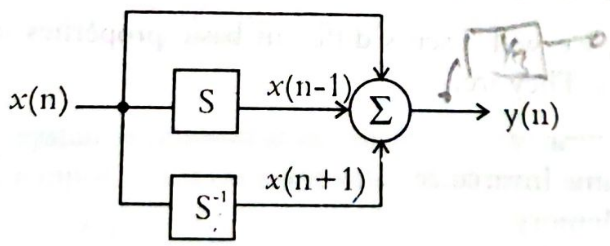


Fig. P1.64.1

$$\therefore H = \frac{1}{3}(S^{-1}+1+S)$$

Example 1.65 A system consists of several subsystems connected as shown in Fig. P1.65. Find the operator H relating $x(t)$ to y(t) for the subsystem operators given by,

$$H_1 : y_1(t) = x_1(t)\, x_1(t-1)$$

$$H_2 : y_2(t) = |x_2(t)|$$

Solution : The block diagram representation is shown in Fig. P1.63.1.

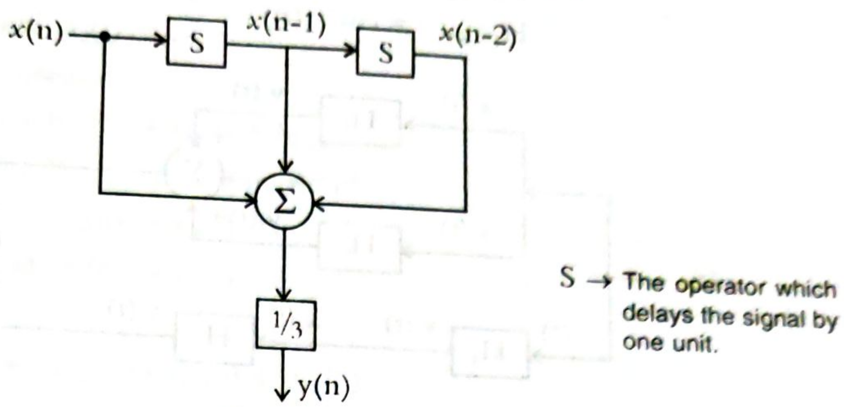


Fig. P1.63.1

where H = S is the shift operator.

$$H = \frac{1}{3}(1+S+S^2)$$

Example 1.64 Find the overall operator of a system whose output signal y(n) is given by,

$$y(n) = \frac{1}{3}\left[x(n+1) + x(n) + x(n-1)\right]$$

Also draw the block diagram representation.

Solution : The block diagram representation is shown in Fig. P1.64.1

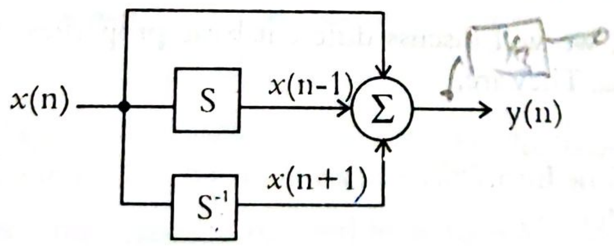


Fig. P1.64.1

$$\therefore H = \frac{1}{3}(S^{-1}+1+S)$$

Example 1.65 A system consists of several subsystems connected as shown in Fig. P1.65. Find the operator H relating $x(t)$ to y(t) for the subsystem operators given by,

