Unit - 3

DIR - I
DIR II (Canonic)
Cascade
Parallel
FIR Linear
FIR Lattice
IIR Lattice

*) DIR I

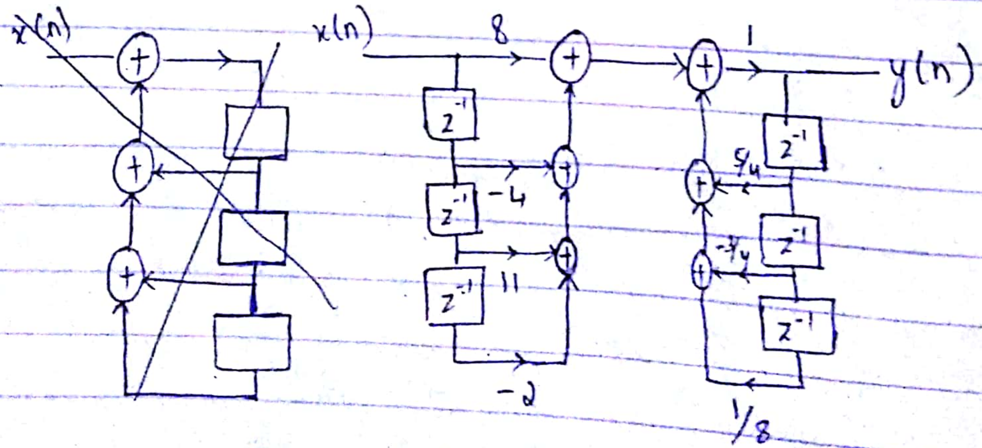
1) $H(z) = \dfrac{8z^3 - 4z^2 + 11z - 2}{(z - 1/4)(z^2 - z + 1/2)}$

$$= \frac{8z^3 - 4z^2 + 11z - 2}{z^3 - z^2 + z/2 - 1/4 z^2 + 1/4 z - 1/8}$$

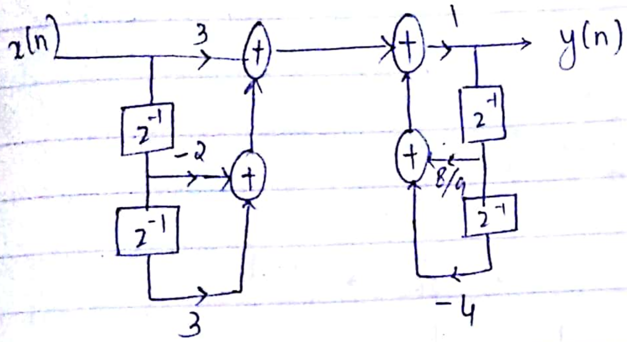
$$= \frac{8z^3 - 4z^2 + 11z - 2}{z^3 - 5/4 z^2 + 3/4 z - 1/8}$$

$$H(z) = \frac{8 - 4z^{-1} + 11z^{-2} - 2z^{-3}}{1 - \frac{5}{4}z^{-1} + 3/4 z^{-2} - 1/8 z^{-3}}$$

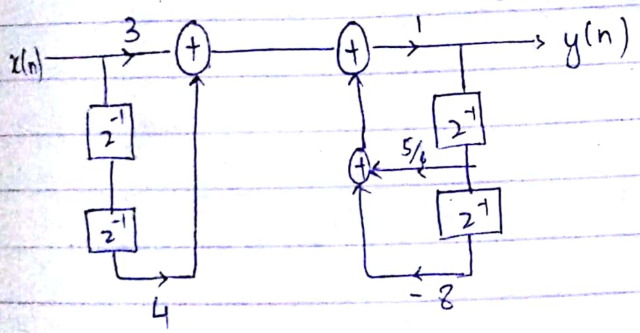
Direct Form I. (DIR I)



2) $H(z) = \dfrac{3 - 2z^{-1} + 3z^{-2}}{1 - \frac{8}{9}z^{-1} + 4z^{-2}}$



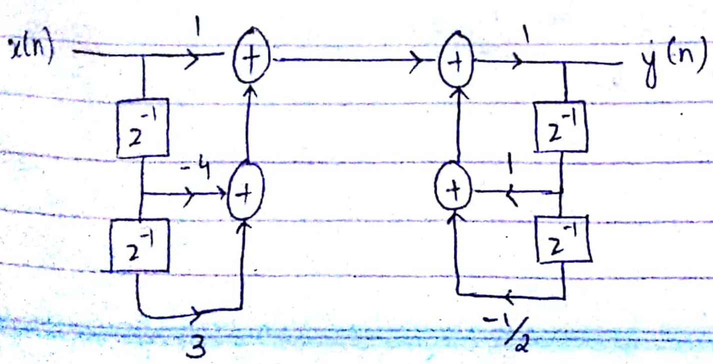
3) $H(z) = \dfrac{3 + 4z^{-2}}{1 - \frac{5}{6}z^{-1} + 8z^{-2}}$



4) $H(z) = \dfrac{(z-1)(z-3)}{\left(z - \left(\frac{1}{2} + j\frac{1}{2}\right)\right)\left(z - \left(\frac{1}{2} - j\frac{1}{2}\right)\right)}$

$$= \frac{z^2 - 4z + 3}{z^2 - 2 \times \frac{1}{2} z + \frac{1}{4} + \frac{1}{4}}$$

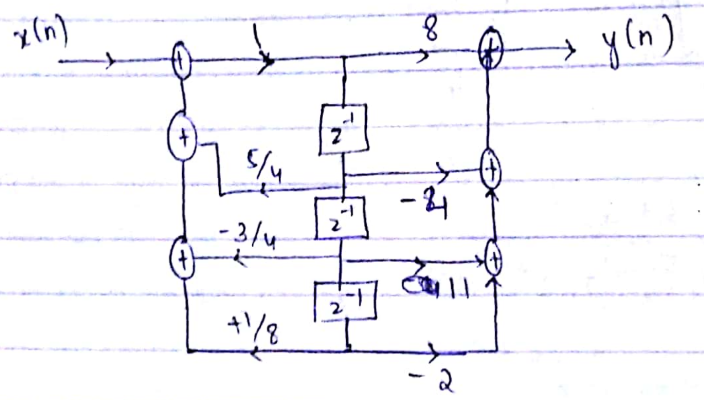
$$= \frac{1 - 4z^{-1} + 3z^{-2}}{1 - z^{-1} + \frac{1}{2}z^{-2}}$$



*) Direct form 2 (DIR 2 or Canonic)

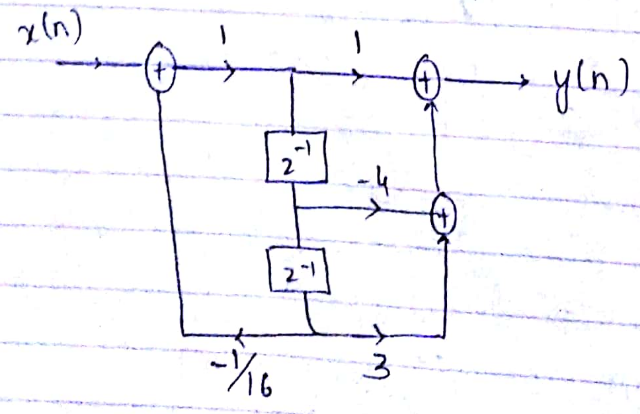
$$H(z) = \frac{8z^3 - 4z^2 + 11z - 2}{(z - 1/4)(z^2 - z + 1/2)}$$

$$H(z) = \frac{8 - 4z^{-1} + 11z^{-2} - 2z^{-3}}{1 - \frac{5}{4}z^{-1} + \frac{3}{4}z^{-2} - \frac{1}{8}z^{-3}}$$



$$H(z) = \frac{(z-1)(z-3)}{(z - j\,1/4)(z + j\,1/4)} = \frac{z^2 - 4z + 3}{z^2 + 1/16}$$

$$= \frac{1 - 4z^{-1} + 3z^{-2}}{1 + \frac{1}{16}z^{-2}}$$



Scanned by CamScanner

$$H(z) = \frac{(z-1)(z-2)(z+1)z}{[z-(\frac{1}{2}+j\frac{1}{2})][z-(\frac{1}{2}-j\frac{1}{2})](z-j\frac{1}{4})(z+j\frac{1}{4})}$$

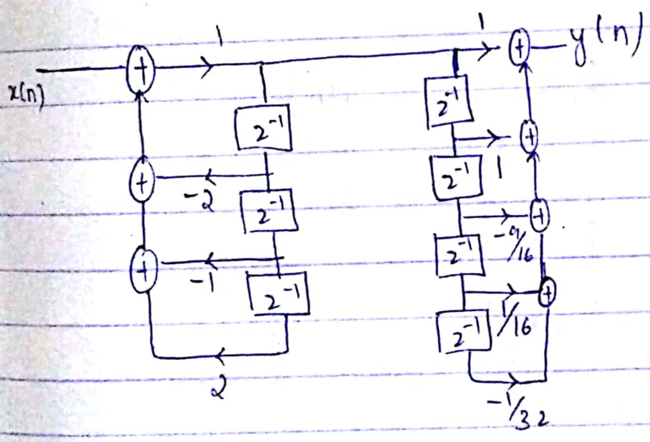
$$H(z) = \frac{(z^2-1)(z^2-2z)}{(z^2-z+\frac{1}{2})(z^2+\frac{1}{16})}$$

$$= \frac{z^4-2z^3-z^2+2z}{z^4+\frac{1}{16}z^2-z^3-\frac{1}{16}z+\frac{1}{2}z^2+\frac{1}{32}}$$

