KLE Technological University
Creating Value
Leveraging Knowledge

KLE TECH.

Earlier known as
B. V. B. College of Engineering & Technology

School of Electronics & Communication Engineering

Question Paper Title: MINOR-I

Total Duration (H:M): 1:15	**Course: Digital Signal Processing**	**Course code: 17EECC303**
Date:25/09/2019	**USN**	**Maximum Marks :40**

Note : Answer any two full questions.

Q.No.	Questions	Marks	CO	BL	PO	PI Code
1a	State and prove the circular convolution in time DFT property.	6	CO1	L2	1	1.1.3
1b	Let X(k) denote a 6-point DFT of a real valued sequence, $x(n)=\cos\left(\frac{\pi n}{2}\right)$, $0 \leq n < 6$. Determine y(n), whose 6-point DFT is given by, $Y(k)=W_3^{2k}X(k)$.	6	CO1	L3	2	2.1.2
1c	Compute the discrete frequency spectrum components of a real valued sequence x(n)={4, 0, 2, 2, 1, 1, 0,1} using radix-2 DIF-FFT algorithm.	8	CO1	L3	1	1.1.3
2a	Determine the response of a linear filter with impulse response h(n)={1, 2, -1} to the input sequence x(n)={1, 4, 1, 4, 2, 8, 1, 3, 1, 1} using overlap save method and justify the results using linear convolution.	6	CO1	L3	1	1.1.3
2b	Explain the relationship of the DFT to the Fourier Series coefficients of a periodic sequence.	6	CO1	L2	1	1.1.3
2c	Determine the circular convolution of the impulse response h(n)={1, 2, 3} and input sequence x(n)={1, 0, 2, 3} in time domain and frequency domain approach using DIT-FFT and IFFT.	8	CO1	L3	2	2.1.2
3a	Explain the chirp-z transform for computing the DFT using linear filtering.	6	CO1	L2	1	1.1.3
3b	Find the N-point DFT of the sequence, $x(n) = e^{\frac{j2\pi mn}{N}}, \quad 0 \leq n \leq N-1$	6	CO1	L3	1	1.1.3
3c	Find the time domain representation of the signal X(k)={4, 2.41-j2.41, -j2, -0.41-j0.41, 0, -0.41+j0.41, j2, 2.41+j2.41}, using computationally efficient DIT-IFFT algorithm.	8	CO1	L3	2	2.1.2

Scheme and Solution

Course: Digital Signal Processing

Faculty: Gjireesha. H.M, Rohit Kalyani, R.M. Banakar. Course Code: 17EECC303

Date: 24/09/2019

1a. Statement: The two finite duration sequences of length N, $x_1(n)$ and $x_2(n)$. Their respective N-point DFT's are

$$X_1(k)=\sum_{n=0}^{N-1} x_1(n)\, e^{\frac{-j2\pi kn}{N}} \quad \text{and} \quad X_2(k)=\sum_{n=0}^{N-1} x_2(n)\, e^{\frac{-j2\pi kn}{N}} \quad , k=0,1\dots N-1$$

If we multiply the two DFTs together, the result is a DFT, say $X_3(k)$ of a sequence $x_3(n)$ of length N.

Proof:

The relationship between $x_3(n)$ and the sequences $x_1(n)$ and $x_2(n)$.

We have $\quad X_3(k)=X_1(k)X_2(k) \quad , k=0,1\dots N-1$

The IDFT of $X_3(k)$ is $\quad x_3(m)=\frac{1}{N}\sum_{k=0}^{N-1} X_3(k)\, e^{\frac{j2\pi km}{N}} = \frac{1}{N}\sum_{k=0}^{N-1} X_1(k)X_2(k)\, e^{\frac{j2\pi km}{N}}$

$$x_3(m)=\frac{1}{N}\sum_{k=0}^{N-1}\left[\sum_{n=0}^{N-1} x_1(n)\, e^{\frac{-j2\pi kn}{N}}\right]\left[\sum_{l=0}^{N-1} x_2(l)\, e^{\frac{-j2\pi kl}{N}}\right] e^{\frac{j2\pi km}{N}}$$

$$x_3(m)=\frac{1}{N}\sum_{n=0}^{N-1} x_1(n)\sum_{l=0}^{N-1} x_2(l)\left[\sum_{k=0}^{N-1} e^{\frac{j2\pi k}{N}(m-n-l)}\right]$$