$$H_1 : y_1(t) = x_1(t)\, x_1(t-1)$$

$$H_2 : y_2(t) = |x_2(t)|$$

$$H_3 : y_3(t) = 1+2x_3(t)$$
$$H_4 : y_4(t) = \cos(x_4(t))$$

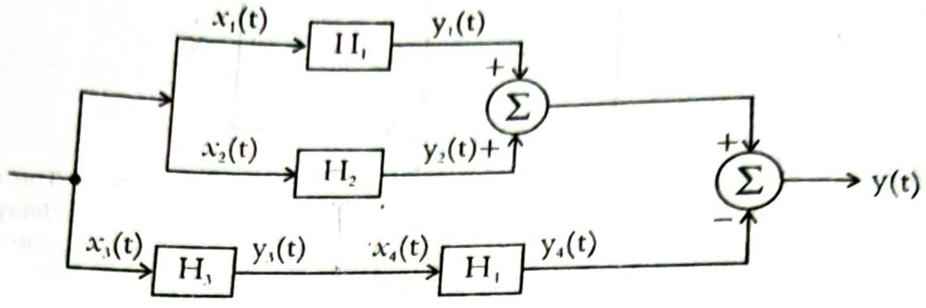


Fig. P1.65

Solution : From Fig. P1.65 we get,

$$y(t) = \{y_1(t) + y_2(t)\} - y_4(t)$$
$$\therefore y(t) = \{x_1(t)\, x_1(t-1) + |x_2(t)|\} - \cos(x_4(t))$$
$$= \{x_1(t)\, x_1(t-1) + |x_2(t)|\} - \cos(y_3(t))$$
$$y(t) = \{x_1(t)\, x_1(t-1) + |x_2(t)|\} - \cos(1+2x_3(t))$$

We have $x_1(t) = x_2(t) = x_3(t) = x(t)$

$$\therefore H : y(t) = x(t)\, x(t-1) + |x(t)| - \cos(1+2x(t))$$

1.8 PROPERTIES OF SYSTEMS

In this section, we will discuss different basic properties of continuous-time and discrete-time systems. They are,

(i) Linearity
(ii) Time Invariance
(iii) Memory
(iv) Causality
(v) Stability
(vi) Invertibility

(i) Linearity :

A system is said to be *linear* if it satisfies the principle of superposition, i.e., if an input consists of the weighted sum of several signals, then the output is the weighted sum of the responses of the system to each of those signals.

Let the input $x_1(t)$ applied to a continuous-time system results in output $y_1(t)$ and another input $x_2(t)$ results in output $y_2(t)$. Then if the system gives output $y_1(t) + y_2(t)$ for the input $x_1(t) + x_2(t)$, the system is said to be linear.

Alternatively,

$$\text{If } x_1(t) \rightarrow y_1(t)$$
$$\text{and } x_2(t) \rightarrow y_2(t)$$

then the system is linear if,

$$ax_1(t) + bx_2(t) \rightarrow ay_1(t) + by_2(t)$$

Similarly, consider a discrete-time system with,

$$x_1(n) \rightarrow y_1(n)$$
$$\text{and } x_2(n) \rightarrow y_2(n)$$

then the system is linear if,

$$ax_1(n) + bx_2(n) \rightarrow ay_1(n) + by_2(n)$$

(ii) Time Invariance :

A *time-invariant* system is one for which a time shift of the input signal causes a corresponding time shift in the output signal. The shift may be advance or delay. Specifically, suppose that a continuous-time system gives output $y(t)$ for an input $x(t)$, then the system is said to be time-invariant if the input $x(t-t_o)$ gives output $y(t-t_o)$.

i.e., $\quad$ If $x(t) \rightarrow y(t)$

then the system is time-invariant if,

$$x(t-t_o) \rightarrow y(t-t_o)$$

Similarly, a discrete-time system with,

$$x(n) \rightarrow y(n)$$

is said to be time-invariant if,

$$x(n-n_o) \rightarrow y(n-n_o)$$

(iii) Memory :

A continuous-time system is referred to as *memoryless* if the output $y(t)$ at every value of 't' depends only on the input $x(t)$ at the same value of 't'.

Similarly, a discrete-time system is referred to as memoryless if the output $y(n)$ at every value of 'n' depends only on the input $x(n)$ at the same value of 'n'.

(iv) Causality :

A continuous-time system is *causal* if present value of output $y(t)$ depends only on the past and/or present value of the input $x(t)$. Similarly, a discrete-time system is causal if present value of output signal $y(n)$ depends only on the past and/or present value of the input signal $x(n)$.

(v) Stability :

A system is said to be *bounded input bounded output* (BIBO) stable if and only if every bounded input results in a bounded output.

Consider a continuous-time system which gives output $y(t)$ for input $x(t)$. Then, system is stable if the output $y(t)$ satisfies the condition,

$$|y(t)| \le B_y < \infty \qquad ;\text{ for all 't'}$$

whenever the input signal $x(t)$ satisfies the condition,

$$|x(t)| \le B_x < \infty \qquad ;\text{ for all 't'}$$

where B_x and B_y are some finite positive numbers.

