Chapter 4. Introduction to Sampling Process

Analog to digital communication.

Analog data: Speech signal, television signal etc.
Digital data: computer data, Telegraph signal

→ Reasons why people prefer digital communication

① Encryption
② high speed computers, very powerful software tools
③ Compatibility ir with internet.

→ Advantages of digital communicatn over analog communicatn
* Digital communication is simple and cost is less.
* " " is adaptive to advanced technologies like DSP, image & data processing.
* Noise is very less.

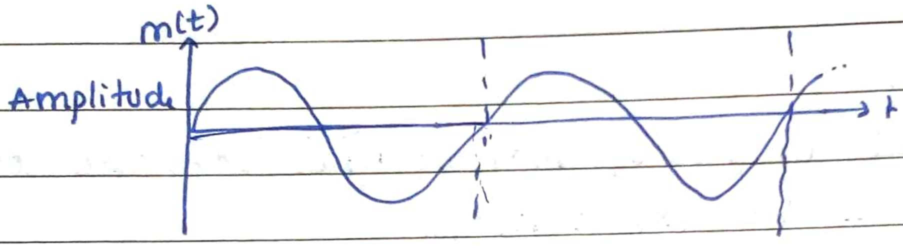
→ disadvantages of digital communication.
* It requires high bandwidth
* Synchronisation.

→ Analog to digital conversion:

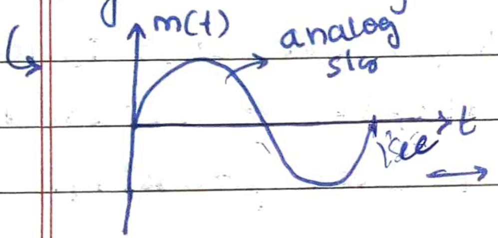
↳ operations / or steps:

① Sampling
② Quantization
③ Encoding

④ ANALOG signal → Sampling → Quantising → Encoding → DIGITAL signal



Let us take a segment of this message signal and try to understand



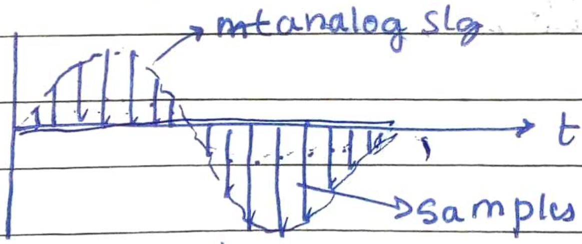
→ Let the info 1sec telephone call

→ continuous time sig

① **Sampling**: The process in which continuous time signal is converted into discrete time signal

→ discrete time sig is nothing collection of discrete time samples

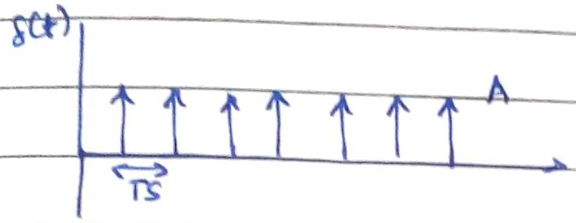
↳



↳ sampling rate = 15 samples per sec
15 Hz

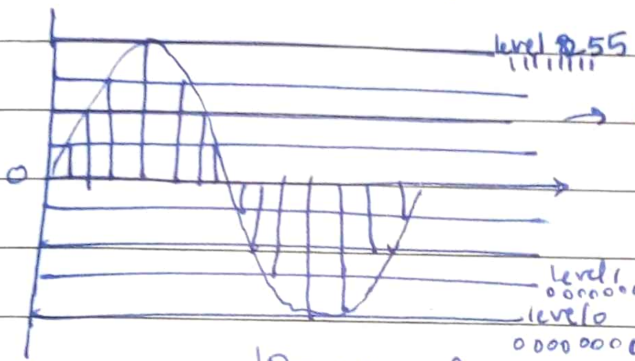
* Sampling ~~Frequency~~ rate: number of sampler per second.

→ We have to multiply message sig of with δ(t) to sample the signal.