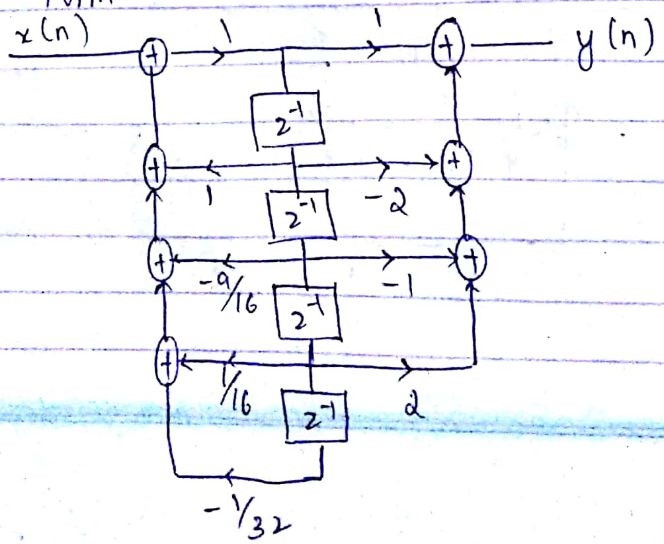
$$= \frac{z^4-2z^3-z^2+2z}{z^4-z^3+\frac{9}{16}z^2-\frac{1}{16}z+\frac{1}{32}}$$

$$H(z) = \frac{1-2z^{-1}-z^{-2}+2z^{-3}}{1-z^{-1}+\frac{9}{16}z^{-2}-\frac{1}{16}z^{-3}+\frac{1}{32}z^{-4}}$$

DIR Form I



DIR II Form



$$H(z) = 3 + \frac{4z}{z-1/2} + \frac{2}{z-1/4}$$

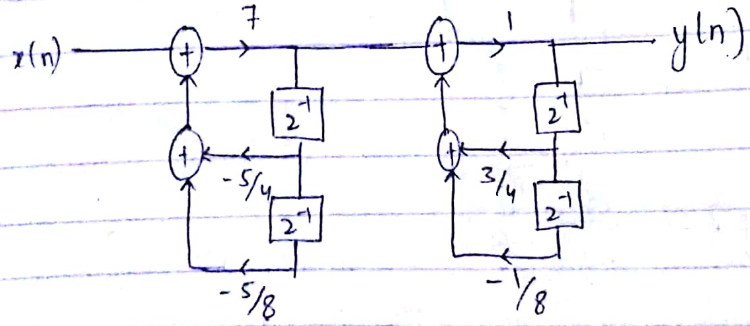
$$= \frac{3(z-1/2)(z-1/4) + 4z(z-1/4) + 2(z-1/2)}{(z-1/2)(z-1/4)}$$

$$= \frac{3(z^2-3/4\,z+1/8) + 4z^2 - z + 2z - 1}{z^2 - 3/4\,z + 1/8}$$

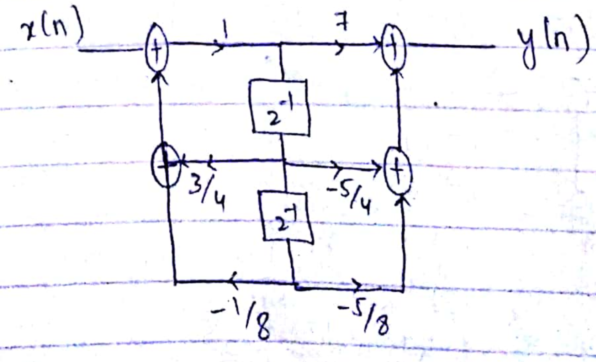
$$= \frac{7z^2 - 5/4\,z - 5/8}{z^2 - 3/4\,z + 1/8}$$

$$H(z) = \frac{7 - 5/4\,z^{-1} - 5/8\,z^{-2}}{1 - 3/4\,z^{-1} + 1/8\,z^{-2}}$$

DIR I



DIR II

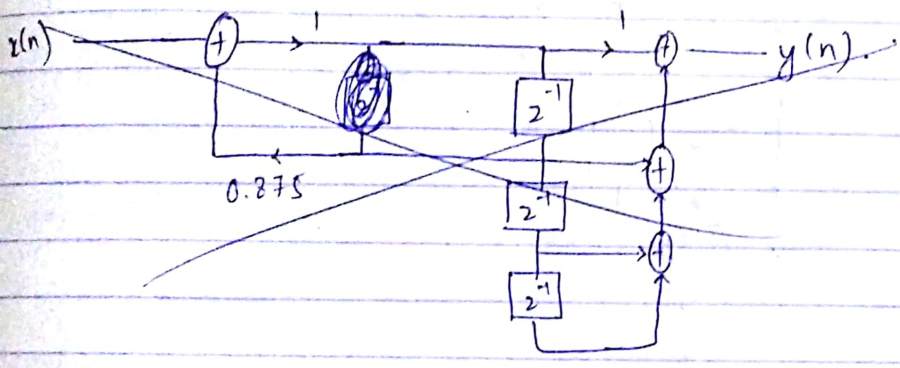


$$H(z) = \frac{1+0.875z^{-1}}{(1+0.2z^{-1}+0.9z^{-2})(1-0.7z^{-1})}$$

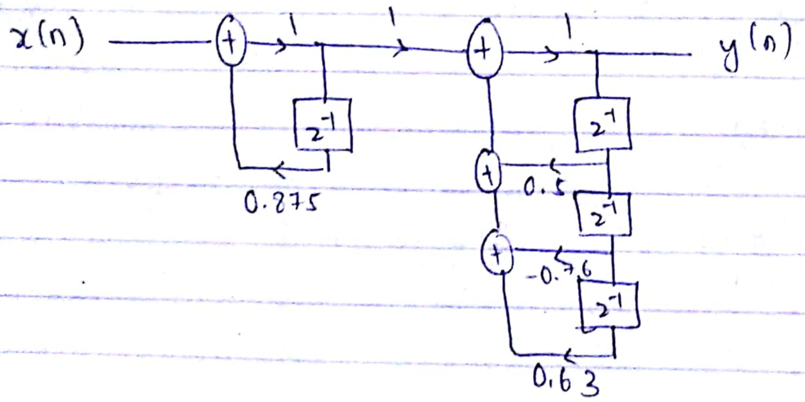
$$= \frac{1+0.875z^{-1}}{(1-0.7z^{-1}+0.2z^{-1}-0.14z^{-2}+0.9z^{-2}-0.63z^{-3})}$$

$$H(z) = \frac{1+0.875z^{-1}}{1-0.5z^{-1}+0.76z^{-2}-0.63z^{-3}}$$

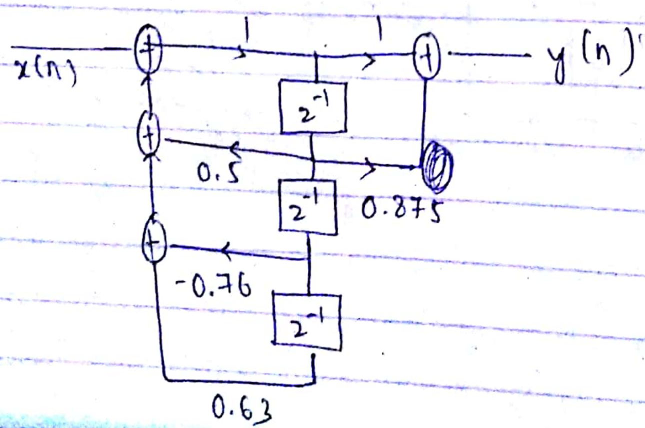
~~DIR I~~



DIR I

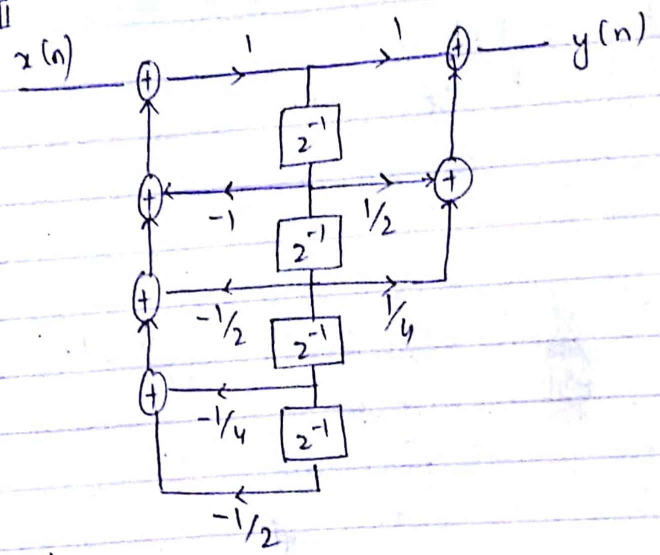


DIR II



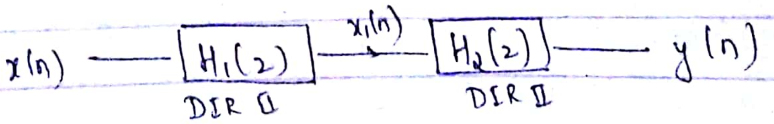
$$H(z) = \frac{1 + \frac{1}{2} z^{-1} + \frac{1}{4} z^{-2}}{1 + z^{-1} + \frac{1}{2} z^{-2} + \frac{1}{4} z^{-3} + \frac{1}{2} z^{-4}}$$

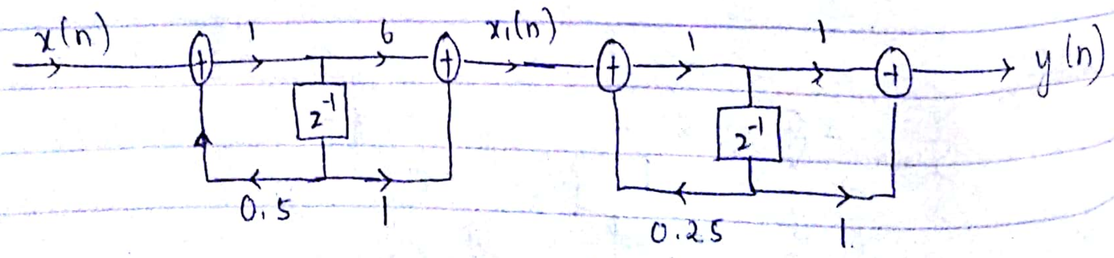
DIR II



*) CASCADE

1) $$H(z) = \underbrace{\frac{(6+z^{-1})}{(1-0.5z^{-1})}}_{H_1(z)} \underbrace{\frac{(1+z^{-1})}{(1-0.25z^{-1})}}_{H_2(z)}$$