Similarly, a discrete-time system is stable if the output $y(n)$ satisfies the condition,

$$|y(n)| \le B_y < \infty \qquad ;\text{ for all 'n'}$$

whenever the input signal $x(n)$ satisfies the condition,

$$|x(n)| \le B_x < \infty \qquad ;\text{ for all 'n'}$$

(vi) Invertibility :

A system is said to be *invertible* if the input of the system can be recovered from the system output. Alternatively, a system is said to be invertible if distinct inputs lead to distinct outputs.

Consider a cascade connection of continuous-time systems shown in Fig. 1.36.

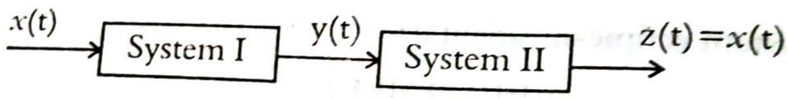


Fig. 1.36

In Fig. 1.36, the system I gives the output $y(t)$ for an input $x(t)$. If the input $x(t)$ to the system-I can be recovered from $y(t)$ by connecting system-II in cascade with system-I, (i.e., the output of system II is equal to the input to system-I) then system-I is said to be invertible and the system-II is called *inverse system*.

Examples

Example 1.66 For the following system, determine whether the system is (i) linear (ii) time-invariant (iii) memoryless (iv) causal (v) stable.

$$H\{x(n)\} = x(n-n_d)$$

Solution : Here 'H' is system operator.

i.e., $\quad y(n) = H\{x(n)\} = x(n-n_d)$

(i) Linearity : Write $x(n)$ as weighted sum of 2 signals $x_1(n)$ and $x_2(n)$.

i.e., $\quad x(n) = ax_1(n) + bx_2(n)$

Then if $H\{ax_1(n) + bx_2(n)\} = a\,H\{x_1(n)\} + b\,H\{x_2(n)\}$, the system is linear otherwise not.

We have $\quad y(n) = H\{x(n)\} = x(n-n_d)$

$$\therefore H\{ax_1(n)+bx_2(n)\} = ax_1(n-n_d)+bx_2(n-n_d)$$
$$= a\,H\{x_1(n)\}+b\,H\{x_2(n)\}$$

$\therefore$ System is linear

(ii) Time-Invariance : If $y(n-n_o) = H\{x(n-n_o)\}$ the system is time invariant, otherwise not.

$$H\{x(n)\} = x(n-n_d)$$
$$H\{x(n-n_o)\} = x(n-n_d-n_o)$$
$$\& \quad y(n-n_o) = x((n-n_o)-n_d) = x(n-n_d-n_o)$$
$$\therefore \quad y(n-n_o) = H\{x(n-n_o)\}$$

$\therefore$ System is time-invariant

(iii) Memoryless : $H\{x(n)\} = x(n-n_d)$

The system is not memoryless unless $n_d = 0$

(iv) Causality : If $n_d \geq 0$, the system causal. Otherwise it is not causal.

(v) Stability : Consider $|x(n)| \leq B_x < \infty$

then $|y(n)| = |x(n-n_d)| \leq B_x < \infty$

$\therefore$ the system stable

Example 1.67 Repeat example 1.66 for the system,

$$T\{x(n)\} = g(n)\,x(n)$$

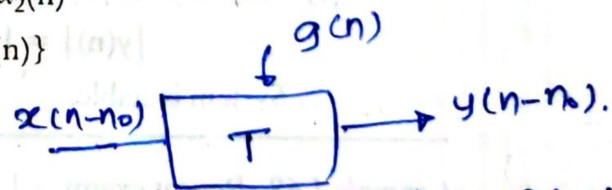
where 'T' is system operator.

Solution :

(i) Linearity : $T\{x(n)\} = g(n)\,x(n)$

$$\therefore T\{ax_1(n) + bx_2(n)\} = g(n)\,(ax_1(n) + bx_2(n))$$
$$= a\,g(n)\,x_1(n) + b\,g(n)\,x_2(n)$$
$$= a\,T\{x_1(n)\} + b\,T\{x_2(n)\}$$

$\therefore$ System is linear



(ii) Time invariance :

$$T\{x(n-n_o)\} = g(n)\,x(n-n_o)$$
$$y(n-n_o) = g(n-n_o)\,x(n-n_o)$$
$$\therefore y(n-n_o) \neq T\{x(n-n_o)\}$$

y(n-no) = g(n) . x(n-no)

$\therefore$ System is not time-invariant

(iii) Momoryless : Since the value of the output depends only on the present value of the input, the system is memoryless.