Sample period = Ts

② Quantization: sampled waveform taken as i/p



→ Quantized waveform.

Amplitude of each samples of sampled sig are being matched with horizontal lines in Quantizing process

Bit depth : no. of bits per sample.

8-bit or 16-bit

8-bits → 2^8 level → 255 levels

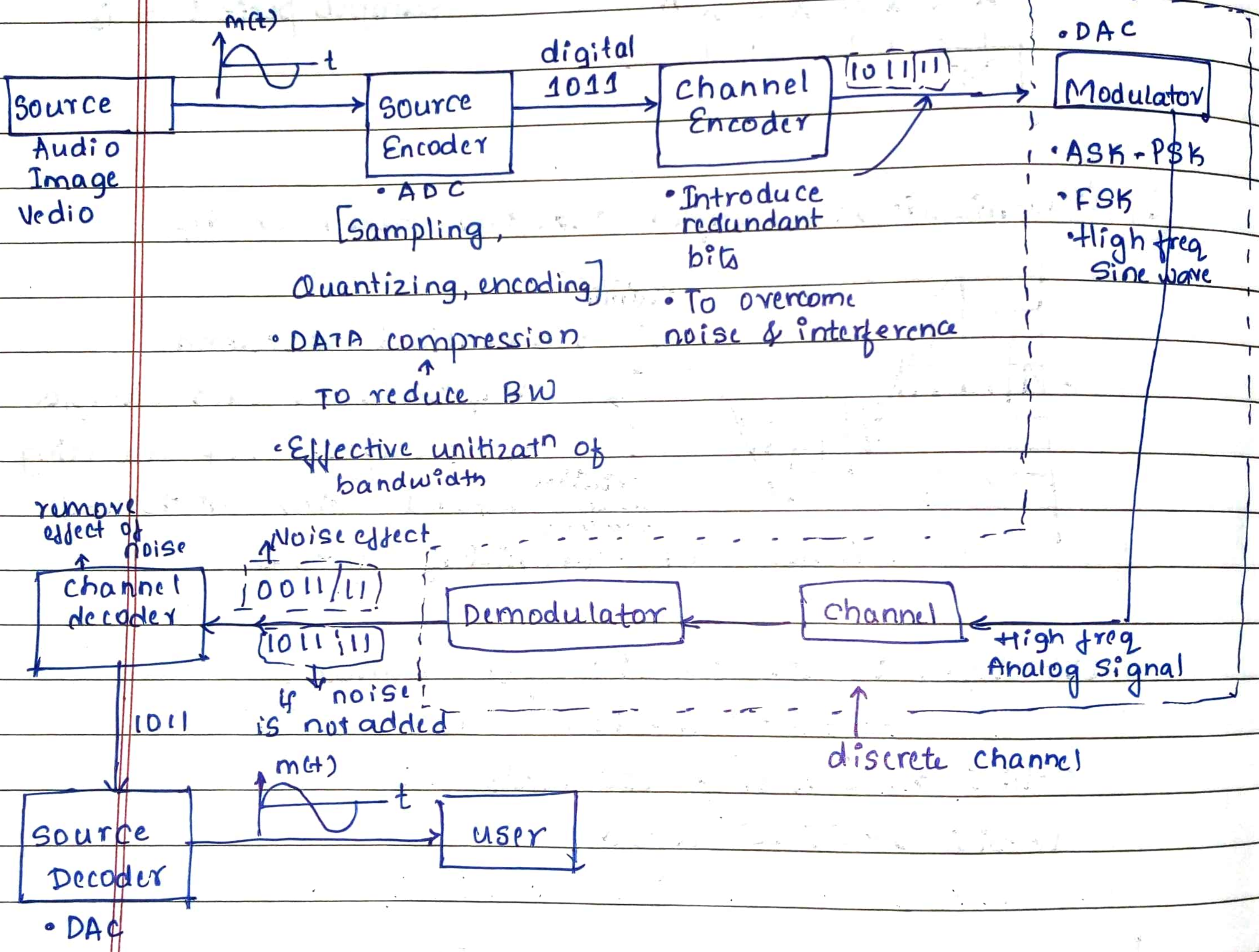
16-bits → 65535 levels

→

③ Encoding : converts each samples from right to level into a binary number.