The inner sum: $\displaystyle\sum_{k=0}^{N-1} a^k = \begin{cases} N & , a=1 \\ \frac{1-a^N}{1-a} & , a\neq 1 \end{cases}$ where $a=e^{\frac{j2\pi}{N}(m-n-l)}$

We observe that $a=1$ when $m-n-l$ is a multiple of N. On the other hand $a^N=1$ for any value of $a\neq 0$.

$$\sum_{k=0}^{N-1} a^k = \begin{cases} N, & l=m-n+pN=((m-n))_N \quad , \text{ p an integer} \\ 0, & \text{otherwise.} \end{cases}$$

$$\therefore\ x_3(m)=\sum_{n=0}^{N-1} x_1(n)\, x_2((m-n))_N \quad , m=0,1\dots N-1$$

The above expression has the form of a circular convolution.

1b

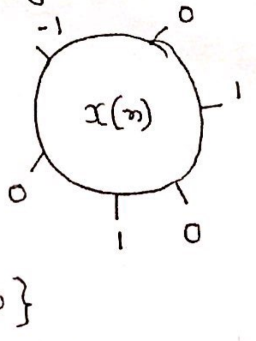
$x(n)=\cos(\pi n/2)$ where $n=0,1,2,3,4$ and 5

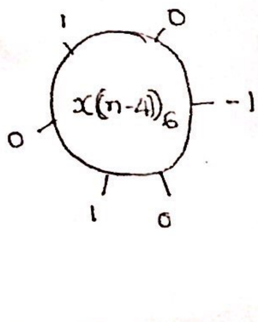
$x(n)=\{1\ 0\ -1\ 0\ 1\ 0\}$

$$Y(k)=W_3^{2k}X(k)=e^{\frac{-j2\pi(2k)}{3}}X(k)=e^{\frac{-j2\pi(4k)}{6}}X(k)$$

According to time shifting DFT property

$$x(n-n_0) \xleftrightarrow[N]{DFT} e^{\frac{-j2\pi kn_0}{N}} X(k)$$





$\therefore\ y(n)=x((n-4))_6$

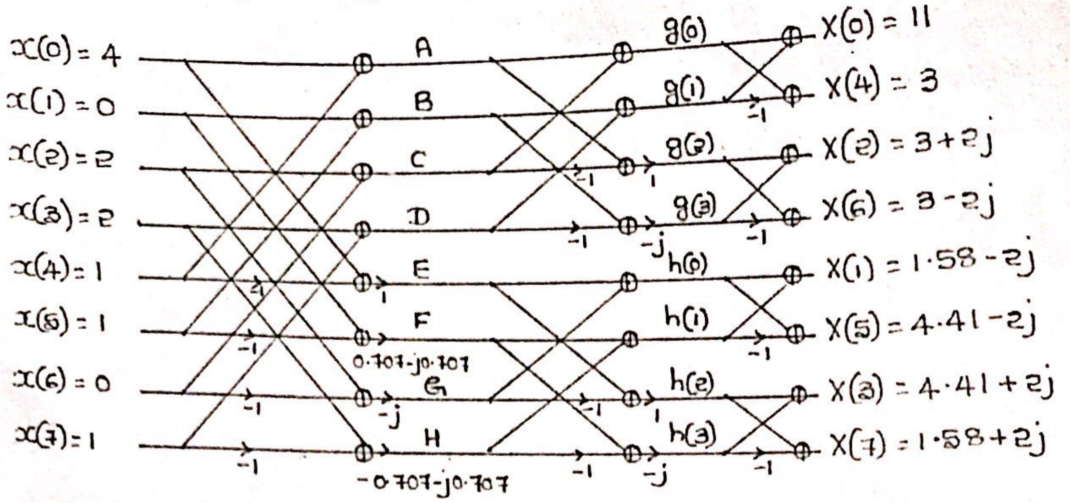
$\cancel{y(n)=\{0,-1,0,1,0,1,0\}}$

$y(n)=x((n-4))_6=\{-1,0,1,0,1,0\}$

Scanned with CamScanner

1C $x(n)=\{4, 0, 2, 2, 1, 1, 0, 1\}$

$N=8=2^3$ radix-2 DIF-FFT algorithm.