(iv) Causality : Since the output depends on only present value of the input, system causal.

(v) Stability : Consider $|x(n)| \leq B_x$

then $|y(n)| = |g(n)\,x(n)| = |g(n)|\,|x(n)|$

$= |g(n)|\ B_x \leq \infty$

$\therefore$ The output is bounded only if g(n) is bounded.

Example 1.68 Repeat example 1.66 for,

$$T\{x(n)\} = x(-n)$$

Solution :

(i) Linearity : $T\{x(n)\} = x(-n)$

$$\therefore T\{ax_1(n) + bx_2(n)\} = ax_1(-n) + bx_2(-n)$$
$$= a\,T\{x_1(n)\} + b\,T\{x_2(n)\}$$

$\therefore$ System is linear

(ii) Time invariance :

$$T\{x(n-n_o\} = x(-n-n_o)$$
$$y(n-n_o) = x(-(n+n_o)) = x(-n+n_o)$$
$$\therefore y(n-n_o) \neq T\{x(n-n_o)\}$$

$\therefore$ System is not time-invariant

(iii) Memoryless : If n>0, the system is not memoryless.

(iv) Causal : We have $T\{x(n)\} = x(-n)$

Consider n=-5, then $T\{x(-5)\} = x(5)$

i.e., the value of the output signal at n=-5 depends on the value of input n=5. Therefore the output depends on the future values of the inp

$\therefore$ System is non-causal.

(v) Stability : If $|x(n)| \leq B_x$ then,

$$|y(n)| = |x(-n)| \leq B_x$$

$\therefore$ System is stable.

Example 1.69 Repeat example 1.66 for,

$$T\{x(n)\} = x(n) + u(n+1)$$

Solution :

(i) Linearity : $T\{x(n)\} = x(n) + u(n+1)$

$$\therefore T\{ax_1(n) + bx_2(n)\} = ax_1(n) + bx_2(n) + u(n+1)$$
$$\neq a\,T\{x_1(n)\} + b\,T\{x_2(n)\}$$

$\therefore$ System is non-linear

(ii) Time-invariance :

$$T\{x(n-n_o)\} = x(n-n_o) + u(n+1)$$
$$y(n-n_o) = x(n-n_o) + u(n-n_o+1)$$
$$\therefore\ y(n-n_o) \neq T\{x(n-n_o)\}$$

$\therefore$ System is not time-invariant

(iii) Memoryless : The output y(n) depends only on the present value of the input.
$\therefore$ Memoryless.

(iv) Causal : Since output doesnot depend on the future values of input, it is causal.

(v) Stability : If $|x(n)| \leq B_x$ then,

$$|y(n)| = |x(n) + u(n+1)|$$
$$\leq |x(n)| + |u(n+1)|$$
$$\leq B_x + |u(n+1)|$$

We know that u(n+1) is bounded

$\therefore$ System is stable.

Example 1.70 Determine whether the system,

$$y(t) = e^{x(t)}$$

is (i) Linear (ii) Time-invariant (iii) Memory (iv) Causal (v) Stable

Solution : Given : $y(t) = e^{x(t)}$

Let $y(t) = T\{x(t)\} = e^{x(t)}$

(i) Linearity : $T\{ax_1(t)+bx_2(t)\} = e^{ax_1(t)+bx_2(t)}$

$$= e^{ax_1(t)} \cdot e^{bx_2(t)}$$
$$\neq a\,T\{x_1(t)\} + b\,T\{x_2(t)\}$$

$\therefore$ System is non-linear.

(ii) Time-invariance :

$$T\{x(t-t_o)\} = e^{x(t-t_o)}$$

and $y(t-t_o) = e^{x(t-t_o)}$

$$\therefore\ y(t-t_o) = T\{x(t-t_o)\}$$

$\therefore$ System is time-invariant

(iii) Memory : $y(t) = e^{x(t)}$

The value of the output signal y(t) depends only on the present value of the input signal $x(t)$.

$\therefore$ the system is memoryless.