→ The o/p of Encoding is digital signal

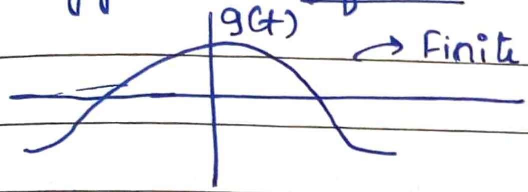
Block diadram of digital Communication System.



Sampling Theorem:

* Sampling process converts continuous time signal into a discrete time signal @ periodic intervals of time
* This discrete time signal to be a complete representation of continuous time signal, sampling ratio should be properly selected. This is done by using sampling theorem.

* Let us consider continuous time signal g(t) with finite energy and infinite duration.



→ Finite energy, Infinite duratⁿ

$g(t) = @ \; t = 0, T_s, 2T_s, 3T_s \ldots$

$g_n(T_s)$ = sampled values for $n = 0, \pm1, \pm2, \pm3 \ldots$

↓ arrive @

→ $g_\delta(t)$ → discrete time signal $= \sum_{n=\infty}^{\infty} g(nT_s)\, \delta(t-nT_s)$ → ①

↓ delta func

By property of delta func

$g_\delta(t) = \sum_{n=-\infty}^{\infty} g(t)\, \delta(t-nT_s)$ → ②

WKT

→ Multiplicatⁿ in time domain is convolutⁿ in fourier Tr

∴ apply FT.

$$G_\delta(F) = G(F) * f_s \sum_{m=-\infty}^{\infty} \delta(f - mf_s) \rightarrow (3)$$

After rearranging using property

$$G_\delta(F) = f_s \sum_{m=\infty}^{\infty} G(F) * \delta(F - mf_s) \rightarrow (4)$$

$$G_\delta(F) = F_s \sum_{m=-\infty}^{\infty} G(F - mF_s) \rightarrow (5)$$

$$\boxed{\text{W.K.T} \quad \delta(t - nT_s) = \exp(-j2\pi nfT_s)}$$

$\therefore$ equat^n (1) becomes

$$g_\delta(t) = \sum_{n=-\infty}^{\infty} g(nT_s) \exp(-j2\pi nfT_s) \rightarrow (6)$$

$$T_s = \frac{1}{2W}, \quad -W < f < W$$

$$g_\delta(t) = \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \exp\left(\frac{-j\pi nf}{W}\right) \rightarrow (7)$$

Let us consider cond^n where $f_s = 2W$ and $-W < F < W$ in equat^n (5)

$$G_\delta(f) = 2W\, G(F)$$

$$G(F) = \frac{1}{2W} G_\delta(f) \rightarrow (8)$$

To reconstruct original signal we use inverse fourier Transform

$$\boxed{g(t) = \int_{t=-\infty}^{\infty} G(F)\exp(j2\pi ft)\,dt} \longrightarrow 9$$

Now substitute 8 in eqn 9

$$g(t) = \int_{t=-\infty}^{\infty} \frac{1}{2W} G_\delta(f)\exp(j2\pi ft)\,dt$$

$$g(t) = \frac{1}{2W}\sum_{m=-\infty}^{\infty}\int_{t=-\infty}^{\infty} g\left(\frac{n}{2W}\right)\exp\left(j\frac{2\pi nf}{W}\right)ex(j2\pi ft)\,dt$$

$$\boxed{g(t) = \sum_{m=-\infty}^{\infty} g\left(\frac{n}{2W}\right)\operatorname{sinc}(2Wt-n)} \longrightarrow 10$$

* Sampling theorem : This theorem is the condn in w the discrete time signal would be complete represen -tatn of continuous time sig by setting sampling frequency $f_s \geq 2W$

Practical aspects of Sampling :