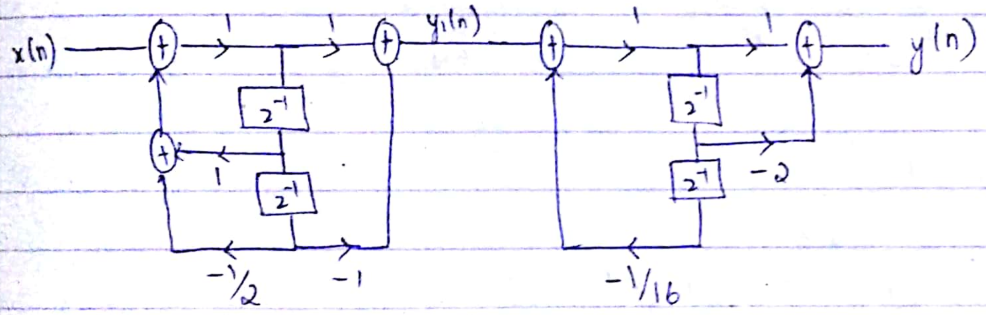
2) Cascade

$$H(z) = \frac{(z-1)(z-2)(z+1)z}{\left[z-\left(\frac{1}{2}+j/2\right)\right]\left[z-\left(\frac{1}{2}-j/2\right)\right]\left(z-j/4\right)\left(z+j/4\right)}$$

$$H_1(z) = \frac{(z^2-1)}{z^2-z+1/2} \qquad H_2(z) = \frac{z^2-2z}{z^2+1/16}$$

$$H_1(z) = \frac{1-z^{-2}}{1-z^{-1}+1/2z^{-1}} \qquad H_2(z) = \frac{1-2z^{-1}}{1+1/16z^{-2}}$$

$$x(n) \longrightarrow \boxed{H_1(z)} \xrightarrow{y_1(n)} \boxed{H_2(z)} \longrightarrow y(n)$$

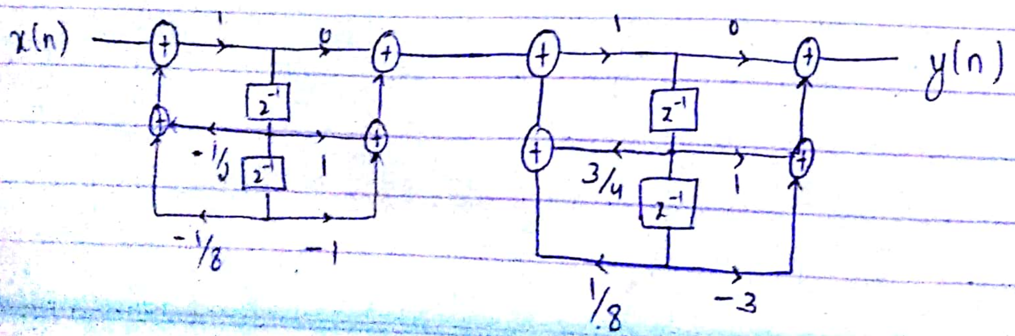


3) Cascade

$$H(z) = \frac{(z-1)(z-3)}{\left[z-\left(\frac{1}{4}+j/4\right)\right]\left[z-\left(\frac{1}{4}-j/4\right)\right]\left(z-j/2\right)\left(z-j/4\right)}$$

$$H_1(z) = \frac{z-1}{z^2-z/2+1/8} \qquad H_2(z) = \frac{z-3}{z^2-zj/4-zj/2-1/8}$$

$$H_1(z) = \frac{z^{-1}-z^{-2}}{1-z^{-1}/2+\frac{1}{8}z^{-2}} \qquad H_2(z) = \frac{z^{-1}-3z^{-2}}{1-\frac{3}{4}jz^{-1}-1/8z^{-2}}$$

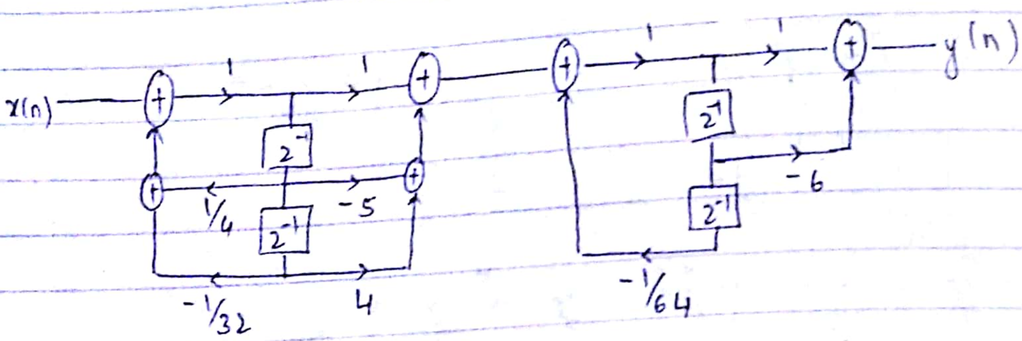


4) Cascade

$$H(z) = \frac{(z-1)(z-4)(z-6)z}{[z-(\frac{1}{8}+j\frac{1}{8})][z-(\frac{1}{8}-j\frac{1}{8})](z-j/8)(z+j/8)}$$

Let $H_1(z) = \frac{\cancel{(z-1)(z-4)}\, z^2-5z+4}{z^2 - \frac{1}{4}z + \frac{1}{32}}$ $\qquad H_2(z) = \frac{z^2-6z}{(z^2+\frac{1}{64})}$

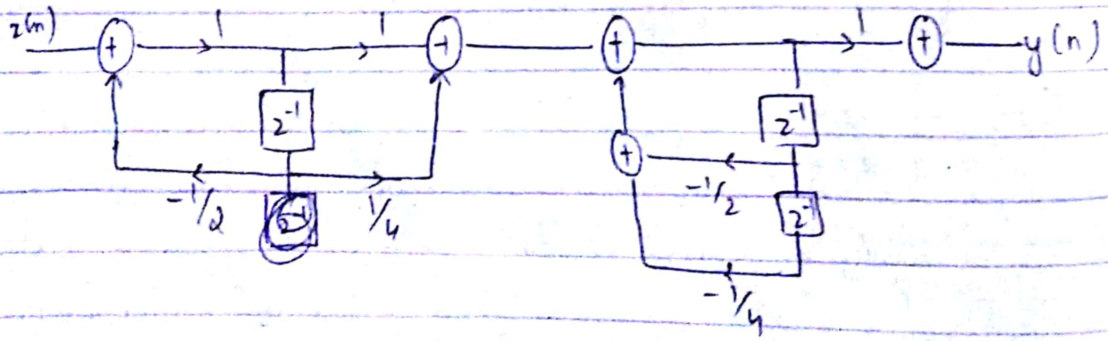
$$H_1(z) = \frac{1-5z^{-1}+4z^{-2}}{1-\frac{1}{4}z^{-1}+\frac{1}{32}z^{-2}} \qquad H_2(z) = \frac{1-6z^{-1}}{1+\frac{1}{64}z^{-2}}$$



5) Cascade

$$H(z) \quad \frac{1+\frac{1}{4}z^{-1}}{(1+\frac{1}{2}z^{-1})(1+\frac{1}{2}z^{-1}+\frac{1}{4}z^{-2})}$$

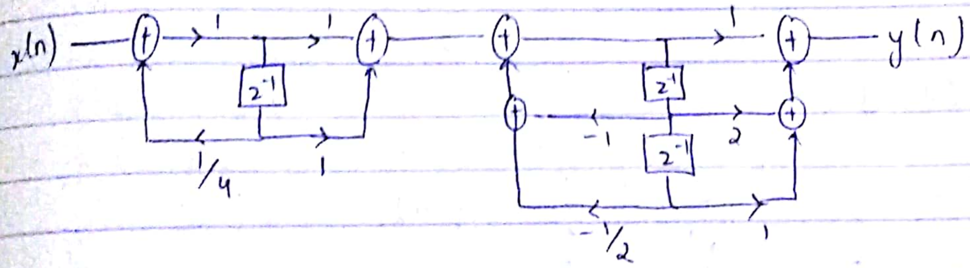
$$H_1(z) = \frac{1+\frac{1}{4}z^{-1}}{(1+\frac{1}{2}z^{-1})} \qquad H_2(z) = \frac{1}{(1+\frac{1}{2}z^{-1}+\frac{1}{4}z^{-2})}$$



6) Cascade

$$H(z) = \frac{(1+z^{-1})^3}{(1-\frac{1}{4}z^{-1})(1+z^{-1}+\frac{1}{2}z^{-2})}$$

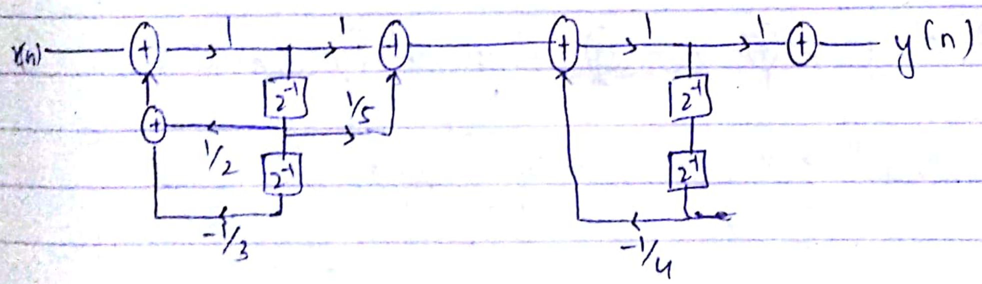
$$H_1(z) = \frac{(1+z^{-1})}{(1-\frac{1}{4}z^{-1})} \qquad H_2(z) = \frac{1+2z^{-1}+z^{-2}}{(1+z^{-1}+\frac{1}{2}z^{-2})}$$