First stage:

$A = 4+1 = 5$

$B = 0+1 = 1$

$C = 2+0 = 2$

$D = 2+1 = 3$

$E = (4-1)1 = 3$

$F = (0-1)(0.707-j0.707)$

$= -0.707+j0.707$

$G = (2-0)(-j) = -2j$

$H = (2-1)(-0.707-j0.707)$

$= -0.707-j0.707.$

Second Stage:

$g(0) = 5+2 = 7$

$g(1) = 1+3 = 4$

$g(2) = (5-2)(1) = 3$

$g(3) = (1-3)(-j) = 2j$

$h(0) = 3+(-2j) = 3-2j$

$h(1) = -0.707+j0.707-0.707-j0.707$

$h(1) = -1.414$

$h(2) = (3+2j)(1) = 3+2j$

$h(3) = (-0.707+j0.707+0.707+j0.707)(-j)$

$h(3) = 1.414$

Third stage:

$X(0) = 7+4 = 11$

$X(4) = 7-4 = 3$

$X(2) = 3+2j$

$X(6) = 3-2j$

$X(1) = 1.58-2j$

$X(5) = 4.41-2j$

$X(3) = 4.41+2j$

$X(7) = 1.58+2j$

$\therefore X(k) = \{11, 1.58-2j, 3+2j, 4.41+2j, 3, 4.41-2j, 3-2j, 1.58+2j\}$

2a $x(n)=\{1\ 4\ 1\ 4\ 2\ 8\ 1\ 3\ 1\ 1\}$ and $h(n)=\{1\ 2\ -1\}$

$M=3$ $L=2$ Using overlap save method.

$x_1(n)=\{0, 0, 1, 4\}$. $x_2(n)=\{1, 4, 1, 4\}$. $x_3(n)=\{1, 4, 2, 8\}$ and $h(n)=\{1, 2, -1, 0\}$

$x_4(n)=\{2, 8, 1, 3\}$. $x_5(n)=\{1, 3, 1, 1\}$. $x_6(n)=\{1, 1, 0, 0\}$

$$y_1(n) = \begin{bmatrix} 1 & 0 & -1 & 2 \\ 2 & 1 & 0 & -1 \\ -1 & 2 & 1 & 0 \\ 0 & -1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 7 \\ -4 \\ 1 \\ 6 \end{bmatrix}$$

$$y_2(n) = \begin{bmatrix} 1 & 0 & -1 & 2 \\ 2 & 1 & 0 & -1 \\ -1 & 2 & 1 & 0 \\ 0 & -1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \\ 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 8 \\ +2 \\ 8 \\ 2 \end{bmatrix}$$

$$y_3(n) = \begin{bmatrix} 1 & 0 & -1 & 2 \\ 2 & 1 & 0 & -1 \\ -1 & 2 & 1 & 0 \\ 0 & -1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \\ 2 \\ 8 \end{bmatrix} = \begin{bmatrix} 15 \\ -2 \\ 9 \\ 8 \end{bmatrix}$$

$$y_4(n) = \begin{bmatrix} 1 & 0 & -1 & 2 \\ 2 & 1 & 0 & -1 \\ -1 & 2 & 1 & 0 \\ 0 & -1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 8 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 7 \\ 9 \\ 15 \\ -3 \end{bmatrix}$$

$$y_5(n) = \begin{bmatrix} 1 & 0 & -1 & 2 \\ 2 & 1 & 0 & -1 \\ -1 & 2 & 1 & 0 \\ 0 & -1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 6 \\ 0 \end{bmatrix} \qquad y_6(n) = \begin{bmatrix} 1 & 0 & -1 & 2 \\ 2 & 1 & 0 & -1 \\ -1 & 2 & 1 & 0 \\ 0 & -1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 1 \\ -1 \end{bmatrix}$$

$y(n) = \{1, 6, 8, 2, 9, 8, 15, -3, 6, 0, 1, -1\}$

Linear Convolution:

$h(n)/x(n)$	1	4	1	4	2	8	1	3	1	1
1	1	4	1	4	2	8	1	3	1	1
2	2	8	2	8	4	16	2	6	2	2
-1	-1	-4	-1	-4	-2	-8	-1	-3	-1	-1

