* Design an analog BPF to meet the following frequency-domain specifications:

(a) a $-3.0103$ dB upper & lower cutoff frequency of 50 Hz & 20 kHz

(b) a stopband attenuation of atleast 20 dB at 20 Hz & 45 kHz &

(c) a monotonic frequency response.

→

The monotonic frequency response can be achieved by using a Butterworth filter.

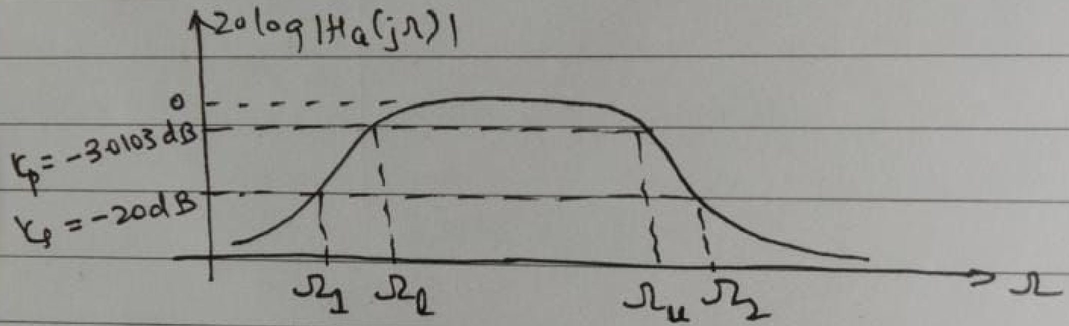



fig: magnitude-frequency response of the specified BPF

We have,

$$\Omega_1 = 2\pi \times 20 = 125.663 \text{ r/sec}$$

$$\Omega_2 = 2\pi \times 45 \times 10^3 = 2.827 \times 10^5 \text{ r/sec}$$

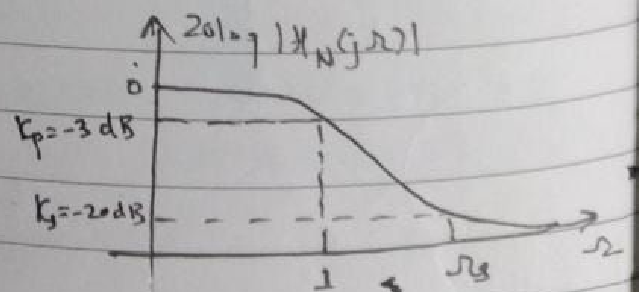
$$\Omega_u = 2\pi \times 20 \times 10^3 = 1.257 \times 10^5 \text{ r/sec}$$

$$\Omega_l = 2\pi \times 50 = 314.159 \text{ r/sec}$$

Let us use the backward design eq$^n$ & find the specification of normalized LPF:

$$A = \frac{-\Omega_1^2 + \Omega_l \Omega_u}{\Omega_1(\Omega_u - \Omega_l)} = 2.51$$

$$B = \frac{\Omega_2^2 - \Omega_l \Omega_u}{\Omega_2(\Omega_u - \Omega_l)} = 2.25$$




$$\therefore \Omega_s = \min(|A|, |B|) = 2.25$$

The order N of the normalized LPF is

$$N = \frac{\log\left[\left(10^{-K_p/10}-1\right)/\left(10^{-K_s/10}-1\right)\right]}{2\log\left(1/\Omega_s\right)} = 2.83 \simeq 3$$

$$\therefore H_3(s) = \frac{1}{s^3+2s^2+2s+1} \Rightarrow H_p(s) = H_3(s)\Big|_{s \to s/\Omega_c = s/1 = s} = H_3(s)$$

Applying the LP → BP transformation

$$H_a(s) = H_p(s)\Big|_{s \to \frac{s^2+\Omega_u\Omega_l}{s(\Omega_u-\Omega_l)}} = \frac{s^2+3.949\times10^7}{s(1.2538\times10^5)}$$

$$\therefore H_a(s) = \frac{19695\times10^{15}\, s^3}{(s^6+2.51\times10^5 s^5+3.154\times10^{10}s^4+1.989\times10^{15}s^3+1.2453\times10^{18}s^2+3.9073\times10^{20}s+6.1529\times10^{22})}$$

* Design a Butterworth analog highpass filter that will meet the following specifications:

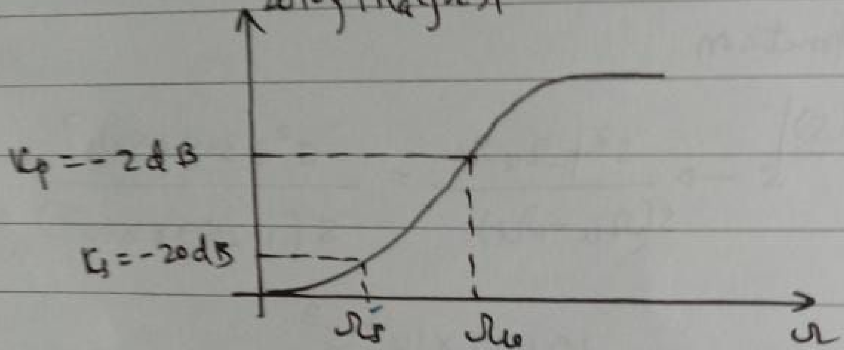
(a) maximum passband attenuation = 2dB

(b) passband edge frequency = 200 rad/sec

(c) minimum stopband attenuation = 20 dB

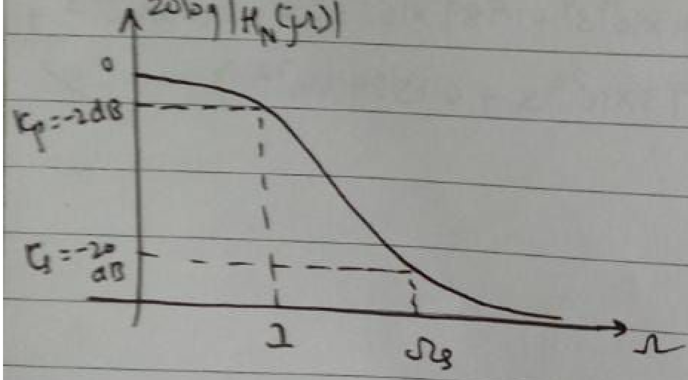
(d) stopband edge frequency = 100 rad/sec

→




$\Omega_s = 100$ rad/sec

$\Omega_u = 200$ rad/sec




$$\Omega_s = \frac{\Omega_u}{\Omega_s'} = \frac{200}{100} = 2$$

$$N = \frac{\log\left[\left(10^{-K_p/10}-1\right)/\left(10^{-K_s/10}-1\right)\right]}{2\log\left(\Omega_p/\Omega_s\right)} = 3.7$$

$$\simeq 4$$

$$H_4(s) = \frac{1}{(s^2+0.765365s+1)(s^2+1.847765s+1)}$$

$$\Omega_c = \frac{\Omega_p}{\left(10^{-K_p/10}-1\right)^{1/2N}} = 1.0693$$

Hence, the lowpass prototype filter is $H_p(s) = H_4(s)\big|_{s \to s/\Omega_c = s/1.0693}$

Now, to get the specified highpass filter $H_a(s)$, let us apply LP → HP transformation on $H_p(s)$. i.e.

$$H_a(s) = H_p(s)\Big|_{s \to \frac{\Omega_u}{s}} = H_p(s)\Big|_{s \to \frac{200}{s}} = H_4(s)\Big|_{s \to \frac{200}{1.0693 s}}$$

$$= H_4(s)\Big|_{s \to \frac{187.031}{s}}$$

$$= \frac{s^4}{(s^2 + 143.1464 s + 34980.7521)(s^2 + 345.5892 s + 34980.7521)}$$

* Let $H(s) = \frac{1}{s^2+s+1}$ represent the TF of a lowpass filter (not Butterworth) with a passband of 1 c/sec. Use frequency transformations to find the TFs of the following analog filters.

(a) A lowpass filter with a passband of $10^c$/sec.

(b) A highpass filter with a cutoff frequency of $1^c$/sec

(c) A highpass filter with a cutoff frequency of $10^c$/sec.