7) Cascade

$$H(z) = \underbrace{\frac{1+\frac{1}{5}z^{-1}}{(1-\frac{1}{2}z^{-1}+\frac{1}{3}z^{-2})}}_{H_1(z)}\underbrace{\frac{1}{(1+\frac{1}{4}z^{-2})}}_{H_2(z)}$$

$\cancel{H(z)}$



8) ~~Cascade~~ a) DIR I b) DIR II c) Cascade

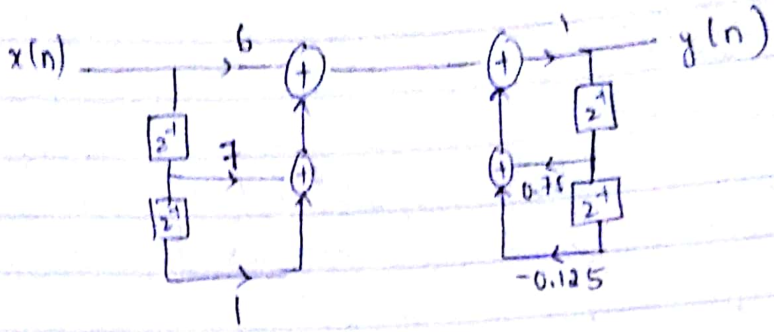
$$y(n) = 0.75\,y(n-1) - 0.125\,y(n-2) + 6\,x(n) + 7\,x(n-1) + x(n-2)$$

Taking z- transform

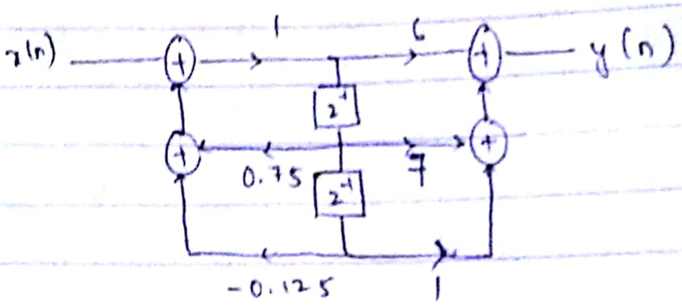
$$Y(z) = 0.75\,z^{-1}Y(z) - 0.125\,z^{-2}Y(z) + 6X(z) + 7z^{-1}X(z) + z^{-2}X(z)$$

$$\frac{Y(z)}{X(z)} = \frac{6+7z^{-1}+z^{-2}}{1-0.75z^{-1}+0.125z^{-2}}$$

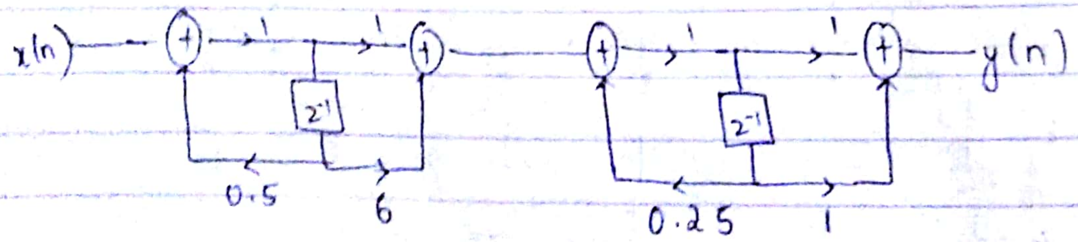
a) DIR 1



b) DIR II



c) $H(z) = \dfrac{(6+z^{-1})(1+z^{-1})}{(1-0.5z^{-1})(1-0.25z^{-1})}$



$a+b=0.5$
ab

$a=0.1$
$b=0.5$

$0.5 \quad 0.25$
$a+b=0.5$
$ab=0.25$
$\Rightarrow 0$
$a+\frac{0.25}{a}=0.5$
$1.25a=$

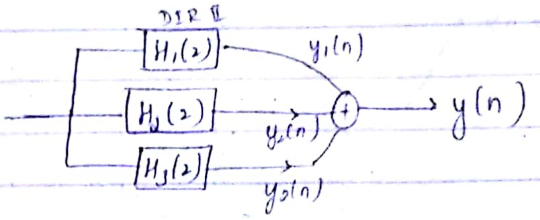
8) Parallel realization

$$H(z)=\frac{1+\frac{1}{4}z^{-1}}{(1+\frac{1}{2})(1+\frac{1}{2}z^{-1}+\frac{1}{4}z^{-2})}$$

$$H(z)=\frac{C_1}{(1+\frac{1}{2}z^{-1})}+\frac{C_2}{1+\frac{1}{2}z^{-1}+\frac{1}{4}z^{-2}}$$

$$1+\frac{1}{4}z^{-1}=C_1\left(1+\frac{1}{2}z^{-1}+\frac{1}{4}z^{-2}\right)+C_2\left(1+\frac{1}{2}z^{-1}\right)$$

→ Summation form
(Partial fraction.



$$H(z)=\frac{1+\frac{1}{4}z^{-1}}{(1+\frac{1}{2}z^{-1})(1+\frac{1}{2}z^{-1}+\frac{1}{4}z^{-2})}$$

$$H(z)=\frac{A}{1+\frac{1}{2}z^{-1}}+\frac{Bz^{-1}+C}{(1+\frac{1}{2}z^{-1}+\frac{1}{4}z^{-2})}$$

$$1+\frac{1}{4}z^{-1}=A\left(1+\frac{1}{2}z^{-1}+\frac{1}{4}z^{-2}\right)+(Bz^{-1}+C)(1+\frac{1}{2}z^{-1})$$
$$(Bz^{-1}+\tfrac{B}{2}z^{-2}+C+\tfrac{C}{2}z^{-1})$$

$\Rightarrow 1=A+C.$

$\frac{1}{4}=\frac{A}{2}+\frac{B}{2}+\frac{C}{2}$

$0=\frac{A}{4}+\frac{B}{2}$ $\Rightarrow$ $\frac{A}{4}=-\frac{B}{2}$ $\Rightarrow A=-2B.$

$\frac{1}{4}=-B+\frac{B}{2}+\frac{C}{2}$ $\Rightarrow$ $-\frac{B}{2}+\frac{C}{2}=\frac{1}{4}$

$\Rightarrow -B+C=\frac{1}{2}$

$-2B+C=1$

$B=-\frac{1}{2}$

$A=1$

$$1+\frac{1}{4}z^{-1}=A\left(1+\frac{1}{2}z^{-1}+\frac{1}{4}z^{-2}\right)+\left(Bz^{-1}+\tfrac{B}{2}z^{-2}+C+\tfrac{C}{2}z^{-1}\right)$$

$1=A+C.$ $\Rightarrow$

$\frac{1}{4}=\frac{A}{2}+\frac{B}{2}+\frac{C}{2}.$ $\Rightarrow$

$0=\frac{A}{4}+\frac{B}{2}$ $\Rightarrow$ $\frac{A}{4}=-\frac{B}{2}\Rightarrow A=-2B.$

$-2B+C=1$ $\qquad \frac{1}{4}=\frac{-2B}{2}+\frac{B}{2}+\frac{C}{2}\Rightarrow -\frac{B}{2}+\frac{C}{2}=\frac{1}{4}$

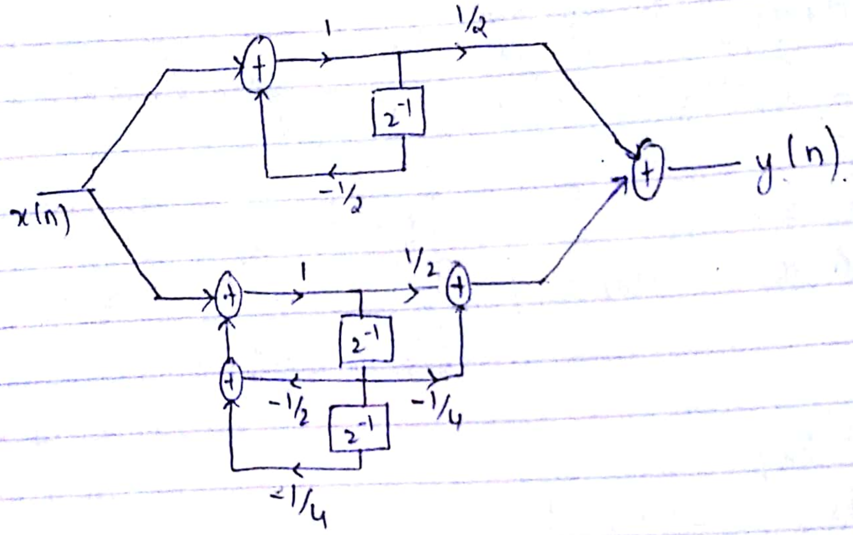
$-B+C=\frac{1}{2}$
$(-)\quad(-)\quad(-)$

$-B=\frac{1}{2}\Rightarrow$

Partial fractions

$$H(z) = \frac{1/2}{(1+1/2\, z^{-1})} + \frac{\frac{1}{2} - 1/4\, z^{-1}}{(1+1/2\, z^{-1} + 1/4\, z^{-2})}$$

$$\qquad H_1(z) \qquad\qquad\qquad H_2(z)$$



$$H(z) = \frac{(1+z^{-1})(1+2z^{-1})}{(1+1/2\, z^{-1})(1-1/2\, z^{-1})(1+1/8\, z^{-1})}$$

$$H(z) = \frac{A}{1+1/2\, z^{-1}} + \frac{B}{1-1/2\, z^{-1}} + \frac{C}{1+1/8\, z^{-1}}$$

$$(1+z^{-1})(1+2z^{-1}) = A(1-1/2\, z^{-1})(1+1/8\, z^{-1}) + B(1+1/2\, z^{-1})(1+1/8\, z^{-1}) + C(1+1/2\, z^{-1})(1-$$

$$1+3z^{-1}+2z^{-2} = A\left[1 - 3/8\, z^{-1} - 1/16\, z^{-2}\right] + B\left[1 + 5/8\, z^{-1} + 1/16\, z^{-2}\right] + C\left[1 - 1/4\, z^{-2}\right]$$