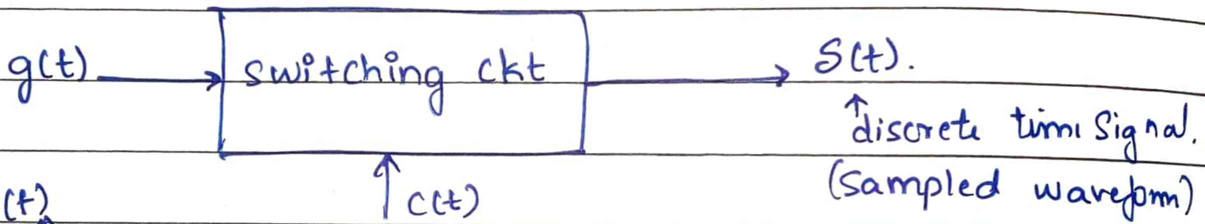
In practical case, sampled wave deviates from sampled wave of ideal case

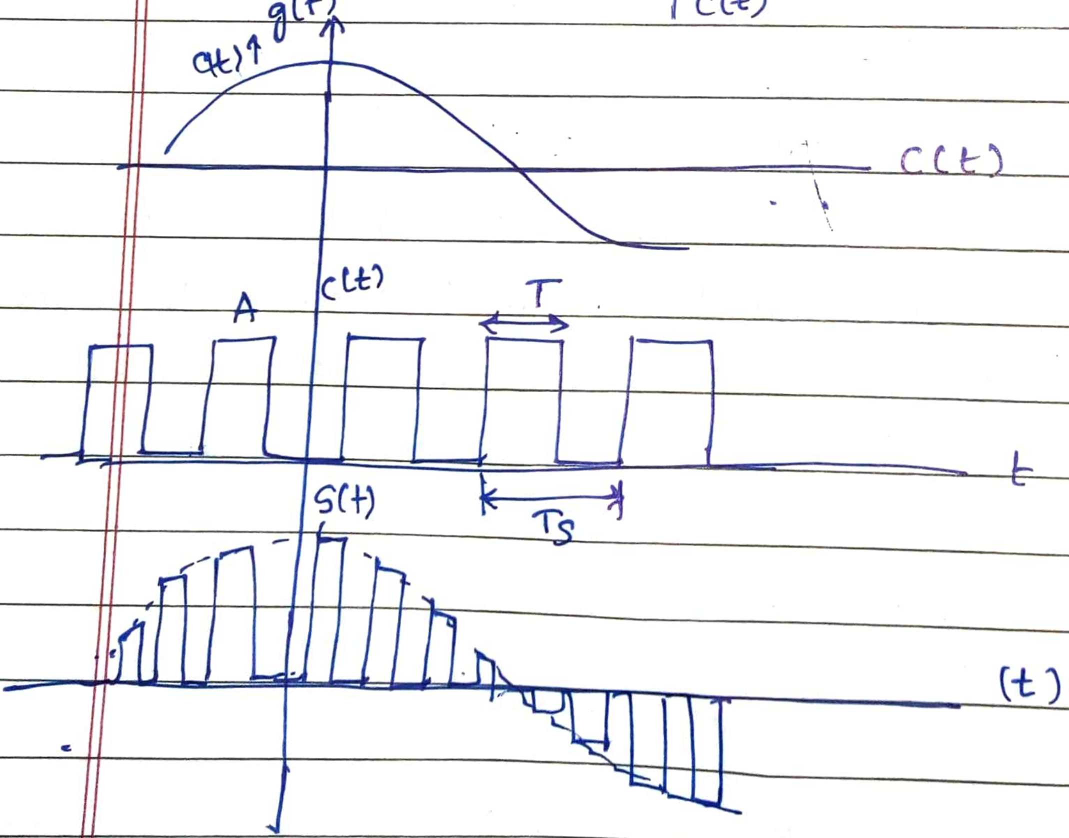
→ Natural Sampling } These two are due to non linearities in
→ flat top pulses } transmitter circuit

① Natural Sampling :-

Let us consider continuous time signal of finite durat^n & apply that to switching circuit.