(d) A bandpass filter with a passband of 10 c/sec & a center frequency of 100 rad/sec

(e) A bandstop filter with a stopband of 2 c/sec & a centre frequency of 10 c/sec

$\longrightarrow$

$$H(s) = \frac{1}{s^2+s+1}$$

(a) LP $\rightarrow$ LP

$s \rightarrow s/\Omega_u$

$$\Rightarrow H_a(s) = H(s)\Big|_{s \rightarrow s/10} = \frac{100}{s^2+10s+100}$$

(b) LP $\rightarrow$ HP

$s \rightarrow \Omega_u/s$

$$\Rightarrow H_a(s) = H(s)\Big|_{s \rightarrow 1/s} = \frac{s^2}{s^2+s+1}$$

(c) LP $\rightarrow$ HP

$s \rightarrow \Omega_u/s$

$$\Rightarrow H_a(s) = H(s)\Big|_{s \rightarrow 10/s} = \frac{s^2}{s^2+10s+100}$$

(d) LP $\rightarrow$ BP

$$s \rightarrow \frac{s^2+\Omega_u\Omega_l}{s(\Omega_u-\Omega_l)} = \frac{s^2+\Omega_0^2}{sB_0}$$

where, $\Omega_0 = \sqrt{\Omega_u\Omega_l} \rightarrow$ centre frequency

$B_0 = \Omega_u - \Omega_l \rightarrow$ width of passband.

$$\Rightarrow H_a(s) = H(s)\Big|_{s \rightarrow \frac{s^2+10^4}{10s}} = \frac{100s^2}{s^4+10s^3+20100s^2+10^5s+10^8}$$

c) LP → BS

$$s \rightarrow \frac{s(\Omega_u - \Omega_l)}{s^2 + \Omega_u \Omega_l} = \frac{sB_0}{s^2 + \Omega_0^2} \Rightarrow H_a(s) = H(s)\Big|_{s \rightarrow \frac{2s}{s^2+100}}$$

$$\therefore H_a(s) = \frac{(s^2+100)^2}{s^4 + 2s^3 + 204s^2 + 200s + 10^4}$$

Consider a fifth-order LPBF with a passband of 1KHz & a maximum passband attenuation of 1dB. What is the attenuation in dB of the lowpass filter at a frequency of 2KHz?

⟶

We know that, $|H(j\Omega)| = \frac{1}{\left[1 + (\Omega/\Omega_c)^{2N}\right]^{1/2}}$

$$\Rightarrow 20\log|H(j\Omega)| = -10\log\left[1 + \left(\frac{\Omega}{\Omega_c}\right)^{2N}\right]$$

at $\Omega = 2\pi \times 1 \times 10^3$ c/sec $20\log|H(j\Omega)| = -1$ & $N = 5$

$$\therefore \Omega_c = 7192.21 \text{ c/sec}$$

$$\therefore 20\log|H(j\Omega)|\Big|_{\Omega = 2\pi \times 2 \times 10^3 \text{ c/sec}} = -24.25\text{dB}$$

∴ stopband attenuation is 24.25dB //

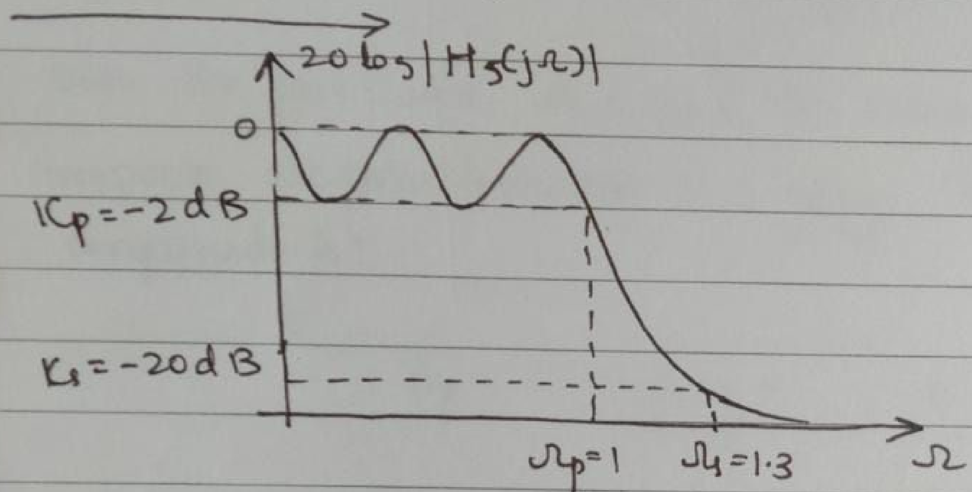
* Design a Chebyshev I filter to meet the following specifications:

a. Passband ripple: $\leq 2$ dB

b. Passband edge: 1 c/sec

c. Stopband attenuation: $\geq 20$ dB

d. Stopband edge: 1.3 c/sec.




We know that,

$$K_p = 20 \log \left( \frac{1}{\sqrt{1+\epsilon^2}} \right) = -2 \quad \Rightarrow \epsilon = 0.76478$$

Also,

$$\delta_p = 1 - \frac{1}{\sqrt{1+\epsilon^2}} = 0.20567$$

$$K_s = 20 \log \delta_s = -20 \quad \Rightarrow \delta_s = 0.1$$

$$d = \sqrt{\frac{(1-\delta_p)^{-2} - 1}{\delta_s^{-2} - 1}} = 0.077$$

$$K = \frac{\Omega_p}{\Omega_s} = \frac{1}{1.3} = 0.769$$

$$\therefore \quad N = \frac{\cosh^{-1}(1/d)}{\cosh^{-1}(1/K)} = 4.3 \simeq 5$$

Now,

$$a = \frac{1}{2}\left(\frac{1+\sqrt{1+\epsilon^2}}{\epsilon}\right)^{1/N} - \frac{1}{2}\left(\frac{1+\sqrt{1+\epsilon^2}}{\epsilon}\right)^{-1/N} = 0.21830398$$

$$b = \frac{1}{2}\left(\frac{1+\sqrt{1+\epsilon^2}}{\epsilon}\right)^{1/N} + \frac{1}{2}\left(\frac{1+\sqrt{1+\epsilon^2}}{\epsilon}\right)^{-1/N} = 1.0235520$$

$$\sigma_k = -a \sin[(2k-1)\pi/2N]$$

$$\Omega_k = b \cos[(2k-1)\pi/2N], \quad k = 1, 2, \ldots, 2N$$

since $N=5$,

$$\sigma_k = -a \sin[(2k-1)\pi/10]$$

$$\Omega_k = b \cos[(2k-1)\pi/10] \quad k = 1, \ldots 10$$

| $k$ | $\sigma_k$ | $\Omega_k$ |
|---|---|---|
| 1 | -0.0674610 | 0.9734557 |
| 2 | -0.1766151 | 0.6016287 |
| 3 | -0.2183083 | 0 |
| 4 | -0.1766151 | -0.6016287 |
| 5 | -0.0674610 | -0.9734557 |

$$H_5(s) = \frac{K_N}{(s-s_1)(s-s_2)(s-s_3)(s-s_4)(s-s_5)}$$

Since, N is odd, $K_N = b_0 = 0.08172$

$$H_5(s) = \frac{0.08172}{s^5 + 0.70646s^4 + 1.4995s^3 + 0.6934s^2 + 0.459349s + 0.08172}$$

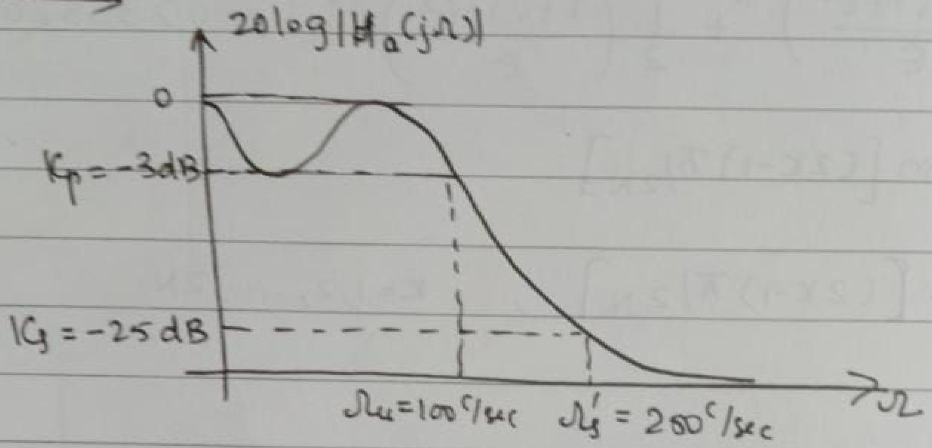
Verification

$$20\log|H_5(j\Omega)|_{\Omega=1} = -2\,dB$$

$$20\log|H_5(j\Omega)|_{\Omega=1.3} = -24.5\,dB$$

* Design a Chebyshev analog lowpass filter that has a $-3\,dB$ cutoff frequency of 100 rad/sec & a stopband attenuation of 25 dB or greater for all radian frequencies past 250 rad/sec. Verify the design