(iv) Causal : The output doesnot depend on the future values of the input. Therefore the system is causal.

(iv) *Causality* : Since the output depends on only present value of the input, system causal.

(v) Stability : Consider $|x(n)| \le B_x$

then $|y(n)| = |g(n)\,x(n)| = |g(n)|\,|x(n)|$

$= |g(n)|\ B_x \le \infty$

$\therefore$ The output is bounded only if g(n) is bounded.

Example 1.68 Repeat example 1.66 for,

$$T\{x(n)\} = x(-n)$$

Solution :

(i) Linearity : $T\{x(n)\} = x(-n)$

$\therefore T\{ax_1(n) + bx_2(n)\} = ax_1(-n) + bx_2(-n)$

$= a\,T\{x_1(n)\} + b\,T\{x_2(n)\}$

$\therefore$ System is linear

(ii) Time invariance :

$T\{x(n-n_o\} = x(-n-n_o)$

$y(n-n_o) = x(-(n-n_o)) = x(-n+n_o)$

$\therefore y(n-n_o) \neq T\{x(n-n_o)\}$

$\therefore$ System is not time-invariant

(iii) Memoryless : If n>0, the system is not memoryless.

(iv) Causal : We have $T\{x(n)\} = x(-n)$

Consider n=-5, then $T\{x(-5)\} = x(5)$

i.e., the value of the output signal at n=-5 depends on the value of input n=5. Therefore the output depends on the future values of the inp

$\therefore$ System is non-causal.

(v) Stability : If $|x(n)| \le B_x$ then,

$|y(n)| = |x(-n)| \le B_x$

$\therefore$ System is stable.

Example 1.69 Repeat example 1.66 for,

$$T\{x(n)\} = x(n) + u(n+1)$$

Solution :

(i) Linearity : $T\{x(n)\} = x(n) + u(n+1)$

$\therefore T\{ax_1(n) + bx_2(n)\} = ax_1(n) + bx_2(n) + u(n+1)$

$\neq a\,T\{x_1(n)\} + b\,T\{x_2(n)\}$

$\therefore$ System is non-linear

(ii) Time-invariance :

$$T\{x(n-n_o)\} = x(n-n_o) + u(n+1)$$
$$y(n-n_o) = x(n-n_o) + u(n-n_o+1)$$
$$\therefore y(n-n_o) \neq T\{x(n-n_o)\}$$

$\therefore$ System is not time-invariant

(iii) Memoryless : The output y(n) depends only on the present value of the input.
$\therefore$ Memoryless.

(iv) Causal : Since output doesnot depend on the future values of input, it is causal.

(v) Stability : If $|x(n)| \leq B_x$ then,

$$|y(n)| = |x(n) + u(n+1)|$$
$$\leq |x(n)| + |u(n+1)|$$
$$\leq B_x + |u(n+1)|$$

We know that u(n+1) is bounded

$\therefore$ System is stable.

Example 1.70 Determine whether the system,

$$y(t) = e^{x(t)}$$

is (i) Linear (ii) Time-invariant (iii) Memory (iv) Causal (v) Stable

Solution : Given : $y(t) = e^{x(t)}$

Let $y(t) = T\{x(t)\} = e^{x(t)}$

(i) Linearity : $T\{ax_1(t)+bx_2(t)\} = e^{ax_1(t)+bx_2(t)}$

$$= e^{ax_1(t)} \cdot e^{bx_2(t)}$$
$$\neq a\,T\{x_1(t)\} + b\,T\{x_2(t)\}$$

$\therefore$ System is non-linear.

(ii) Time-invariance :

$$T\{x(t-t_o)\} = e^{x(t-t_o)}$$

and $y(t-t_o) = e^{x(t-t_o)}$

$$\therefore\ y(t-t_o) = T\{x(t-t_o)\}$$

$\therefore$ System is time-invariant

(iii) Memory : $y(t) = e^{x(t)}$

The value of the output signal y(t) depends only on the present value of the input signal $x(t)$.

$\therefore$ the system is memoryless.

(iv) Causal : The output doesnot depend on the future values of the input. Therefore the system is causal.

(v) **Stability** : Let $|x(t)| \le B_x$,

then $|y(t)| = |e^{x(t)}|$

$\le B_y$

i.e., if input bounded, the output is also bounded. Therefore system is stable.