$-\frac{1}{2} = 4 \quad \frac{2}{3} \quad \frac{4}{3}$

$\Rightarrow z = 1/4 \quad z^{-1} = 1 \quad \frac{1}{2} = 1$

$z^2 = 1/16$

$$H(z) = \frac{(1+z^{-1})(1+2z^{-1})}{(1+\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})(1+\frac{1}{8}z^{-1})}$$

$$1+3z^{-1}+2z^{-2} = A(1-\tfrac{1}{4}z^{-1})(1+\tfrac{1}{8}z^{-1}) + B(1+\tfrac{1}{2}z^{-1})(1+\tfrac{1}{8}z^{-1}) + C[(1+\tfrac{1}{2}z^{-1})(1-\tfrac{1}{4}z^{-1})]$$

$z = 4 \Rightarrow 1+ \quad 1+12+32 = B(3)\tfrac{3}{2} \Rightarrow (\tfrac{9B}{2} = 45)$?

$$1+3z^{-1}+2z^{-2} = A[1-\tfrac{1}{8}z^{-1}+\tfrac{1}{32}z^{-2}] + B[1+\tfrac{5}{8}z^{-1}+\tfrac{1}{16}z^{-2}] + C[1+\tfrac{1}{4}z^{-1}-\tfrac{1}{8}z^{-2}]$$

$1 = A + B + C$

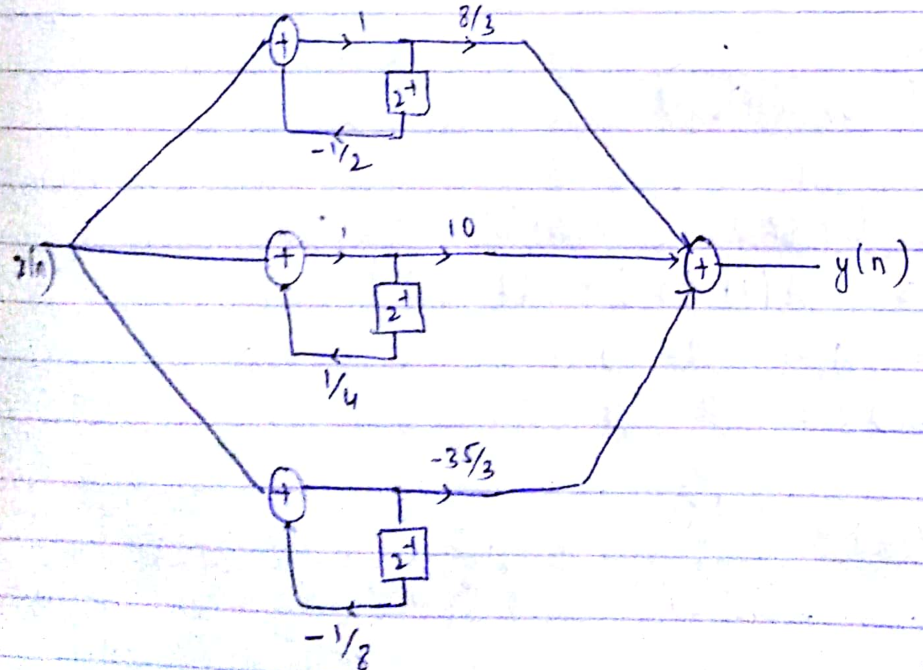
$3 = -\frac{1}{8}A + \frac{5}{8}B + \frac{1}{4}C$

$2 = \frac{1}{32}A + \frac{1}{16}B + -\frac{1}{8}C$

$A = 8/3 \qquad B = 10 \qquad C = -35/3$

~~$A = 2.1818 \; 23/11 \quad B = 9\,17/33 \quad C = -10\,23/33$~~

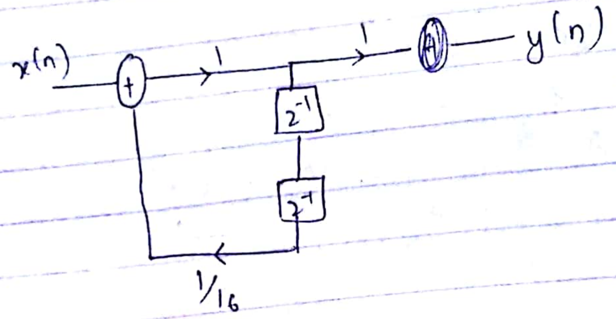
$$H(z) = \frac{8/3}{1+\frac{1}{2}z^{-1}} + \frac{10}{1-\frac{1}{4}z^{-1}} + \frac{-35/3}{1+\frac{1}{8}z^{-1}}$$

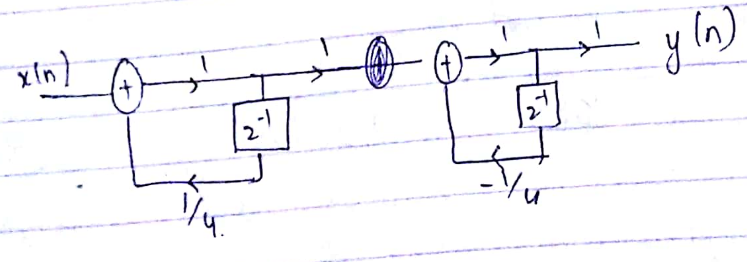


2) $y(n) = \frac{1}{16}y(n-2) + x(n)$

$Y(z) = \frac{1}{16} z^{-2} Y(z) + X(z)$

$\therefore \; H(z) = \frac{1}{1-\frac{1}{16}z^{-2}} = \frac{1}{(1-\frac{1}{4}z^{-2})(1+\frac{1}{4}z^{-1})}$





$H(z) = \frac{1}{(1-\frac{1}{4}z^{-1})(1+\frac{1}{4}z^{-1})}$

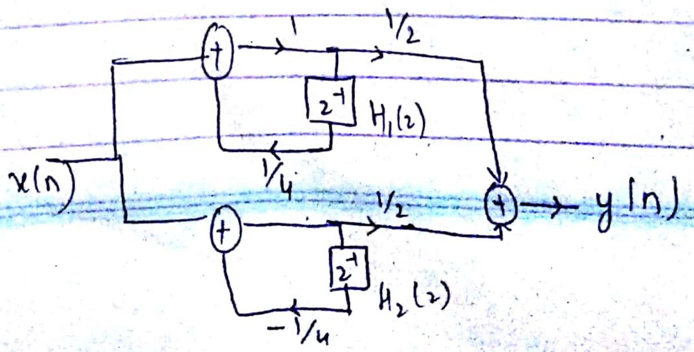
$= \frac{A}{(1-\frac{1}{4}z^{-1})} + \frac{B}{1+\frac{1}{4}z^{-1}}$

$\Rightarrow \; 1 = A(1+\frac{1}{4}z^{-1}) + B(1-\frac{1}{4}z^{-1})$

$1 = 2A \Rightarrow \; A = \frac{1}{2}$

$1 = 2B = \; B = \frac{1}{2}$

$\therefore \; H(z) = \frac{\frac{1}{2}}{(1-\frac{1}{4}z^{-1})} + \frac{\frac{1}{2}}{(1+\frac{1}{4}z^{-1})}$



*) FIR Linear Phase realization

1) $h(n) = \left(\frac{1}{2}\right)^n [u(n) - u(n-4)]$. Realise FIR linear phase realisation

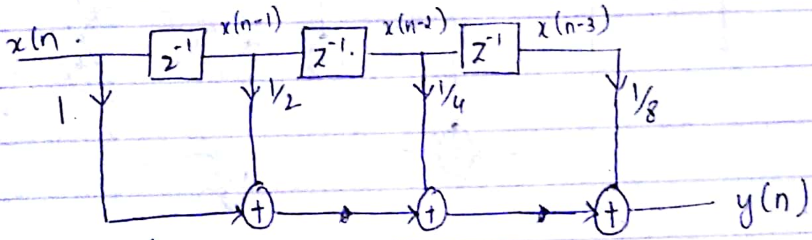
$h\delta(n) = [\delta(n) + \frac{1}{2}\delta(n-1) + \frac{1}{4}\delta(n-2) + \frac{1}{8}\delta(n-3)]$

$H(z) = 1 + \frac{1}{2}z^{-1} + \frac{1}{4}z^{-2} + \frac{1}{8}z^{-3}$

$\Rightarrow Y(z) = X(z)\left[1 + \frac{1}{2}z^{-1} + \frac{1}{4}z^{-2} + \frac{1}{8}z^{-3}\right]$

$y(n) = x(n) + \frac{1}{2}x(n-1) + \frac{1}{4}x(n-2) + \frac{1}{8}x(n-3)$

FIR DIR I

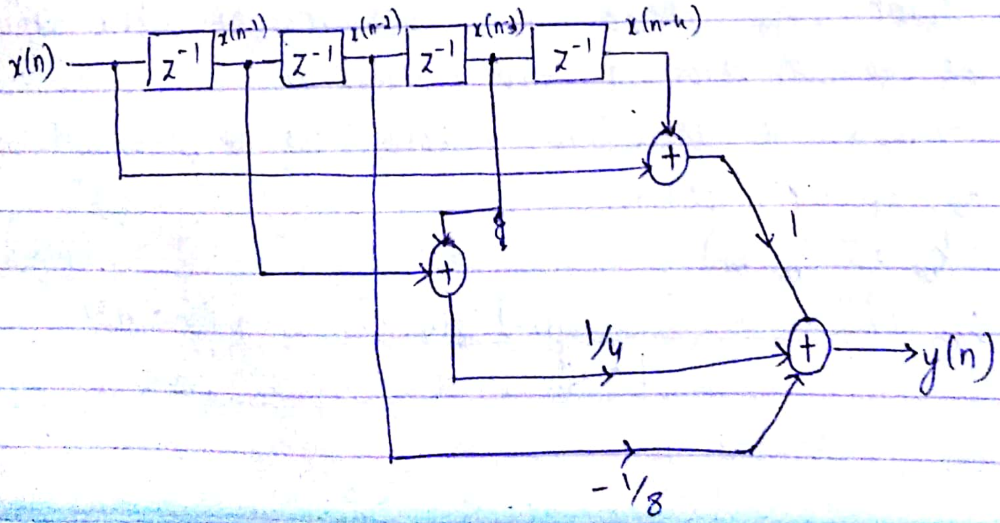


2) $h(n) = \delta(n) + \frac{1}{4}\delta(n-1) - \frac{1}{8}\delta(n-2) + \frac{1}{4}\delta(n-3) + \delta(n-4)$