→

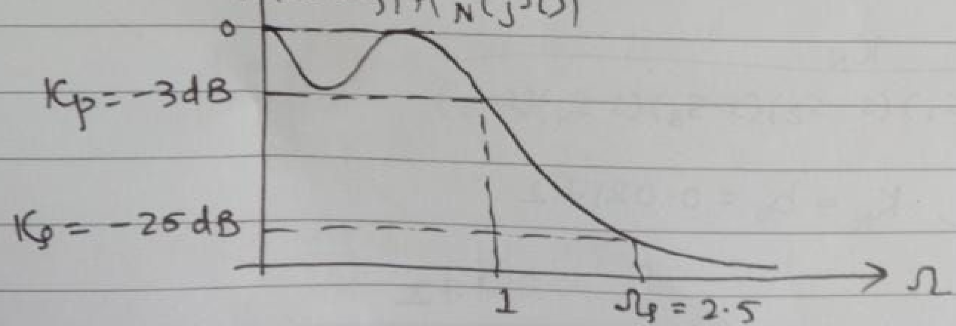



The specified frequency response of the Chebyshev analog lowpass filter is shown in fig.

The specifications of an equivalent normalized lowpass Chebyshev prototype is obtained using the backward design eqn

$$\Omega_s = \frac{\Omega_s'}{\Omega_u} = \frac{250}{100} = 2.5$$

The normalized LP magnitude freq. response is:




We know that,

$$K_p = 20\log\left(\frac{1}{\sqrt{1+\epsilon^2}}\right) = -3$$

$$\Rightarrow \epsilon = 0.99762834$$

Thus, $\delta_p = 1 - \frac{1}{\sqrt{1+\epsilon^2}} = 0.2920542138$

$$r_s = -25\,dB \Rightarrow 20\log\delta_s = -25 \Rightarrow \delta_s = 0.056$$

$$K = \frac{\Omega_p}{\Omega_s} = \frac{1}{2.5} = 0.4$$

$$d = \sqrt{\frac{(1-\delta_p)^{-2}-1}{\delta_s^{-2}-1}} = 0.056$$

$$N = \frac{\cosh^{-1}(1/d)}{\cosh^{-1}(1/k)} = 2.28 \simeq 3$$

$$a = \frac{1}{2}\left(\frac{1+\sqrt{1+\epsilon^2}}{\epsilon}\right)^{1/N} - \frac{1}{2}\left(\frac{1+\sqrt{1+\epsilon^2}}{\epsilon}\right)^{-1/N} = 0.2986202$$

$$b = \frac{1}{2}\left(\frac{1+\sqrt{1+\epsilon^2}}{\epsilon}\right)^{1/N} + \frac{1}{2}\left(\frac{1+\sqrt{1+\epsilon^2}}{\epsilon}\right)^{-1/N} = 1.043635$$

$$\sigma_k = -a\sin\left[(2k-1)\frac{\pi}{2N}\right] = -a\sin\left[(2k-1)\pi/6\right]$$

$$\Omega_k = b\cos\left[(2k-1)\pi/2N\right] = b\cos\left[(2k-1)\pi/6\right] \qquad k = 1, 2, \ldots 6$$

| $k$ | $\sigma_k$ | $\Omega_k$ |
|---|---|---|
| 1 | −0.1493101 | 0.9038144 |
| 2 | −0.2986202 | 0 |
| 3 | −0.1493101 | −0.9038144 |

$$H_3(s) = \frac{K_N}{(s-s_1)(s-s_2)(s-s_3)} = \frac{0.2505943}{s^3+0.5972404s^2+0.928348s+0.2505943}$$

$$\therefore H_a(s) = H_3(s)\big|_{s \to s/\Omega_a = s/100} = \frac{250594.3}{s^3+59.72404s^2+9283.48s+250594.3}$$

Verification

$20\log|H_a(j\Omega)|_{\Omega=100} = -3\,dB$

$20\log|H_a(j\Omega)|_{\Omega=250} = -35\,dB$

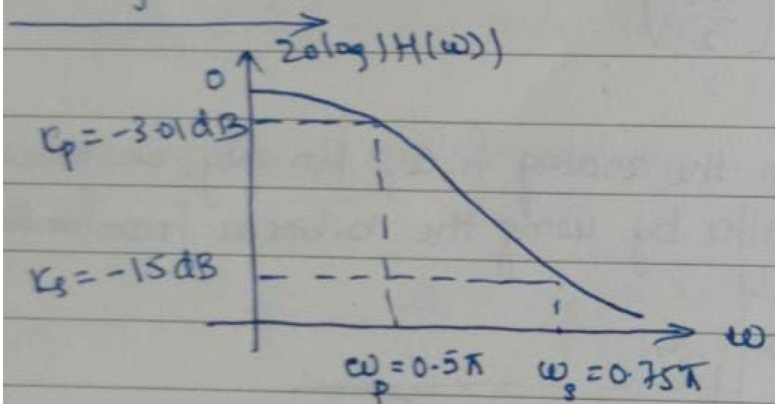
* A digital lowpass filter is required to meet the following specifications:

(a) Monotonic passband & stopband.

(b) $-3.01$ dB cut off frequency of $0.5\pi$ rad

(c) stopband attenuation of atleast 15 dB at $0.75\pi$ rad

Find the system function $H(z)$ & difference eqn realization. Verify the design by checking for the passband & stopband specifications.

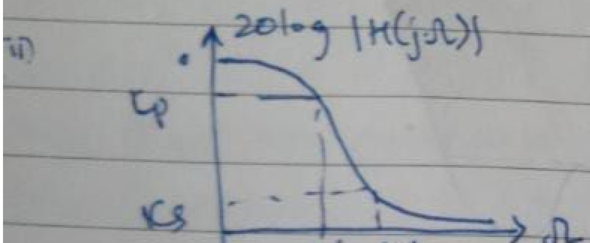



(i) Prewarp the frequencies $\omega_p$ & $\omega_s$ using $T = 1$ sec.

$$\Omega_p' = \frac{2}{T}\tan\left(\frac{\omega_p}{2}\right) = \frac{2}{1}\tan\left(\frac{0.5\pi}{2}\right) = 2$$

$$\Omega_s' = \frac{2}{T}\tan\left(\frac{\omega_s}{2}\right) = \frac{2}{1}\tan\left(\frac{0.75\pi}{2}\right) = 4.8282$$

(ii)




$N = 1.94 \simeq 2$

$$H_2(s) = \frac{1}{s^2 + \sqrt{2}s + 1}$$

$$\Omega_c = \frac{\Omega_p'}{\left(10^{-K_p/10} - 1\right)^{1/2N}} = 2\ \text{r/sec}$$

$$H_a(s) = H_2(s)\Big|_{s \to s/\Omega_c = s/2} = \frac{4}{s^2 + 2\sqrt{2}s + 4}$$

(iii) Using bilinear transformation $(T=1)$

$$H(z) = H_a(s)\Big|_{s \to 2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)} = \frac{(1+z^{-1})^2}{3.4142+0.5858\,z^{-2}}$$

<u>verification of the design</u>

Let $z = e^{j\omega}$

i.e. $$H(e^{j\omega}) = H(\omega) = \frac{(1+e^{-j\omega})^2}{3.4142+0.5858\,e^{-j2\omega}}$$

$$= \frac{\left[(1+\cos\omega) - j\sin\omega\right]^2}{(3.4142+0.5858\cos 2\omega) - j0.5858\sin 2\omega}$$

$\Rightarrow$ $|H(\omega)|$ in dB i.e. $20\log|H(\omega)|\Big|_{\omega=0.5\pi} = -3.01\,dB$

$20\log|H(\omega)|\Big|_{\omega=0.75\pi} = -15.44\,dB$.