Example 1.71 Repeat example 1.70 for

$$y(t) = \frac{dx(t)}{dt}$$

Solution : Let $y(t) = T\{x(t)\} = \frac{dx(t)}{dt}$

(i) Linearity : $T\{ax_1(t)+bx_2(t)\} = \frac{d}{dt}\{ax_1(t)+bx_2(t)\}$

$$= a\,\frac{dx_1(t)}{dt} + b\,\frac{dx_2(t)}{dt}$$

$$= a\,T\{x_1(t)\} + b\,T\{x_2(t)\}$$

∴ System is linear

(ii) Time-invariance : $T\{x(t-t_o)\} = \frac{d}{dt}x(t-t_o)$

$$y(t-t_o) = \frac{d}{dt}x(t-t_o)$$

$$\therefore\ y(t-t_o) = T\{x(t-t_o)\}$$

∴ System is time-invariant

(iii) Memory : Differentiator has memory.

(iv) Causal : The output doesnot depend on the future values of the input. So it is causal.

(v) Stability : If $|x(t)| \le B_x$,

then $|y(t)| = \left|\frac{dx(t)}{dt}\right| \not\le B_y$

∴ System is unstable.

Example 1.72 Repeat example 1.70 for,

$$y(t) = x(t/2)$$

Solution : Let $y(t) = T\{x(t)\} = x(t/2)$

(i) Linearity : $T\{ax_1(t)+bx_2(t)\} = ax_1(t/2)+bx_2(t/2)$

$$= a\,T\{x_1(t)\} + b\,T\{x_2(t)\}$$

∴ System is linear.

(ii) Time-invariance :

$$T\{x(t-t_o)\} = x(\tfrac{t}{2}-t_o)$$

$$y(t-t_o) = x\left(\frac{(t-t_o)}{2}\right)$$

$$\therefore\ y(t-t_o) \neq T\{x(t-t_o)\}$$

∴ System is not time-invariant

(iii) Memoryless : The output depends on the past values of the input. For example, $y(1) = x(0.5)$. So the system has memory.

(iv) Causal : The output depends on the future values of the input. For example, $y(-1) = x(-0.5)$. So the system is non-causal.

(v) Stability : Let $|x(t)| \leq B_x$,

then $|y(t)| = |x(\tfrac{t}{2})| \leq B_x$

∴ System is stable.

Example 1.73 Repeat example 1.70 for,

$$y(t) = \cos(x(t))$$

Solution : Let $y(t) = T\{x(t)\} = \cos(x(t))$

(i) Linearity : $T\{ax_1(t)+bx_2(t)\} = \cos(ax_1(t)+bx_2(t))$

$$\neq a\,T\{x_1(t)\} + b\,T\{x_2(t)\}$$

∴ System is non-linear

(ii) Time-invariance :

$$T\{x(t-t_o)\} = \cos(x(t-t_o))$$

$$y(t-t_o) = \cos(x(t-t_o))$$

$$\therefore\ y(t-t_o) = T\{x(t-t_o)\}$$

∴ System is time-invariant

(iii) Memory : The output depends only on the present values of the input. So the system is memoryless.

(iv) Causality : The output doesnot depend on the future values of the input. So it is causal.

(v) Stability : Let $|x(t)| \leq B_x$,

then $|y(t)| = |\cos(x(t))| = 1 < \infty$

∴ System is stable.

(ii) Time-invariance :

$$T\{x(t-t_o)\} = x(\tfrac{t}{2}-t_o)$$

$$y(t-t_o) = x\left(\frac{(t-t_o)}{2}\right)$$

$$\therefore y(t-t_o) \neq T\{x(t-t_o)\}$$

$\therefore$ System is not time-invariant

(iii) Memoryless : The output depends on the past values of the input. For example, $y(1) = x(0.5)$. So the system has memory.

(iv) Causal : The output depends on the future values of the input. For example, $y(-1) = x(-0.5)$. So the system is non-causal.

(v) Stability : Let $|x(t)| \leq B_x$,

then $|y(t)| = |x(\tfrac{t}{2})| \leq B_x$

$\therefore$ System is stable.