$H(z) = 1 + \frac{1}{4}z^{-1} - \frac{1}{8}z^{-2} + \frac{1}{4}z^{-3} + z^{-4}$

$Y(z) = X(z)\left[1 + \frac{1}{4}z^{-1} - \frac{1}{8}z^{-2} + \frac{1}{4}z^{-3} + z^{-4}\right]$

$y(n) = x(n) + x(n-4) + \frac{1}{4}[x(n-1) + x(n-3)] - \frac{1}{8}x(n-2)$



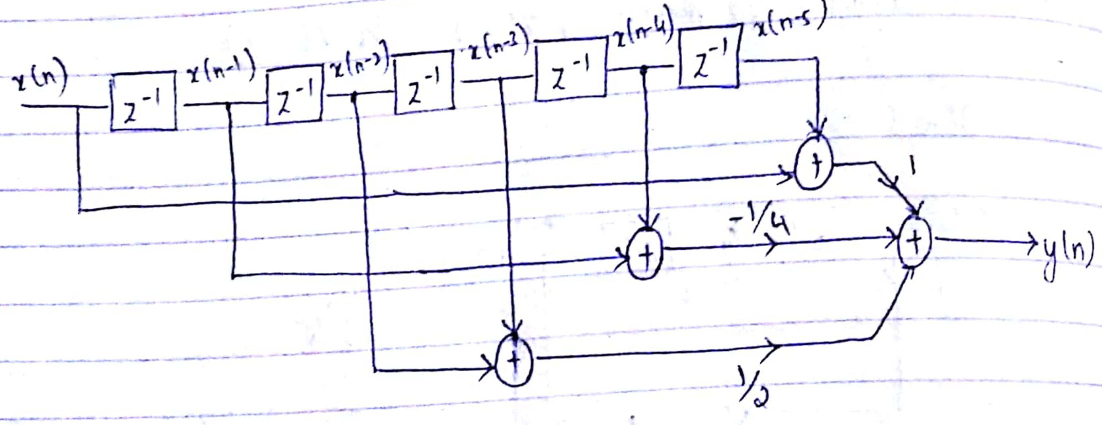
3) Realise Linear phase FIR filter given ~~for~~

$$h(n) = \delta(n) - \tfrac{1}{4}\,\delta(n-1) + \tfrac{1}{2}\,\delta(n-2) + \tfrac{1}{2}\,\delta(n-3) - \tfrac{1}{4}\,\delta(n-4) + \delta(n-5)$$

$$H(z) = 1 - \tfrac{1}{4}(z^{-1} + z^{-4}) + \tfrac{1}{2}\left[z^{-2} + z^{-3}\right] + z^{-5}$$

$$Y(z) = X(z)\left[1 + z^{-5} + \tfrac{1}{2}\left[z^{-2} + z^{-3}\right] - \tfrac{1}{4}(z^{-1} + z^{-4})\right]$$

$$y(n) = x(n) + x(n-5) - \tfrac{1}{4}\left[x(n-1) + x(n-4)\right] + \tfrac{1}{2}\left[x(n-2) + x(n-3)\right]$$



*) FIR Lattice Structure.

FIR Lattice structure is obtained by realising for all zero filter inserting appropriate hardware units. The hardware units are delay elements, multipliers & adders. The design platform is obtained to find the lattice structure coefficients (multiplier coefficients). These coefficients are given by symbol k_m where m is an integer which depends on the no. of zeros in the transfer fn.

For example: if there are 3 zeros, the designer will find k_1, k_2, k_3 coefficients.

$$k_m = a_m(m)$$

$$a_{m-1}(i) = \frac{a_m(i) - a_m(m)\,a_m(m-i)}{1 - k_m^2} \qquad 1 \le i \le m-1$$

1) Given FIR filter with following difference eqn

$y(n) = x(n) + 3.1\,x(n-1) + 5.5\,x(n-2) + 4.2\,x(n-3) + 2.3\,x(n-4)$

Sketch the FIR lattice realization of filter

$\rightarrow\; Y(z) = X(z)\left[1 + 3.1z^{-1} + 5.5z^{-2} + 4.2z^{-3} + 2.3z^{-4}\right]$

$H(z) = 1 + 3.1z^{-1} + 5.5z^{-2} + 4.2z^{-3} + 2.3z^{-4}$

$M = 4$

$k_m = a_m(m)$

$a_{m-1}(i) = \dfrac{a_m(i) - a_m(m)\,a_m(m-i)}{1 - k_m^2} \qquad 1 \le i \le m-1$

For $m = 4$, $\quad k_4 = a_4(4) = 2.3$

$a_3(i) = \dfrac{a_4(i) - a_4(4)\,a_4(4-i)}{1 - k_4^2} \qquad 1 \le i \le 3.$

$i = 1,\quad a_3(1) = \dfrac{a_4(1) - a_4(4)\,a_4(3)}{1 - k_4^2} = \dfrac{3.1 - 2.3 \times 4.2}{1 - (2.3)^2} = 1.529.$

$i = 2,\quad a_3(2) = \dfrac{a_4(2) - a_4(4)\,a_4(2)}{1 - (2.3)^2} = \dfrac{5.5 - 2.3 \times 5.5}{1 - (2.3)^2} = 1.667$

$i = 3\quad a_3(3) = \dfrac{a_4(3) - a_4(4)\,a_4(1)}{1 - (2.3)^2} = \dfrac{4.2 - 2.3 \times 3.1}{1 - (2.3)^2} = 0.683.$

For $m = 3$

$k_3 = a_3(3) = 0.683.$

$i = 1,\quad a_2(1) = \dfrac{a_3(1) - a_3(3)\,a_3(2)}{1 - k_3^2} = \dfrac{1.529 - 0.683 \times 1.667}{1 - 0.683^2} = 0.732$

$i = 2,\quad a_2(2) = \dfrac{a_3(2) - a_3(3)\,a_3(1)}{1 - k_3^2} = \dfrac{1.667 - 0.683 \times 1.529}{1 - 0.683^2} = 1.167$

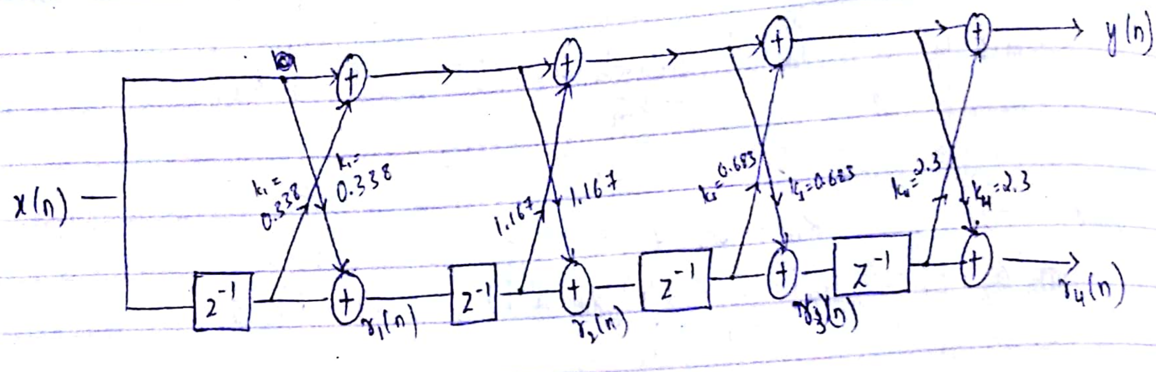
For $m = 2$

$$k_2 = a_2(2) = 1.167$$

$$i=1,\quad a_1(1) = \frac{a_2(1) - a_2(2)\, a_2(1)}{1 - k_2^2} = \frac{0.732 - 1.167 \times 0.732}{1 - 1.167^2} = 0.33$$

For $m = 1$

$$k_1 = a_1(1) = 0.338$$



2) $y(n) = x(n) + 2.4\,x(n-1) + 3.6\,x(n-2) + 2.9\,x(n-3) + 4.1\,x(n-4)$

$$Y(z) = X(z)\left[1 + 2.4z^{-1} + 3.6z^{-2} + 2.9z^{-3} + 4.1z^{-4}\right]$$

$$H(z) = \frac{Y(z)}{X(z)} = 1 + 2.4z^{-1} + 3.6z^{-2} + 2.9z^{-3} + 4.1z^{-4}$$

$M = 4$

For $m = 4$

$$k_4 = a_4(4) = 4.1$$

$$a_3(i) = \frac{a_4(i) - a_4(4)\, a_4(4-i)}{1 - k_4^2} \qquad 1 \le i \le 3$$

$$i=1,\quad a_3(1) = \frac{a_4(1) - k_4\, a_4(3)}{1 - k_4^2} = \frac{2.4 - 4.1 \times 2.9}{1 - 4.1^2} = 0.600$$

$$i=2\quad a_3(2) = \frac{a_4(2) - k_4\, a_4(2)}{1 - k_4^2} = \frac{3.6 - 4.1 \times 3.6}{1 - 4.1^2} = 0.706$$

$$i=3\quad a_3(3) = \frac{a_4(3) - k_4\, a_4(1)}{1 - k_4^2} = \frac{2.9 - 4.1 \times 2.4}{1 - 4.1^2} = 0.439$$

For $m = 3$

$k_3 = a_3(3) = 0.439$

$a_2(i) = \dfrac{a_3(i) - a_3(3)a_3(3-i)}{1-k_3^2} \qquad 1 \le i \le 2$

$i=1, \quad a_2(1) = \dfrac{a_3(1) - k_3 a_3(2)}{1-k_3^2} = \dfrac{0.6 - 0.439 \times 0.706}{1-0.439^2} = 0.359$

$i=2 \quad a_2(2) = \dfrac{a_3(2) - k_3 a_3(1)}{1-k_3^2} = \dfrac{0.706 - 0.439 \times 0.6}{1-0.439^2} = 0.548$

For $m = 2$.