<u>Difference eq^n realization:</u>

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1+2z^{-1}+z^{-2}}{0.5858\,z^{-2}+3.4142}$$

$\Rightarrow$ $x(n) + 2x(n-1) + x(n-2) = 0.5858\,y(n-2) + 3.4142\,y(n)$

$\therefore$ $y(n) = -0.1715\,y(n-2) + 0.2928\,x(n) + 0.5857\,x(n-1) + 0.2928\,x(n-2)$

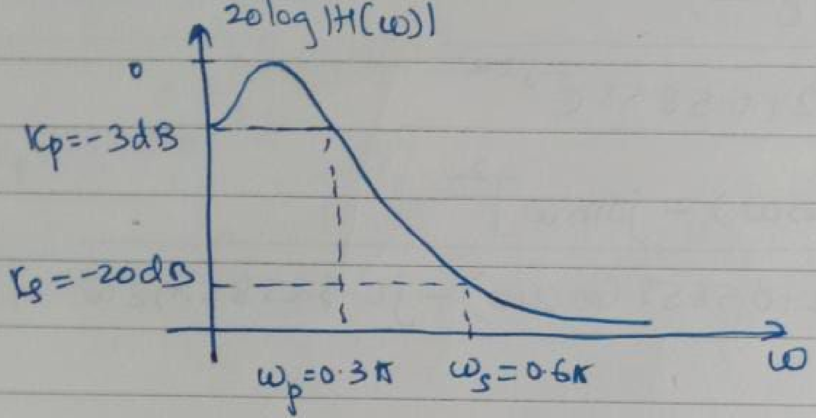
* Determine the system function $H(z)$ of the lowest-order Chebyshev filter that meets the following specifications:

$$-3 \leq 20\log|H(j\omega)| \leq 0 \qquad 0 \leq |\omega| \leq 0.3\pi$$

$$20\log|H(j\omega)| \leq -20\,dB \qquad 0.6\pi \leq |\omega| \leq \pi.$$

Use the bilinear transformation.

$\longrightarrow$

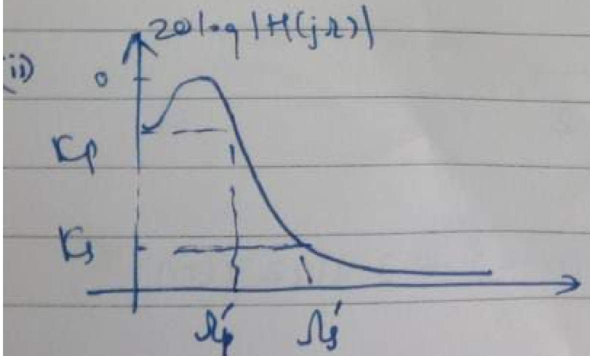



(i) Prewarping the frequencies $\omega_p$ & $\omega_s$ using $T = 1$ sec

$$\Omega_p' = \frac{2}{T}\tan\left(\frac{\omega_p}{2}\right) = \frac{2}{1}\tan\left(\frac{0.3\pi}{2}\right) = 1.019$$

$$\Omega_s' = \frac{2}{T}\tan\left(\frac{\omega_s}{2}\right) = \frac{2}{1}\tan\left(\frac{0.6\pi}{2}\right) = 2.75$$

(ii)




$d = 0.1$

$K = 0.3705$

$N = 1.8 \simeq 2$

$a$

$b$

$\tilde{\varepsilon}$

$\Omega_k$

$$H_2(s) = \frac{K_N}{s^2+0.6448s+0.7079}, \quad K_N = \frac{b_0}{\sqrt{1+\epsilon^2}}$$

$$\therefore H_2(s) = \frac{0.50119}{s^2+0.6448s+0.7079}$$

$$H_a(s) = H_2(s)\Big|_{s \to s/\Omega_p' = \frac{s}{1.019}} = \frac{0.52}{s^2+0.6575s+0.7351}$$

(iii) $$H(z) = H_a(s)\Big|_{s \to \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)} = \frac{0.52(1+z^{-1})^2}{6.0494-6.53z^{-1}+3.4208z^{-2}}$$

* A third order Butterworth lowpass filter has the transfer function

$$H(s) = \frac{1}{(s+1)(s^2+s+1)}$$

Design $H(z)$ using impulse invariant technique.

$\longrightarrow$

$$H(s) = \frac{1}{(s+1)(s^2+s+1)} = \frac{1}{(s+1)(s+0.5-j0.866)(s+0.5+j0.866)}$$

$$= \frac{C_1}{s+1} + \frac{C_2}{s+0.5-j0.866} + \frac{C_2^*}{s+0.5+j0.866}$$

Using partial fraction expansion, we find

$$C_1 = 1, \quad C_2 = 0.577e^{-j2.62} \quad \& \quad C_2^* = 0.577e^{j2.62}$$

$$H(s) = \frac{1}{s+1} + \frac{0.577e^{-j2.62}}{s+0.5-j0.866} + \frac{0.577e^{j2.62}}{s+0.5+j0.866}$$

Since we know that,

$$H(z) = \sum_{i=1}^{3} \frac{c_i}{1-e^{s_i T} z^{-1}} \quad \& \quad C_3 = C_2^*$$

$$\therefore \; H(z) = \frac{1}{1-e^{-T} z^{-1}} + \frac{0.577e^{-j2.62}}{1-e^{(-0.5+j0.866)T} z^{-1}} + \frac{0.577e^{j2.62}}{1-e^{(-0.5-j0.866)T} z^{-1}}$$

$$H(z) = \frac{b_0 z^{-1} + b_1 z^{-2}}{1-a_1 z^{-1} - a_2 z^{-2} - a_3 z^{-3}}$$

where,

$$b_0 = -2e^{-0.5T}\cos(0.866T) + e^{-T} + 1.154\, e^{-0.5T}\cos\left(\frac{5\pi}{6} + 0.866T\right)$$

$$b_1 = e^{-T} + 1.154\, e^{-1.5T}\cos\left(\frac{5\pi}{6} + 0.866T\right)$$

$$a_1 = e^{-T} + 2e^{-0.5T}\cos(0.866T)$$

$$a_2 = -e^{-T} - 2e^{-1.5T}\cos(0.866T)$$

$$a_3 = e^{-2T}$$

* Let $H_a(s) = \frac{b}{(s+a)^2+b^2}$ be a causal second-order analog TF.

Show that the causal second-order digital TF $H(z)$ obtained from $H_a(s)$ through impulse invariance method is given by

$$H(z) = \frac{e^{-aT}\sin bT\, z^{-1}}{1-2e^{-aT}\cos bT\, z^{-1}+e^{-2aT}z^{-2}}$$

Also, find $H(z)$ when $H_a(s) = \frac{1}{s^2+2s+2}$

$\longrightarrow$

$$(s+a)^2+b^2 = 0 \Rightarrow s = -a \pm bj$$

$$\therefore H_a(s) = \frac{b}{(s+a-jb)(s+a+jb)} = \frac{C_1}{s+a-jb} + \frac{C_2}{s+a+jb}$$

$$\Rightarrow C_1 = \left.\frac{b}{s+a+jb}\right|_{s=-a+jb} = \frac{1}{2j}$$

$$C_2 = C_1^* = -1/j2$$

$$\therefore H(z) = \sum_{i=1}^{N} \frac{C_i}{1-e^{s_iT}z^{-1}} = \frac{1}{2j}\left(\frac{1}{1-e^{(-a+jb)T}z^{-1}} - \frac{1}{1-e^{(-a-jb)T}z^{-1}}\right)$$

$$= \frac{e^{-aT}\sin bT\, z^{-1}}{1-2e^{-aT}\cos bT\, z^{-1}+e^{-2aT}z^{-2}}$$

When,

$$H_a(s) = \frac{1}{s^2+2s+2} = \frac{1}{(s+1)^2+1^2} \Rightarrow a=1 \;\&\; b=1$$

$$\Rightarrow H(z) = \frac{e^{-T}\sin T\, z^{-1}}{1-2e^{-T}\cos T\, z^{-1}+e^{-2T}z^{-2}}$$

* Let $H_a(s) = \frac{s+a}{(s+a)^2+b^2}$ be a causal second-order analog TF. Show that the causal second-order digital transfer function $H(z)$ is obtained from $H_a(s)$ through impulse invariance method is given by,

$$H(z) = \frac{1 - e^{-aT} \cos bT \, z^{-1}}{1 - 2\cos bT \, e^{-aT} z^{-1} + e^{-2aT} z^{-2}}$$

$\longrightarrow$

A digital lowpass filter is required to meet the following specifications.

$$20\log|H(\omega)|_{\omega=0.2\pi} \geq -1.9328\ \text{dB}$$

$$20\log|H(\omega)|_{\omega=0.6\pi} \leq -13.9794\ \text{dB}$$

The filter must have a maximally flat frequency response. Find $H(z)$ to meet the above specifications using impulse invariant transformation.