Example 1.73 Repeat example 1.70 for,

$$y(t) = \cos(x(t))$$

Solution : Let $y(t) = T\{x(t)\} = \cos(x(t))$

(i) Linearity : $T\{ax_1(t)+bx_2(t)\} = \cos(ax_1(t)+bx_2(t))$

$$\neq a\,T\{x_1(t)\} + b\,T\{x_2(t)\}$$

$\therefore$ System is non-linear

(ii) Time-invariance :

$$T\{x(t-t_o)\} = \cos(x(t-t_o))$$

$$y(t-t_o) = \cos(x(t-t_o))$$

$$\therefore y(t-t_o) = T\{x(t-t_o)\}$$

$\therefore$ System is time-invariant

(iii) Memory : The output depends only on the present values of the input. So the system is memoryless.

(iv) Causality : The output doesnot depend on the future values of the input. So it is causal.

(v) Stability : Let $|x(t)| \leq B_x$,

then $|y(t)| = |\cos(x(t))| = 1 < \infty$

$\therefore$ System is stable.

Example 1.74 Repeat example 1.70 for,

$$y(t) = \frac{d}{dt}\{e^{-t}x(t)\}$$

Solution : Let $y(t) = T\{x(t)\} = \frac{d}{dt}\{e^{-t}x(t)\}$

(i) Linearity : $T\{ax_1(t)+bx_2(t)\} = \frac{d}{dt}\{e^{-t}(ax_1(t)+bx_2(t))\}$

$$= a\frac{d}{dt}\{e^{-t}x_1(t)\} + b\frac{d}{dt}\{e^{-t}x_2(t)\}$$

$$= a\,T\{x_1(t)\} + b\,T\{x_2(t)\}$$

$\therefore$ System is linear.

(ii) Time-invariance :

$$T\{x(t-t_o)\} = \frac{d}{dt}[e^{-t}x(t-t_o)]$$

$$y(t-t_o) = \frac{d}{dt}[e^{-(t-t_o)}x(t-t_o)]$$

$$\therefore\; y(t-t_o) \neq T\{x(t-t_o)\}$$

$\therefore$ System is not time-invariant.

(iii) Memory : System has memory.

(iv) Causality : The output does not depend on the future values of the input. So it is causal.

(v) Stability : Let $|x(t)| \le B_x$,

then $|y(t)| = \left|\frac{d\,e^{-t}}{dt}x(t)\right| \le B_y$

Therefore system is stable.

Example 1.75 Repeat example 1.70 for,

$$y(n) = 2x(n)\,u(n)$$

Solution : Let $y(n) = T\{x(n)\} = 2x(n)\,u(n)$

(i) Linearity : $T\{ax_1(n)+bx_2(n)\} = 2\,(ax_1(n)+bx_2(n))\,u(n)$

$$= 2\,ax_1(n)\,u(n) + 2\,bx_2(n)\,u(n)$$

$$= a\,T\{x_1(n)\} + b\,T\{x_2(n)\}$$

$\therefore$ System is linear.

(ii) Time-invariance :

$$T\{x(n-n_o)\} = 2\,x(n-n_o)\,u(n)$$

$$y(n-n_o) = 2x(n-n_o)\,u(n-n_o)$$

$$\therefore y(n-n_o) \neq T\{x(n-n_o)\}$$

∴ System is time-variant.

(iii) Memory : System is memoryless.

(iv) Causality : The output doesnot depend on the future values of the input. So it is causal.

(v) Stability : Let $|x(n)| \leq B_x$,

then $|y(n)| = |2x(n)\,u(n)| \leq B_x$

∴ System is stable.

Example 1.76 Repeat example 1.70 for,

$$y(n) = x(n) \sum_{k=-\infty}^{\infty} \delta(n-2k)$$

Solution :

Let $y(n) = T\{x(n)\} = x(n) \sum_{k=-\infty}^{\infty} \delta(n-2k)$

(i) Linearity : $T\{ax_1(n)+bx_2(n)\}$

$$= \{ax_1(n)+bx_2(n)\} \sum_{k=-\infty}^{\infty} \delta(n-2k)$$

$$= ax_1(n) \sum_{k=-\infty}^{\infty} \delta(n-2k) + bx_2(n) \sum_{k=-\infty}^{\infty} \delta(n-2k)$$

$$= a\,T\{x_1(n)\} + b\,T\{x_2(n)\}$$

∴ System is linear.