$k_2 = a_2(2) = 0.548$

$a_1(i) = \dfrac{a_2(i) - a_2(2)a_2(2-i)}{1-k_2^2} \quad 1 \le i \le 1$

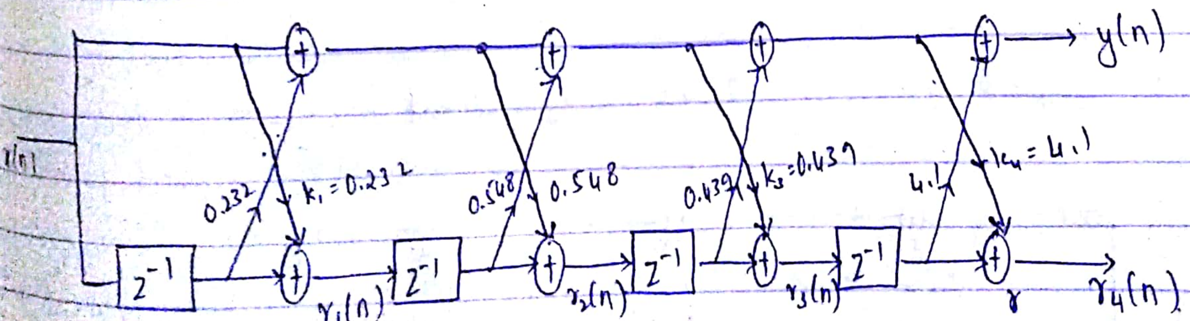
$i=1 \quad a_1(1) = \dfrac{a_2(1) - a_2(2)\, a_2(1)}{1-k_2^2} = \dfrac{0.359 - 0.548 \times 0.359}{1-0.548^2} = 0.232$

For $m = 1$

$k_1 = a_1(1) = 0.232$

$\therefore$

$k_4 = 4.1 \qquad k_3 = 0.439 \qquad k_2 = 0.548 \qquad k_1 = 0.232$



3) $y(n) = x(n) + 1.16\,x(n-1) + 2.41\,x(n-2) + 3.43\,x(n-3)$

$Y(z) = X(z)\left[1 + 1.16z^{-1} + 2.41z^{-2} + 3.43z^{-3}\right]$

$H(z) = 1 + 1.16z^{-1} + 2.41z^{-2} + 3.43z^{-3}$

$M = 3$

For $m = 3$

$k_3 = a_3(3) = 3.43$

$a_2(i) = \dfrac{a_3(i) - k_3 a_3(3-i)}{1 - k_3^2} \qquad 1 \le i \le 2.$

$i=1 \quad a_2(1) = \dfrac{a_3(1) - k_3 a_3(2)}{1-k_3^2} = \dfrac{1.16 - 3.43 \times 2.41}{1 - 3.43^2} = 0.66$

$i=2 \quad a_2(2) = \dfrac{a_3(2) - k_3 a_3(1)}{1-k_3^2} = \dfrac{2.41 - 3.43 \times 1.16}{1 - 3.43^2} = 0.146$

For $m = 2$

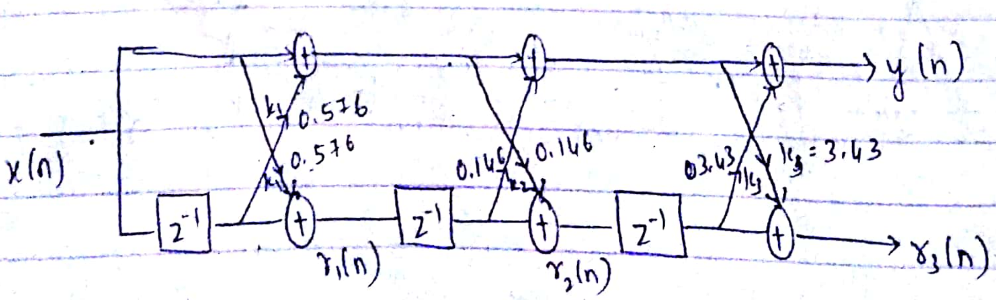
$k_2 = a_2(2) = 0.146$

$a_1(1) = \dfrac{a_2(1) - k_2 a_2(1)}{1 - k_2^2} = \dfrac{0.66 - 0.146 \times 0.66}{1 - 0.146^2} = 0.576$

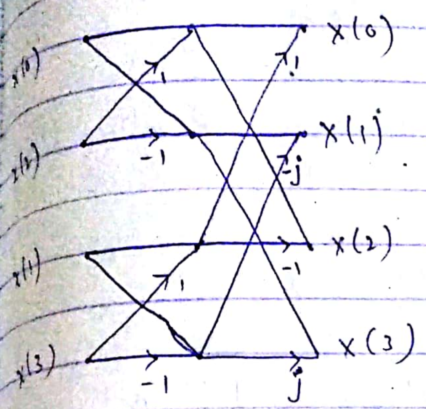
For $m = 1$

$k_1 = a_1(1) = 0.576$

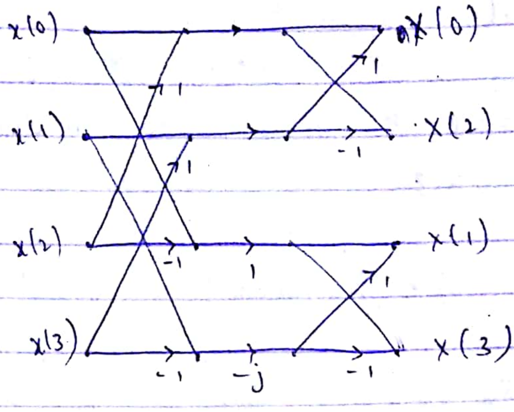
$\therefore\ k_1 = 0.576 \qquad k_2 = 0.146 \qquad k_3 = 3.43$



N=4 , DIT FFT

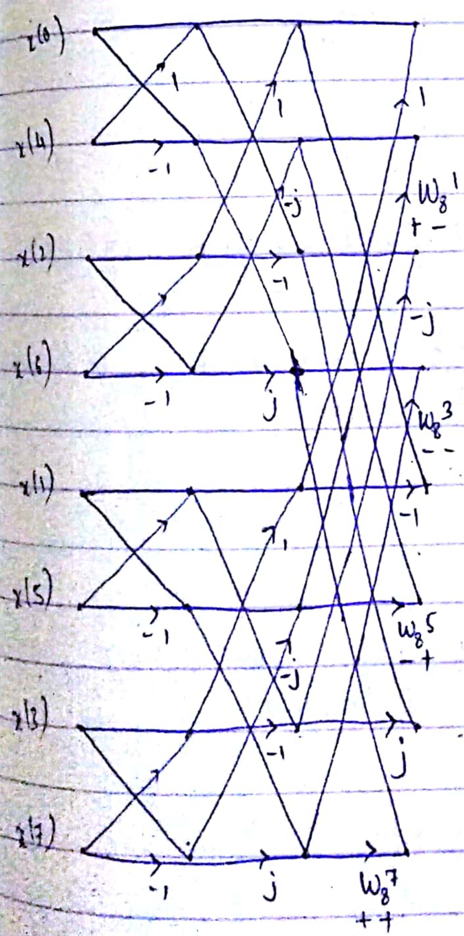


N=4 , DIF FFT



N= 8

DIT FFT



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*) A LTI system is defined by ~~$H(z) = 0.129 + 0.3867z^{-1} + 0$~~

$$H(z) = \frac{0.129 + 0.3867z^{-1} + 0.3869z^{-2} + 0.129z^{-3}}{1 - 0.2971z^{-1} + 0.3564z^{-2} - 0.0276z^{-3}}$$

Realise the given IIR transfer fn. using IIR lattice ladder structure

→ i) $\beta_i = b_i - \sum_{m=i+1}^{M} \beta_m a_m(m-i)$ $\quad i = M, M-1, \dots 0.$

ii) $k_m = a_m(m)$ $\quad a_{m-1}(i) = \dfrac{a_m(i) - a_m(m)\, a_m(m-i)}{1-k_m^2}$ $\quad 1 \le i \le m-1$

Step 1: Lattice coefficients for denominator.

$M = 3$

For $m = 3$

$k_3 = a_3(3) = -0.0276.$

$$a_2(i) = \frac{a_3(i) - a_3(3)\, a_3(3-i)}{1-k_3^2}$$

$i=1$ $\quad a_2(1) = \dfrac{a_3(1) - a_3(3)a_3(2)}{1-k_3^2} = \dfrac{-0.2971 + 0.0276 \times 0.3564}{1-(0.0276)^2} = -0.2875$

$i=2$ $\quad a_2(2) = \dfrac{a_3(2) - a_3(3)a_3(1)}{1-k_3^2} = \dfrac{0.3564 - 0.0276 \times 0.2971}{1-(0.0276)^2} = 0.34845$

For $m = 2$.

$k_2(2) = a_2(2) = 0.3485$

$a_1(1) = \dfrac{a_2(1) - a_2(2)\, a_2(1)}{1-k_2^2} = \dfrac{-0.2875 + 0.2875 \times 0.3485}{1-(0.3485)^2} = -0.213$

For $m = 1$

$k_1(1) = -0.2132$

$\therefore$ $k_3 = -0.0276$

$k_2 = 0.3485$

$k_1 = -0.2132$

Step 2: Forward tap coefficients for numerator.