* Design a digital Chebyshev I filter that satisfies the following constraints

$$0.8 \le |H(\omega)| \le 1 \qquad 0 \le \omega \le 0.2\pi$$

$$|H(\omega)| \le 0.2 \qquad 0.6\pi \le \omega \le \pi$$

Use impulse invariant transformation.

* A LPF is to be designed with the following desired frequency response

$$H_d(e^{j\omega}) = H_d(\omega) = \begin{cases} e^{-j2\omega} & |\omega| < \pi/4 \\ 0 & \pi/4 < |\omega| < \pi \end{cases}$$

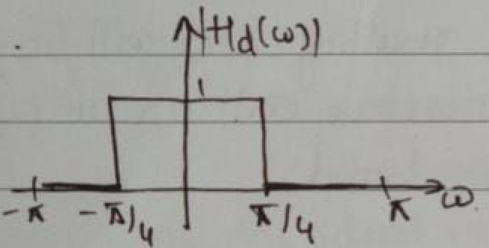
Determine the filter coefficients $h_d(n)$ & $h(n)$ if $\omega(n)$ is a rectangular window defined as follows:

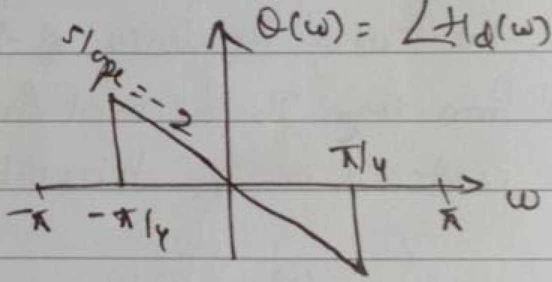
$$\omega_R(n) = \begin{cases} 1 & 0 \le n \le 4 \\ 0 & \text{otherwise} \end{cases}$$

Also, find the frequency response $H(\omega)$ of the resulting FIR filter.

$\longrightarrow$

Magnitude response ~~&~~ the phase response of the desired Lowpass IIR filter is:




$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(\omega) e^{j\omega n} d\omega = \frac{1}{2\pi}\int_{-\pi/4}^{\pi/4} e^{-j2\omega} e^{j\omega n} d\omega$$

$$= \frac{\sin\left[\pi/4(n-2)\right]}{\pi(n-2)} \quad n \neq 2$$

$$h_d(2) = \frac{1}{2\pi}\int_{-\pi/4}^{\pi/4} e^{0} d\omega = \frac{1}{2\pi}\frac{\pi}{2} = 1/4$$

Given, $\omega_R(n) = \begin{cases} 1 & 0 \le n \le 4 \\ 0 & \text{otherwise} \end{cases}$

Hence, $h(n) = h_d(n)\,\omega_R(n) = \begin{cases} \frac{\sin\left[\pi/4(n-2)\right]}{\pi(n-2)} \times 1 & n = 0,1,3,4 \\ \frac{1}{4} \times 1 & n = 2 \end{cases}$

The filter coefficients are:

| $n$ | $h_d(n)$ | $\omega_R(n)$ | $h(n) = h_d(n)\,\omega_R(n)$ |
|---|---|---|---|
| 0 | 0.159 | 1 | 0.159 |
| 1 | 0.225 | 1 | 0.225 |
| 2 | 0.25 | 1 | 0.25 |
| 3 | 0.225 | 1 | 0.225 |
| 4 | 0.159 | 1 | 0.159 |

Since N is odd, the frequency response of the centre symmetry FIR filter is computed using:

$$H(\omega) = e^{-j\omega\left(\frac{N-1}{2}\right)}\left[h\left(\frac{N-1}{2}\right) + \sum_{n=0}^{\frac{(N-3)}{2}} 2h(n)\cos\left[\omega\left(n-\left(\frac{N-1}{2}\right)\right)\right]\right]$$

Here N=5, hence,

$$H(\omega) = e^{-j2\omega}\left(h(2) + \sum_{n=0}^{1} 2h(n)\cos[\omega(n-2)]\right)$$

$$= e^{-j2\omega}\left(0.25 + 0.318\cos 2\omega + 0.45\cos\omega\right)$$

* A filter is to be designed with the following desired frequency response

$$H_d(\omega) = \begin{cases} 0 & -\pi/4 < \omega < \pi/4 \\ e^{-j2\omega} & \frac{\pi}{4} < |\omega| < \pi \end{cases}$$

Find the frequency response of the FIR filter designed using a rectangular window defined below:

$$\omega_R(n) = \begin{cases} 1 & 0 \leq n \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

$\longrightarrow$

IDFT of $H_d(\omega)$ is $h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(\omega)\, e^{j\omega n}\, d\omega$

$$= \frac{1}{2\pi} \int_{-\pi}^{-\pi/4} e^{-j2\omega} e^{j\omega n} d\omega + \frac{1}{2\pi} \int_{\pi/4}^{\pi} e^{-j2\omega} e^{j\omega n} d\omega$$

$$= \frac{1}{\pi(n-2)} \left[ \sin[\pi(n-2)] - \sin\left(\frac{\pi}{4}(n-2)\right) \right] \quad n \neq 2$$

$$h_d(2) = \frac{1}{2\pi} \int_{-\pi}^{-\pi/4} e^{0} d\omega + \frac{1}{2\pi} \int_{\pi/4}^{\pi} e^{0} d\omega = \frac{1}{2\pi}\left[\frac{3\pi}{4} + \frac{3\pi}{4}\right] = \frac{3}{4}$$

The impulse response of the FIR filter is:

$$h(n) = h_d(n)\, \omega_R(n) \quad 0 \leq n \leq 4$$

$$\omega_R(n) = \begin{cases} 1 & 0 \leq n \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

| n | $h_d(n)$ | $\omega_R(n)$ | $h(n)$ |
|---|---|---|---|
| 0 | $-0.159$ | 1 | $-0.159$ |
| 1 | $-0.225$ | 1 | $-0.225$ |
| 2 | $0.75$ | 1 | $0.75$ |
| 3 | $-0.225$ | 1 | $-0.225$ |
| 4 | $-0.159$ | 1 | $-0.159$ |

Since N is odd,

$$H(\omega) = e^{-j\omega\left(\frac{N-1}{2}\right)}\left(h\left(\frac{N-1}{2}\right) + \sum_{n=0}^{\frac{N-3}{2}} 2h(n)\cos\left[\omega\left(n-\frac{N-1}{2}\right)\right]\right)$$

$$= e^{-j2\omega}\left(h(2) + 2h(0)\cos 2\omega + 2h(1)\cos\omega\right)$$

$$= e^{-j2\omega}\left(0.75 - 0.318\cos 2\omega - 0.45\cos\omega\right)$$

* The desired frequency response of a lowpass filter is given by,

$$H_d(\omega) = \begin{cases} e^{-j3\omega} & |\omega| < 3\pi/4 \\ 0 & 3\pi/4 < |\omega| < \pi \end{cases}$$

Determine the frequency response of the FIR filter if Hamming window is used with $N=7$