$y(n) = \{1, 6, 8, 2, 9, 8, 15, -3, 6, 0, 1, -1\}$

2b Relationship between DFT and The Fourier Series

A periodic sequence $x_p(n)$ with fundamental period N can be represented in a Fourier Series of the form

$$x_p(n) = \sum_{k=0}^{N-1} C_k e^{\frac{j2\pi kn}{N}} \quad \text{where } \infty < n < \infty$$

and

$$C_k = \frac{1}{N} \sum_{n=0}^{N-1} x_p(n) e^{\frac{-j2\pi kn}{N}}, \quad k = 0, 1 \ldots N-1.$$

DFT pairs: $X(k) = \sum_{n=0}^{N-1} x(n) e^{\frac{-j2\pi kn}{N}}, \quad k = 0, 1 \ldots N-1$

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{\frac{j2\pi kn}{N}}, \quad n = 0, 1 \ldots N-1$$

From the above equations, we can observe that the formula for the Fourier Series coefficients has the form of a DFT. If we define a sequence $x(n) = x_p(n)$, $0 \le n \le N-1$.

$\therefore X(k) = NC_k$.

2c

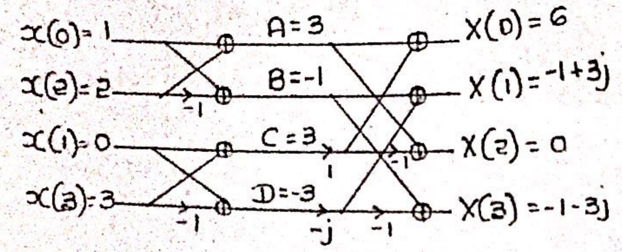
$x(n) = \{1, 0, 2, 3\}$ and $h(n) = \{1, 2, 3\}$

Circular convolution in time domain:

$$\begin{bmatrix} 1 & 0 & 3 & 2 \\ 2 & 1 & 0 & 3 \\ 3 & 2 & 1 & 0 \\ 0 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 13 \\ 11 \\ 5 \\ 7 \end{bmatrix}$$

$\therefore y(n) = \{13, 11, 5, 7\}$

Circular Convolution in frequency domain using DIT method



$A = 1+2 = 3$

$B = 1-2 = -1$

$C = 0+3 = 3$

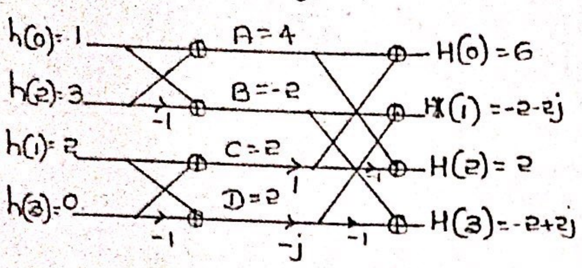
$D = 0-3 = -3$

$X(0) = 3+3 = 6$

$X(1) = -1+(-3)(-j) = -1+3j$

$X(2) = 3-3 = 0$

$X(3) = -1-(-3)(-j) = -1-3j$



$A = 1+3 = 4$

$B = 1-3 = -2$

$C = 2+0 = 2$

$D = 2-0 = 2$

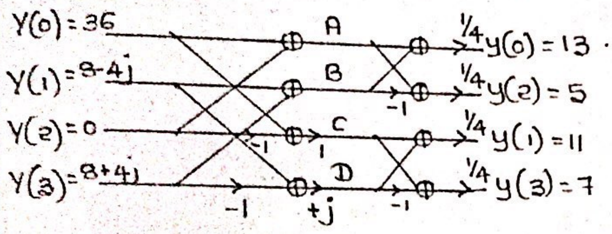
$H(0) = 4+2 = 6$

$H(1) = -2+2(-j) = -2-2j$

$H(2) = 4-2 = 2$

$H(3) = -2-2(-j) = -2+2j$

$$Y(k) = \{6, -1+3j, 0, -1-3j\}\{6, -2-2j, 2, -2+2j\} = \{36, 8-4j, 0, 8+4j\}$$



$A = 36+0 = 36$

$B = 8-4j+8+4j = 16$

$C = 36-0 = 36$

$D = (8-4j-8-4j)(j) = 8$

$y(0) = \frac{1}{4}[36+16] = 13$

$y(2) = \frac{1}{4}[36-16] = 5$

$y(1) = \frac{1}{4}[36+8] = 11$

$y(3) = \frac{1}{4}[36-8] = 7$

$$y(n) = \{\underset{\uparrow}{13}, 11, 5, 7\}$$

3a The chirp z-transform

The chirp z-transform is used for evaluating z-transform of a sequence of M points in the z-plane which lie on circular or spiral contours beginning at any arbitrary point in the z-plane.