Numerator: $\overset{b_0}{0.129} + \overset{b_1}{0.3867 z^{-1}} + \overset{b_2}{0.3869 z^{-2}} + \overset{b_3}{0.129 z^{-3}}$

$M = 3$

$$\beta_i = b_i - \sum_{m=i+1}^{M} \beta_m a_m(m-i)$$

$$\beta_3 = b_3 = 0.129$$

$$\beta_2 = b_2 - \sum_{m=3}^{3} \beta_m a_m(m-2) = b_2 - \beta_3 a_3(1)$$

$$\beta_2 = 0.3869 - 0.129 \times (-\overset{0.2971}{\cancel{0.0296}}) = 0.4252$$

$$\beta_1 = b_1 - \sum_{m=2}^{3} \beta_m a_m(m-1) = b_1 - \beta_2 a_2(1) - \beta_3 a_3(2)$$

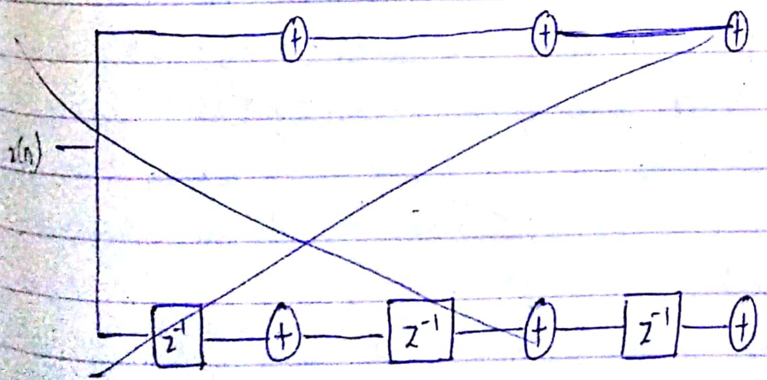
$$= 0.3867 + 0.4252 \times 0.2875 - 0.129 \times 0.3564$$

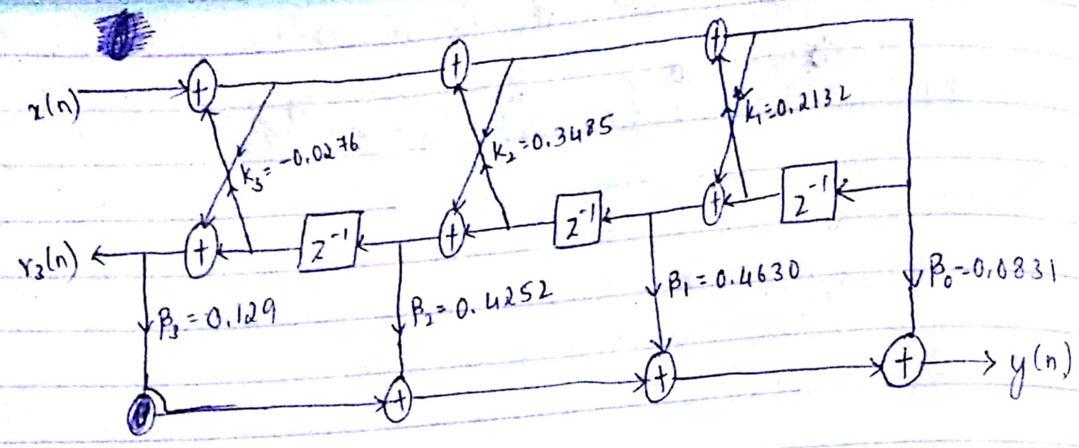
$$\beta_1 = 0.4630$$

$$\beta_0 = b_0 - \sum_{m=1}^{3} \beta_m a_m(m-0) = b_0 - \beta_1 a_1(1) - \beta_2 a_2(2) - \beta_3 a_3(3)$$

$$= 0.129 + 0.4630 \times 0.2132 - 0.4252 \times 0.3485 + 0.129 \times 0.0276$$

$$\beta_0 = 0.0831$$





*) Find IIR Lattice ladder structure for filter given by following difference eqn.

$$y(n) + \tfrac{3}{4}\, y(n-1) + \tfrac{1}{4}\, y(n-2) = x(n) + 2x(n-1)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 + 2z^{-1}}{1 + \frac{3}{4}z^{-1} + \frac{1}{4}z^{-2}}$$

→ Step 1: Lattice coefficients for denominator.

$M = 2$

For $m = 2$

$$k_2 = a_2(2) = 0.25$$

$$a_1(1) = \frac{a_2(1) - a_2(2)\, a_2(1)}{1 - k_2^2} = \frac{0.75 - 0.75 \times 0.25}{1 - 0.25^2} = 0.6$$

For $m = 1$

$k_1 = a_1(1) = 0.6$

$k_2 = 0.25$

Step 2: Forward tap coefficients for numerator.

$$\overset{b_0}{1} + \overset{b_1}{2}z^{-1}$$

$M = 2$

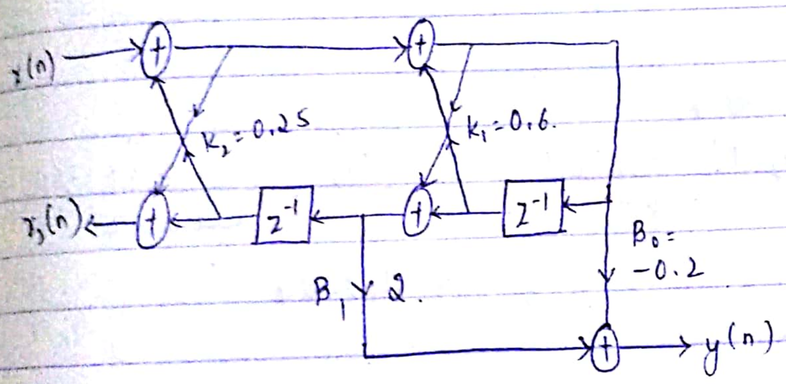
$$\beta_2 = b_2 + \beta_3 = 0.$$

$$\beta_1 = b_1 - \sum_{m=2}^{2} \beta_m a_m(m-1) = b_1 = 2.$$

$$\beta_0 = b_0 - \sum_{m=1}^{2} \beta_m a_m(m-0) = b_0 - \beta_1 a_1(1)$$

$$= 1 - 2\times 0.6$$

$$\beta_0 = -0.2$$



3) Find lattice ladder structure for an IIR filter having transfer fn.

$$H(z) = \frac{1+z^{-1}+z^{-2}}{(1+0.5z^{-1})(1+0.3z^{-1})(1+0.4z^{-1})}$$

$$= \frac{1+z^{-1}+z^{-2}}{(1+0.8z^{-1}+0.15z^{-2})(1+0.4z^{-1})}$$

$$= \frac{1+z^{-1}+z^{-2}}{(1+1.2z^{-1}+0.15z^{-2}+0.32z^{-2}+0.06z^{-3})}$$

$$= \frac{1+z^{-1}+z^{-2}}{1+1.2z^{-1}+0.47z^{-2}+0.06z^{-3}}$$

Step 1: Lattice coefficients for denominator. $\cancel{1+0.}$ $1+1.2+0.47+0.06$.

$M=3$

For $m=3$

$k_3 = a_3(3) = 0.06$.

$$a_2(i) = \frac{a_3(i) - a_3(3)\,a_3(3-i)}{1-k_3^2}$$

$i=1$ $\quad a_2(1) = \dfrac{a_3(1) - a_3(3)\,a_3(2)}{1-k_3^2} = \dfrac{1.2 - 0.06\times 0.47}{1-0.06^2} = 1.1760$

$i=2$ $\quad a_2(2) = \dfrac{a_3(2) - a_3(3)\,a_3(1)}{1-k_3^2} = \dfrac{0.47 - 0.06\times 1.2}{1-0.06^2} = 0.3994$

For $m=2$.

$k_2 = a_2(2) = 0.3994$

$$a_1(1) = \frac{a_2(1) - a_2(2)\,a_2(1)}{1-k_2^2} = \frac{1.1760 - 0.3994\times 1.1760}{1-0.3994^2}$$

$$= 0.8404.$$

For $m=1$

$k_1 = a_1(1) = 0.8404$

$k_2 = 0.3994$

$k_3 = 0.06$.

Step 2: Forward tap coefficients for numerator.

Numerator: $1+z^{-1}+z^{-2}$

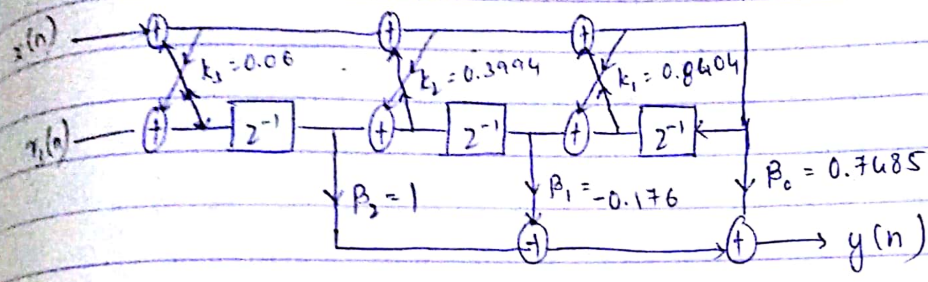
$\beta_2 = b_2 = 1$

$\beta_1 = b_1 - \beta_2 a_2(1) = 1 - 1\times 1.1760 = -0.176$

$\beta_0 = b_0 - \sum_{m=1}^{2} \beta_m a_m(m) = b_0 - \beta_1 a_1(1) - \beta_2 a_2(2)$

$= 1 + 0.176\times 0.8404 - 1\times 0.3994$

$= 0.7485$


x(n)
$x_1(n)$
$k_3 = 0.06$
$k_2 = 0.3994$
$k_1 = 0.8404$
z^{-1}
z^{-1}
z^{-1}
$\beta_3 = 1$
$\beta_1 = -0.176$
$\beta_0 = 0.7485$
y(n)