$\longrightarrow$

$$\text{IDTFT of } H_d(\omega) = h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(\omega)\, e^{j\omega n}\, d\omega = \frac{1}{2\pi}\int_{-3\pi/4}^{3\pi/4} e^{-j3\omega} e^{j\omega n}\, d\omega$$

$$= \frac{\sin\left[3\pi/4\,(n-3)\right]}{\pi(n-3)} \qquad n \neq 3$$

$$h_d(3) = \frac{1}{2\pi}\int_{-3\pi/4}^{3\pi/4} 1\, d\omega = 3/4$$

The impulse response of the FIR filter is:

$$h(n) = h_d(n)\, \omega_{Ham}(n) \qquad 0 \le n \le 6$$

where,

$$\omega_{Ham}(n) = 0.54 - 0.46\cos\left(\frac{2\pi n}{N-1}\right) \qquad 0 \le n \le N-1$$

$$= 0.54 - 0.46\cos\left(\frac{2\pi n}{6}\right) \qquad 0 \le n \le 6$$

Hence, the impulse respon of the FIR filter for $0 \le n \le 6$ is:

$$h(n) = \begin{cases} \dfrac{\sin\left[3\pi/4\,(n-3)\right]}{\pi(n-3)}\left[0.54 - 0.46\cos\left(\frac{2\pi n}{6}\right)\right] & n \neq 3 \\ \dfrac{3}{4} \times \left[0.54 - 0.46\cos\left(\frac{2\pi n}{6}\right)\right] & n = 3 \end{cases}$$

| n | $h_d(n)$ | $\omega_{Ham}(n)$ | $h(n)$ |
|---|---|---|---|
| 0 | 0.075 | 0.08 | 0.006 |
| 1 | -0.159 | 0.31 | -0.049 |
| 2 | 0.225 | 0.77 | 0.173 |
| 3 | 0.75 | 1 | 0.75 |
| 4 | 0.225 | 0.77 | 0.173 |
| 5 | -0.159 | 0.31 | -0.049 |
| 6 | 0.025 | 0.08 | 0.006 |

Since N is odd,

$$H(\omega) = e^{-j\omega\left(\frac{N-1}{2}\right)}\left(h\left(\frac{N-1}{2}\right) + \sum_{n=0}^{\frac{N-3}{2}} 2h(n) \cos\left[\omega\left(n - \frac{N-1}{2}\right)\right]\right)$$

$$= e^{-j3\omega}\left(0.75 + 0.012 \cos 3\omega - 0.098 \cos 2\omega + 0.346 \cos\omega\right)$$

* Obtain the coefficients of an FIR filter to meet the specifications given below using the window method.

Passband edge frequency: 1.5 kHz

Stopband edge frequency: 2 kHz

Minimum stopband attenuation: 50 dB

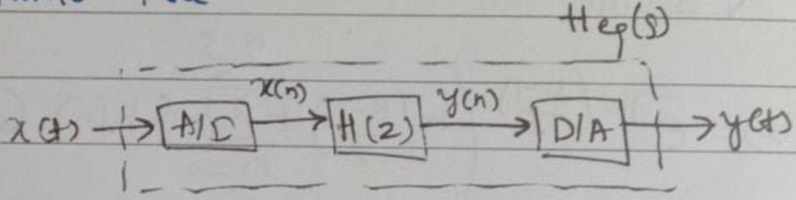
Sampling frequency: 8 kHz

$\longrightarrow$

$\Omega_p = 2\pi \times 1.5 \times 10^3 = 3\pi \times 10^3$ c/sec

$\Omega_s = 2\pi \times 2 \times 10^3 = 4\pi \times 10^3$ c/sec

$K_s = -A_s = -50$ dB




Converting the above analog specification into equivalent digital specifications using the formula $\omega = \Omega T$ with $T = \frac{1}{8 \times 10^3}$ sec, we get

$$\omega_p = \Omega_p T = \frac{2\pi \times 1.5 \times 10^3}{8 \times 10^3} = 0.375\pi^c$$

$$\omega_s = \Omega_s T = \frac{2\pi \times 2 \times 10^3}{8 \times 10^3} = 0.5\pi \text{ rad}$$

Step 1: Since $K_s = -50$ dB, either we can choose Hamming or Blackman window since it satisfies the stopband attenuation requirement.

~~Step 2~~ We will select Hamming window as it has a lower transition width than the Blackman window & thus giving the smaller value of N.

Step 2: The length of the window is selected using the expression,

$$\omega_s - \omega_p \geq K \frac{2\pi}{N}$$

($\because$ $8\pi/N$ is the width of transition band)

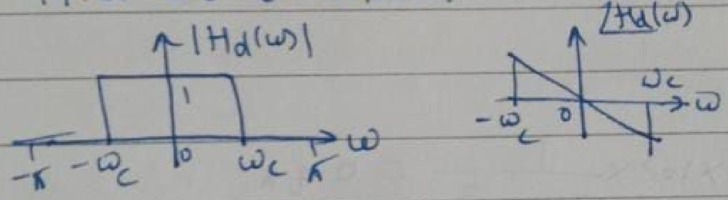
Since $K = 4$ for the Hamming window, we get

$$0.5\pi - 0.375\pi \geq 8\pi/N$$

$$\Rightarrow N \geq 64$$

Let $N = 65$, so that $\alpha = \frac{N-1}{2}$ is an integer.

Step 3: Let the magnitude & phase responses of the desired lowpass filter be as shown.




Then,

$$H_d(\omega) = \begin{cases} e^{-j\omega\alpha} & |\omega| < \omega_c \\ 0 & \omega_c < |\omega| < \pi \end{cases}$$

Step 4: $h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(\omega) e^{j\omega n} d\omega = \frac{\sin[\omega_c(n-\alpha)]}{\pi(n-\alpha)}$ $n \neq \alpha$

$$h_d(\alpha) = \frac{\omega_c}{\pi}$$

Step 5: Select $\omega_c' = \omega_p + \frac{\Delta\omega}{2} = 0.375\pi + \frac{(0.5\pi - 0.375\pi)}{2}$

$$= 0.4375\pi^c$$

$$\alpha = \frac{N-1}{2} = 32$$

Step 6: $h(n) = h_d(n)\, w_{Ham}(n)$

where, $w_{Ham}(n) = 0.54 - 0.46\cos\left(\frac{2\pi n}{N-1}\right) = 0.54 - 0.46\cos\left(\frac{2\pi n}{64}\right)$ $0 \leq n \leq 64$

$$h(n)=\begin{cases}\dfrac{\sin[0.4375\pi(n-32)]}{\pi(n-32)}\times\left[0.54-0.46\cos\left(\dfrac{2\pi n}{64}\right)\right] & n\neq 32\\ 0.4375\times\left[0.54-0.46\cos\left(\dfrac{2\pi n}{64}\right)\right] & n=32\end{cases}$$

* An analog signal contains frequencies upto 10kHz. The signal is sampled at 50KHz. Design an FIR filter having a linear-phase characteristic & a transition band of 5KHz. The filter should provide minimum 50dB attenuation at the end of transition band.

$\longrightarrow$

$\Omega_p = 2\pi\times10\times10^3$ c/sec

$\Omega_s = 2\pi\times(10+5)\times10^3 = 2\pi\times15\times10^3$ c/sec

$K_s = -50$ dB

$T = 1/50\times10^3$ sec

$\omega_p = \Omega_p T = 2\pi\times10\times10^3\times\dfrac{1}{50\times10^3} = 0.4\pi$

$\omega_s = \Omega_s T = 2\pi\times15\times10^3\times\dfrac{1}{50\times10^3} = 0.6\pi$

**step 1:** Window selected: Hamming window

**step 2:** $\omega_s-\omega_p \geq \dfrac{K2\pi}{N} \Rightarrow 0.6\pi-0.4\pi \geq \dfrac{8\pi}{N} \Rightarrow N\geq 40$

$\therefore N = 41$ & $\alpha = \dfrac{N-1}{2} = 20$

**step 3:**

$$H_d(\omega)=\begin{cases}e^{-j\omega\alpha} & |\omega|<\omega_c\\ 0 & \omega_c<|\omega|<\pi\end{cases}$$