(ii) Time-invariance :

$$T\{x(n-n_o)\} = x(n-n_o) \sum_{k=-\infty}^{\infty} \delta(n-2k)$$

$$y(n-n_o) = x(n-n_o) \sum_{k=-\infty}^{\infty} \delta(n-n_o-2k)$$

$$\therefore y(n-n_o) \neq T\{x(n-n_o)\}$$

∴ System is time-variant.

(iii) Memory : It is memoryless.

(iv) Causality : The output doesnot depend on the future values of the input. So is causal.

(v) Stability : Let $|x(n)| \leq B_x$.

$$\text{then } |y(n)| = |x(n)\sum_{k=-\infty}^{\infty}\delta(n-2k)| = |x(n)|\,|\sum_{k=-\infty}^{\infty}\delta(n-2k)| \leq B$$

$\therefore$ System is stable.

Example 1.77 Repeat example 1.70 for,

$$y(n) = \log_{10}(|x(n)|)$$

Solution : Let $y(n) = T\{x(n)\} = \log_{10}(|x(n)|)$

(i) Linearity : $T\{ax_1(n)+bx_2(n)\} = \log_{10}\{|a\,x_1(n)+b\,x_2(n)|\}$

$$\neq a\,T\{x_1(n)\}+b\,T\{x_2(n)\}$$

$\therefore$ System is non-linear

(ii) Time Invariance : $T\{x(n-n_o)\} = \log_{10}(|x(n-n_o)|)$

$$y(n-n_o) = \log_{10}(|x(n-n_o)|)$$

$$\therefore y(n-n_o) = T\{x(n-n_o)\}$$

$\therefore$ System is time-invariant

(iii) Memory : Since the output depends only on the present value of the input, i memoryless.

(iv) Causality : The output doesnot depend on the future values of the input. S is causal.

(v) Stability : Let $|x(n)| \leq B_x$

then $|y(n)| = |\log_{10}(|x(n)|)| \leq B_y$

$\therefore$ System is stable.

Example 1.78 Repeat example 1.70 for,

$$y(n) = n\,x(n)$$

Solution : Let $y(n) = T\{x(n)\} = n\,x(n)$

(i) Linearity : $T\{ax_1(n) + bx_2(n)\} = n\{ax_1(n) + bx_2(n)\}$

$$= anx_1(n) + bnx_2(n)$$

$$= a\,T\{x_1(n)\} + b\,T\{x_2(n)\}$$

$\therefore$ System is linear.

(ii) Time Invariance

$$T\{x(n-n_o)\} = n\,x(n-n_o)$$
$$y(n-n_o) = (n-n_o)\,x(n-n_o)$$
$$\therefore\ y(n-n_o) \neq T\{x(n-n_o)\}$$

$\therefore$ Time-Variant

(iii) Memory : Since the output depends only on the present value of input, it is memoryless.

(iv) Causality : The output doesnot depend on the future values of the input. So it is causal.

(v) Stability : Let $|x(n)| \leq B_x$

then $|y(n)| = |nx(n)|$

$= |n|\,|x(n)| = |n|\,B_x$

Since 'n' is not bounded, system is unstable.

Example 1.79 Repeat example 1.70 for,

$$y(n) = \sum_{k=-\infty}^{n} x(k+2)$$

Solution :

(i) Linearity : Let $y(n) = T\{x(n)\} = \sum_{k=-\infty}^{n} x(k+2)$

$$T\{ax_1(n)+bx_2(n)\} = \sum_{k=-\infty}^{n} \{ax_1(k+2)+bx_2(k+2)\}$$

$$= a\sum_{k=-\infty}^{n} x_1(k+2) + b\sum_{k=-\infty}^{n} x_2(k+2)$$

$$= a\,T\{x_1(n)\} + b\,T\{x_2(n)\}$$

$\therefore$ System is linear

(ii) Time Invariance :

$$T\{x(n-n_o)\} = \sum_{k=-\infty}^{n} x(k+2-n_o)$$

$$y(n-n_o) = \sum_{k=-\infty}^{n-n_o} x(\ldots 2) = \sum_{k=-\infty}^{n} x(k+2-n_o)$$

$\therefore$ y(n–n$_o$) = T...

$\therefore$ System is ...