The z-transform of the sequence $x(n)$, which for discrete set of values of z_k can be written as

$$X(z_k) = \sum_{n=0}^{N-1} x(n) z_k^{-n}, \quad k = 0, 1 \ldots M-1 \qquad \text{where } z_k = e^{\frac{j2\pi k}{N}} = W_N^k$$

let z_k be a point on the spiral centered about the origin.

$$\therefore z_k = AB^{-k} \qquad \text{where } A = A_0 e^{j\theta_0} \text{ and } B = B_0 e^{-j\phi_0}$$

$$X(z_k) = \sum_{n=0}^{N-1} x(n) A^{-n} B^{kn}, \quad 0 \le k \le M-1 \quad \text{and } nk = \tfrac{1}{2}[n^2 + k^2 - (k-n)^2]$$

$$X(z_k) = \sum_{n=0}^{N-1} x(n) A^{-n} B^{n^2/2} B^{k^2/2} B^{-(k-n)^2/2}$$

let us define two functions: $g(n) = x(n) A^{-n} B^{n^2/2}$ and $h(n) = B^{-n^2/2}$

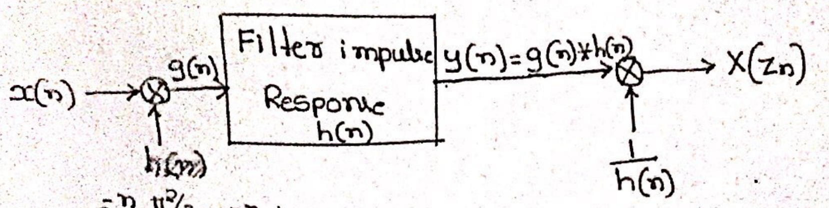
$$X(z_k) = \sum_{n=0}^{N-1} g(n) \frac{1}{h(n)} h(k-n), \quad 0 \le k \le M-1$$

Replacing k by n and n by k.

$$X(z_n) = \sum_{k=0}^{N-1} g(k) \frac{1}{h(n)} h(n-k), \quad 0 \le n \le M-1$$

The equation is recognized as the convolution of the sequences $h(n)$ and $x(n)$. hence

$$X(z_n) = \frac{1}{h(n)}\left[g(n) * h(n)\right]$$



$A^{-n} B^{n^2/2} = A^{-n} \frac{1}{h(n)}$

Block diagram representation for generating chirp z-transform.

If A and B_0 are unity, the sequence $h(n)$ is given by

$$h(n) = \left[e^{-j\phi_0}\right]^{-n^2/2} = e^{jn^2\phi_0/2} = e^{j\omega n}$$

$$\omega = \frac{n\phi_0}{2}$$

The above sequence can be considered as a complex exponential sequence with linearly increasing frequency.

3b $\quad x(n) = e^{\frac{j2\pi mn}{N}}, \quad 0 \le n \le N-1$

DFT: $X(k) = \sum_{n=0}^{N-1} x(n)\, e^{\frac{-j2\pi kn}{N}}, \quad n = 0, 1 \ldots N-1$

$$DFT\{x(n)\} = \sum_{n=0}^{N-1} e^{\frac{j2\pi mn}{N}} e^{\frac{-j2\pi kn}{N}}$$

$$= \sum_{n=0}^{N-1} W_N^{-mn} W_N^{kn}$$

$$X(k) = \sum_{n=0}^{N-1} W_N^{(k-m)n} \qquad \because \sum_{n=0}^{N-1} b^n = \frac{b^N - 1}{b-1}, \; b \ne 1$$

$$X(k) = \frac{W_N^{(k-m)N} - 1}{W_N^{(k-m)} - 1}, \quad k \ne m$$

$$X(k) = \frac{W_N^{kN} W_N^{-mN} - 1}{W_N^{(k-m)} - 1} = \frac{1-1}{W_N^{(k-m)} - 1} = 0, \quad k \ne m$$