**step 4:**

$$h_d(n)=\begin{cases}\dfrac{\sin[\omega_c(n-\alpha)]}{\pi(n-\alpha)} & n\neq\alpha\\ \dfrac{\omega_c}{\pi} & n=\alpha\end{cases}$$

step 5:

$$\omega_c = \omega_p + \frac{\Delta\omega}{2} = 0.4\pi + \frac{0.2\pi}{2} = 0.5\pi^c$$

$$\alpha = \frac{N-1}{2} = 20$$

step 6: $h(n) = h_d(n)\,\omega_{Ham}(n) \qquad 0 \le n \le N-1$

$$\omega_{Ham}(n) = 0.54 - 0.46\cos\left(\frac{2\pi n}{40}\right) \qquad 0 \le n \le 40$$

$$h(n) = \begin{cases} \dfrac{\sin[0.5\pi(n-20)]}{\pi(n-20)} \times \left[0.54 - 0.46\cos\left(\dfrac{2\pi n}{40}\right)\right] & n \neq 20 \\[2ex] \dfrac{0.5\pi}{\pi} \times \left[0.54 - 0.46\cos\left(\dfrac{2\pi n}{40}\right)\right] & n = 20 \end{cases}$$

* Design a LPF with a cutoff frequency $\omega_c = \frac{\pi}{4}$, a transition width, $\Delta\omega = 0.02\pi$ & a stopband ripple $\delta_s = 0.01$. Use Kaiser window

$\longrightarrow$

Express Stopband ripple in dB, $A = -20\log \delta_s = 40\,dB$

For $A = 40\,dB$,

$$\beta = 0.5842(A-21)^{0.4} + 0.07886(A-21)$$

$$\beta = 0.5842(40-21)^{0.4} + 0.07886(40-21)$$

$$= 3.4$$

$$\Delta f = \frac{\Delta\omega}{2\pi} = \frac{0.02\pi}{2\pi} = 0.01$$

The length of the window: $N \geq \dfrac{A - 7.95}{14.36\,\Delta f} \Rightarrow N \geq 223.189$

$$\therefore N = 225$$

$$\Rightarrow \alpha = \frac{N-1}{2} = 112$$

Hence the Kaiser window function is:

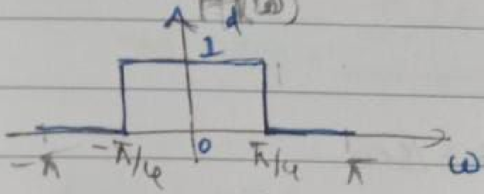
$$\omega(n) = \frac{I_0\left\{\beta\sqrt{1-\left(\frac{n-\alpha}{\alpha}\right)^2}\right\}}{I_0\{\beta\}} \quad 0 \leq n \leq \cancel{N}\,4$$

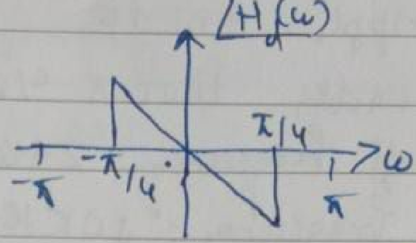
$$= \frac{I_0\left\{3.4\sqrt{1-\left(\frac{n-112}{112}\right)^2}\right\}}{I_0\{3.4\}} \quad 0 \leq n \leq 224$$

where, the Bessel function: $I_0\{x\} = 1 + \sum_{n=1}^{\infty}\left[\left(\frac{x}{2}\right)^n \frac{1}{n!}\right]^2$

The frequency response of the desired LPF is:

$$H_d(\omega) = \begin{cases} e^{-j\omega\alpha} & |\omega| < \omega_c \\ 0 & \omega_c < |\omega| < \pi \end{cases} \qquad \begin{array}{l} \alpha = 112 \\ \omega_c = \pi/4 \end{array}$$




$$h_d(n) = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{-j\omega\alpha} e^{j\omega n}\, d\omega = \begin{cases} \dfrac{\sin[\omega_c(n-\alpha)]}{\pi(n-\alpha)} & n \neq \alpha \\ \dfrac{\omega_c}{\pi} & n = \alpha \end{cases}$$

Selecting the cutoff frequency, which is in the middle of transition band

$$\omega_c' = \omega_c + \frac{\Delta\omega}{2} = \frac{\pi}{4} + \frac{0.02\pi}{2} = 0.26\pi$$

$$\therefore\ h_d(n) = \begin{cases} \dfrac{\sin[0.26\pi(n-112)]}{\pi(n-112)} & n \neq 112 \\ 0.26 & n = 112 \end{cases}$$

$$h(n) = h_d(n)\,\omega(n) \qquad 0 \le n \le N-1$$

$$= \begin{cases} \dfrac{\sin[0.26\pi(n-112)]}{\pi(n-112)} \times \dfrac{I_0\left\{3.4\sqrt{1-\left(\frac{n-112}{112}\right)^2}\right\}}{I_0\{\beta\}} & n \neq 112 \\ 0.26 \times \dfrac{I_0\left\{3.4\sqrt{1-\left(\frac{n-112}{112}\right)^2}\right\}}{I_0\{\beta\}} & n = 112 \end{cases}$$

$0 \le n \le 224$

* Find an expression for the impulse response $h(n)$ of a linear-phase LP FIR filter using the Kaiser window to satisfy the following magnitude response specification for the equivalent analog filter.

stopband attenuation: 40 dB

Passband ripple: 0.01 dB

Transition width: $1000\pi$ c/sec

Ideal cut off frequency: $2400\pi$ c/sec

Sampling frequency: 10KHz

→

$$A = -20\log\delta_s = 40 \Rightarrow \delta_s = 0.01$$

$$20\log(1-\delta_p) = 0.01 \Rightarrow \delta_p = 0.00115$$

$$\Delta\Omega = 1000\pi \text{ c/sec}$$

$$\Omega_c = 2400\pi \text{ c/sec}$$

We are required to design $H(z)$ & $h(n)$ so that the specification of $H(\Omega)$ are met.

$$\Delta\omega = \Delta\Omega \times T = 0.1\pi \text{ c}$$

$$\omega_c = \Omega_c T = 0.24\pi \text{ c}$$

Select $\delta = \min[\delta_p, \delta_s] = 0.00115$

This means the stopband attenuation is more than actually required, in this case

$$A = -20\log(0.00115) = 58.8 \text{ dB}$$

&

$$N \geq \frac{A - 7.95}{14.36\Delta f} \geq \frac{58.8 - 7.95}{14.36 \times \frac{0.1\pi}{2\pi}} \geq 70.82$$

$$\therefore N = 71 \Rightarrow \alpha = \frac{N-1}{2} = 35$$

Ripple parameter $\beta = 0.1102(58.8 - 8.7) = 5.48$

Hence, Kaiser window function $\omega(n) = \dfrac{I_0\left\{\beta\sqrt{1-\left(\frac{n-\alpha}{\alpha}\right)^2}\right\}}{I_0\{\beta\}}$ $0 \le n \le N-1$

$$\Rightarrow \omega(n) = \frac{I_0\left\{5.48\sqrt{1-\left(\frac{n-35}{35}\right)^2}\right\}}{I_0\{5.48\}} \quad 0 \le n \le 70$$

Let desired LP be

$$H_d(\omega) = \begin{cases} e^{-j\omega\alpha} & |\omega| < \omega_c \\ 0 & \omega_c < |\omega| < \pi \end{cases}$$

$$\Rightarrow h_d(n) = \frac{\sin[\omega_c(n-\alpha)]}{\pi(n-\alpha)} \quad n \ne \alpha$$

$$= \frac{\omega_c}{\pi} \quad n = \alpha$$

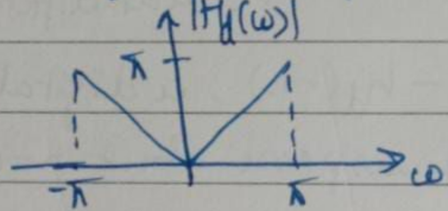
$$\omega_c' = \omega_c + \frac{\Delta\omega}{2} = 0.24\pi + \frac{0.1\pi}{2} = 0.29\pi$$

$$\therefore h_d(n) = \begin{cases} \dfrac{\sin[0.29\pi(n-35)]}{\pi(n-35)} & n \ne 35 \\ 0.29 & n = 35 \end{cases}$$

then $h(n) = h_d(n)\,\omega(n)$

$$= \begin{cases} \dfrac{\sin[0.29\pi(n-35)]}{\pi(n-35)} \times \dfrac{I_0\left\{5.48\sqrt{1-\left(\frac{n-35}{35}\right)^2}\right\}}{I_0\{5.48\}} & n \ne 35,\ 0 \le n \le 70 \\ 0.29 \times \dfrac{I_0\left\{5.48\sqrt{1-\left(\frac{n-35}{35}\right)^2}\right\}}{I_0\{5.48\}} & n = 35 \end{cases}$$

* Use the window method with a Hamming window to design a 21-point differentiator. The magnitude response of an ideal differentiator is as shown in fig.