The Switching ckt is controlled by c(t) → sampling func





c(t) is sampling funct^n

In the Natural Sampling, the amplitude of each samples will be different. so flat top pulses is preferred.

from the fig

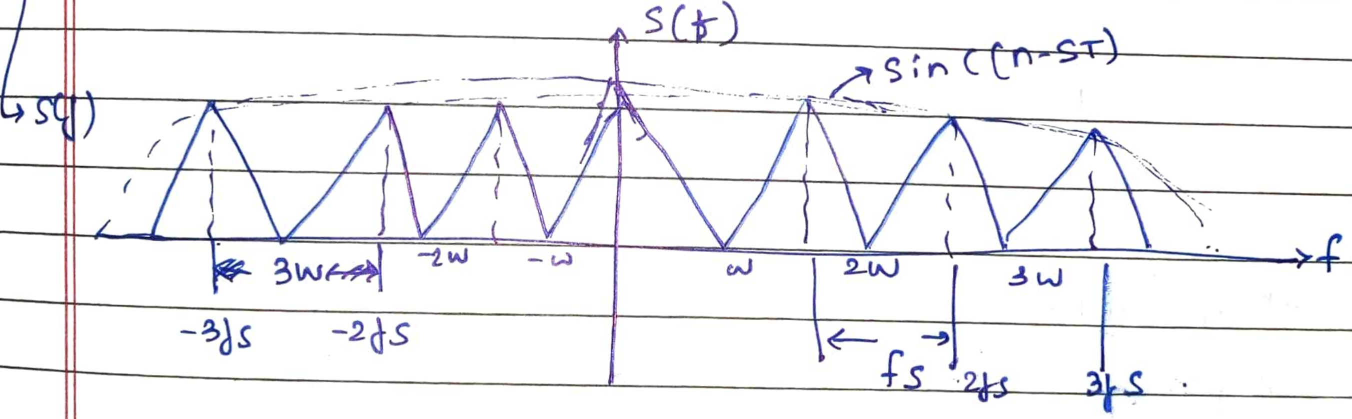
$$S(t) = g(t)c(t) \rightarrow ①$$

$$c(t) = f_s TA \sum_{n=-\infty}^{\infty} \text{sinc}(nf_sT)\exp(j2\pi nf_st)$$

$$\boxed{\therefore\ S(t) = f_sTA \sum_{n=-\infty}^{\infty} \text{sinc}(nf_sT)\exp(j2\pi nf_st)\,g(t)} \quad ③$$

Take F.T of ③

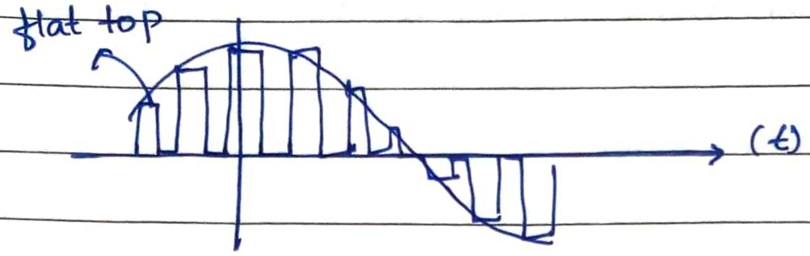
$$\boxed{S(f) = f_sTA \sum_{m=-\infty}^{\infty} \text{sinc}(n\pi f_sT)\,G(f - mf_s)} \rightarrow ④$$



→ upon increasing frequency the amplitude ↓

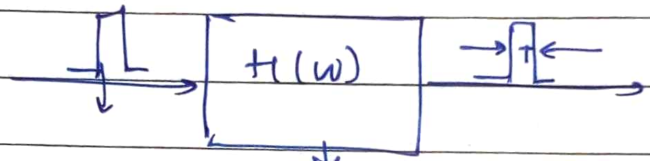
∴ amplitudes scaled by $\frac{T_s}{N}$. this the problem with natural sampling