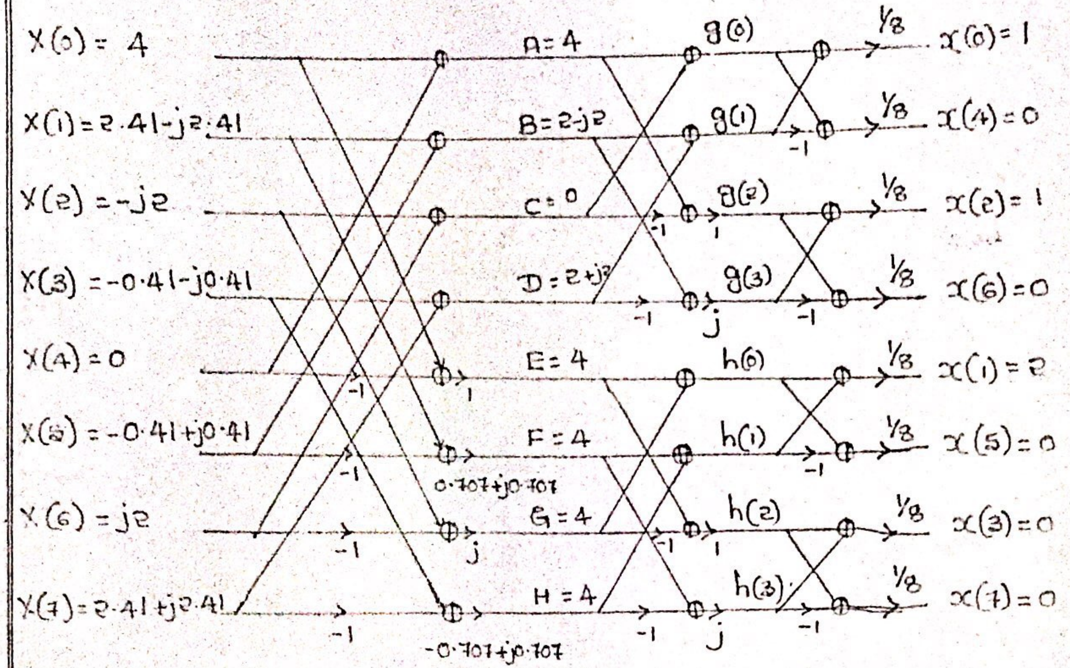
$$X(k) = \sum_{n=0}^{N-1} 1 = N, \quad k = m$$

$$\therefore X(k) = \begin{cases} 0, & k \ne m \\ N, & k = m \end{cases}$$

$$X(k) = N\delta(k-m), \quad 0 \le m \le N-1.$$

$X(k) = \{4,\ 2.41-j2.41,\ -j2,\ -0.41-j0.41,\ 0,\ -0.41+j0.41,\ j2,\ 2.41$

DIT-IFFT algorithm.



First stage:

$A = 4+0 = 4$

$B = 2.41-j2.41-0.41+j0.41$

$= 2-j2$

$C = -j2+j2 = 0$

$D = -0.41-j0.41+2.41+j2.41$

$= 2+j2$

$E = 4-0 = 4$

$F = (2.41-j2.41+0.41-j0.41)(0.707+j0.707)$

$F = 4$

$G = (-j2-j2)(j) = 4$

$H = (-2.82-j2.82)(-0.707+j0.707) = 4$

Second stage:

$g(0) = 4+0 = 4$

$g(1) = 2-j2+2+j2 = 4$

$g(2) = 4-0 = 4$

$g(3) = (2-j2-2-j2)\,j = 4$

$h(0) = 4+4 = 8$

$h(1) = 4+4 = 8$

$h(2) = 4-4 = 0$

$h(3) = (4-4)(j) = 0$

Third stage:

$x(0) = \frac{1}{8}[4+4] = 1$

$x(4) = \frac{1}{8}[4-4] = 0$

$x(2) = \frac{1}{8}[4+4] = 1$

$x(6) = \frac{1}{8}[4-4] = 0$

$x(1) = \frac{1}{8}[8+8] = 2$

$x(5) = \frac{1}{8}[8-8] = 0$

$x(3) = \frac{1}{8}[0+0] = 0$

$x(7) = \frac{1}{8}[0-0] = 0$

$x(n) = \{1,\ 2,\ 1,\ 0,\ 0,\ 0,\ 0,\ 0\}$