→

The impulse response of an ideal IIR differentiator having antisymmetry about $n=\alpha$ is

$$h_d'(n) = \begin{cases} \dfrac{\cos[\pi(n-\alpha)]}{n-\alpha} & n \neq \alpha \\ 0 & n = \alpha \end{cases}$$

$$\alpha = \frac{N-1}{2} = \frac{21-1}{2} = 10$$

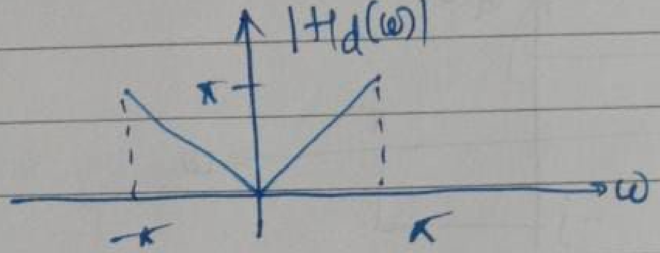
$$\therefore h_d'(n) = \begin{cases} \dfrac{\cos[\pi(n-10)]}{n-10} & n \neq 10 \\ 0 & n = 10 \end{cases}$$

The impulse response of the FIR differentiator is:

$$h(n) = h_d'(n)\, \omega_{Ham}(n) \qquad 0 \le n \le N-1$$

$$\omega_{Ham}(n) = 0.54 - 0.46 \cos\left[\frac{2\pi n}{N-1}\right] \qquad 0 \le n \le N-1$$

* Use the window method with a Hamming window to design a 7-tap differentiator. The magnitude response of an ideal differentiator is as shown in fig. Compute & plot the magnitude response of the resulting FIR differentiator.




→

$$h_d'(n) = \begin{cases} \dfrac{\cos[\pi(n-\alpha)]}{n-\alpha} & n \neq \alpha \\ 0 & n = \alpha \end{cases} \qquad \alpha = \frac{N-1}{2} = \frac{7-1}{2} = 3$$

$$h_d'(n) = \begin{cases} \dfrac{\cos[\pi(n-3)]}{n-3} & n \neq 3 \\ 0 & n = 3 \end{cases}$$

The impulse response of the FIR differentiator is:

$$h(n) = h_d'(n)\, \omega_{Ham}(n) \qquad 0 \le n \le N-1$$

$$\omega_{Ham}(n) = \begin{cases} 0.54 - 0.46\cos\left(\frac{2\pi n}{N-1}\right) & 0 \le n \le N-1 \\ 0 & \text{otherwise} \end{cases}$$

| $n$ | $h_d'(n)$ | $\omega_{Ham}(n)$ | $h(n)$ |
|---|---|---|---|
| 0 | 1/3 | 0.08 | 0.0267 |
| 1 | −1/2 | 0.31 | −0.155 |
| 2 | 1 | 0.77 | 0.77 |
| 3 | 0 | 1 | 0 |
| 4 | −1 | 0.77 | −0.77 |
| 5 | 1/2 | 0.31 | 0.155 |
| 6 | −1/3 | 0.08 | −0.0267 |

$$H_r(\omega) = 2\sum_{n=0}^{\frac{N-3}{2}} h(n) \cos\left[\omega\left(\left(\frac{N-1}{2}\right) - n\right)\right]$$

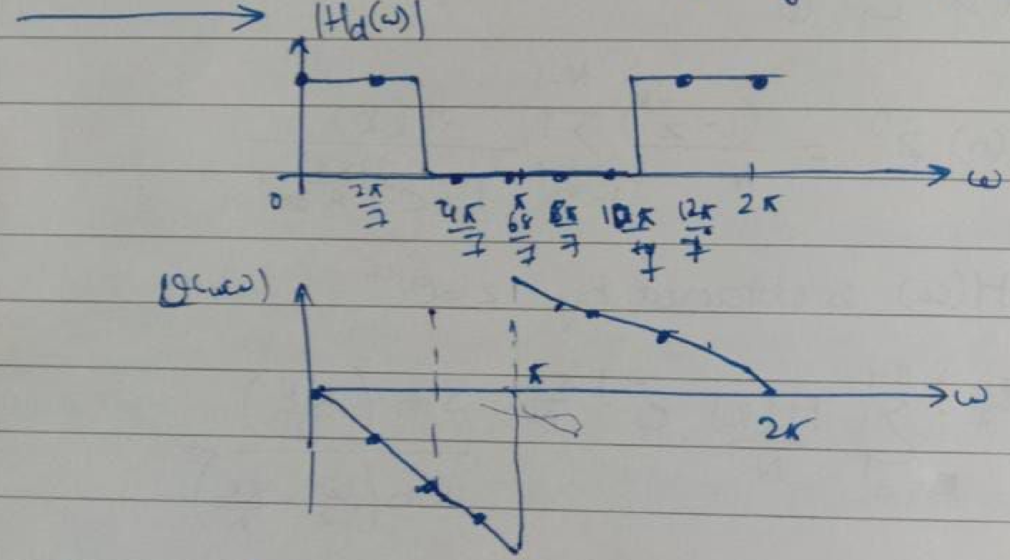
$$= 2\sum_{n=0}^{2} h(n) \sin[\omega(3-n)]$$

$$= 0.0534 \sin 3\omega - 0.31 \sin 2\omega + 1.54 \sin\omega$$

* A LPF has the desired frequency response

$$H_d(\omega) = \begin{cases} e^{-j3\omega} & 0 < \omega < \pi/2 \\ 0 & \pi/2 < \omega < \pi \end{cases}$$

Determine h(n) based on frequency sampling technique. Take N=7




Let the ideal response of a linear-phase LPF is

$$H_d(\omega) = \begin{cases} e^{-j\left(\frac{N-1}{2}\right)\omega} & 0 < \omega < \pi/2 \\ 0 & \pi/2 < \omega < \pi \end{cases} = \begin{cases} e^{-j3\omega} & 0 < \omega < \pi/2 \\ 0 & \frac{\pi}{2} < \omega < \pi \end{cases}$$

The ideal magnitude & frequency response is taken to be symmetric about π while the ideal phase response is taken to be antisymmetric about π. The ideal magnitude & phase response with samples taken for N = 7.

From fig:

$$|H(k)| = \begin{cases} 1 & k=0,1 \\ 0 & k=2,3,4,5 \\ 1 & k=6 \end{cases}$$

Also,

$$\theta_k = -3\omega_k = -3 \times \frac{2\pi}{N}k = -\frac{6\pi k}{7} \qquad k=0,1,2,3$$

$$\theta_k = -\frac{6\pi}{7}(k-7) \qquad k=4,5,6$$

$\therefore$ $H(k) = |H(k)| . e^{j\theta_k}$

$$= \begin{cases} e^{-\frac{6\pi k}{7}} & k=0,1, \\ 0 & k=2,3,4,5 \\ e^{-j\frac{6\pi}{7}(k-7)} & k=6 \end{cases}$$

The inverse DFT of $H(k)$ is

$$h(n) = \frac{1}{N}\left[H(0) + 2\sum_{k=1}^{\left(\frac{N-1}{2}\right)} Re\left[H(k)e^{\frac{j2\pi nk}{N}}\right]\right]$$

$$= \frac{1}{7}\left[1 + 2\cos\left(\frac{2\pi}{7}(n-3)\right)\right] \qquad 0 \le n \le 6$$

| $n$ | $h(n)$ |
|---|---|
| 0 | -0.11456 |
| 1 | 0.7928 |
| 2 | 0.320997 |
| 3 | 0.42857 |
| 4 | 0.320997 |
| 5 | 0.07923 |
| 6 | -0.4486 |