→ ~~But~~ (b) Flat-top Sampling



We prefer flat top more than Natural Sampling

→ where we have const amplitude



$$h(t) = \begin{cases} 1 & t < T \\ 0 & \text{otherwise} \end{cases}$$

$$\boxed{s(t) = \sum_{n=-\infty}^{\infty} g(nT_s)\, h(t-nT_s)} \rightarrow (1)$$

→ Instantaneous Sample

$$\text{sampled} \leftarrow g_\delta(t) = \sum_{n=-\infty}^{\infty} g(nT_s)\, \delta(t-nT_s) \rightarrow (2)$$

$$\boxed{s(t) = g_\delta(t) * h(t)}$$

Apply F.T

$$S(f) = G_\delta(f) \cdot H(\omega) \rightarrow (4)$$

$$G_\delta(f) = f_s \sum_{m=-\infty}^{\infty} G(f-mf_s) \rightarrow 5$$

5 in 4 $$\boxed{S(f) = f_s \sum_{m=-\infty}^{\infty} G(f-mf_s)\, H(f)}$$

spectrum of sampled wave in flat-top samples.

H(f) is freq response of rectangular.

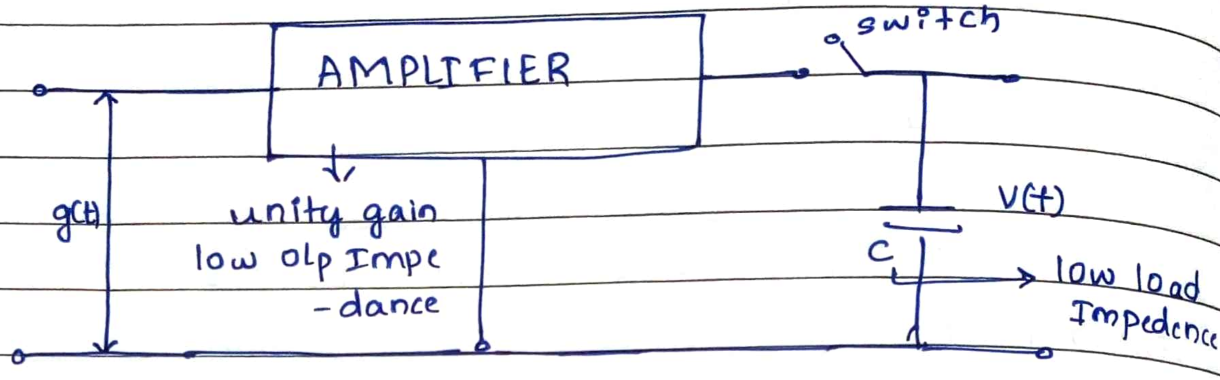
$$H(f) = T\,\text{sinc}(fT)\,\exp(-j\pi f t) \longrightarrow (7)$$

In this also amplitude get scaled by $\frac{T_s}{N}$.

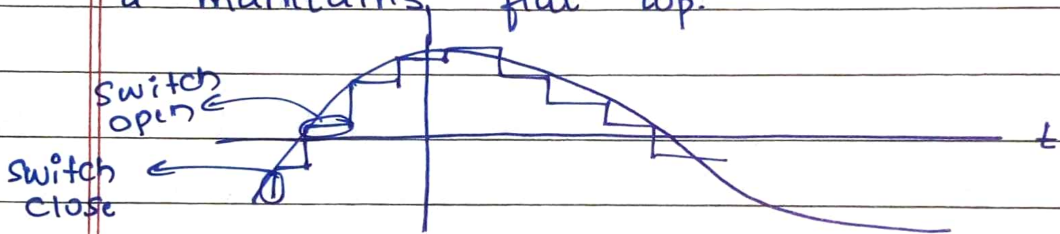
To overcome the problem from Natural sampling & flat-top sampling we go for sample & hold circuit.

→ Apperture effect : error introduced if we use flat top sampling.

Sample and hold Circuit.



When switch is closed that time capacitor rapidly charges upto the voltage level of each sample.
& when switch is open the capacitor retains voltage & maintains flat top.

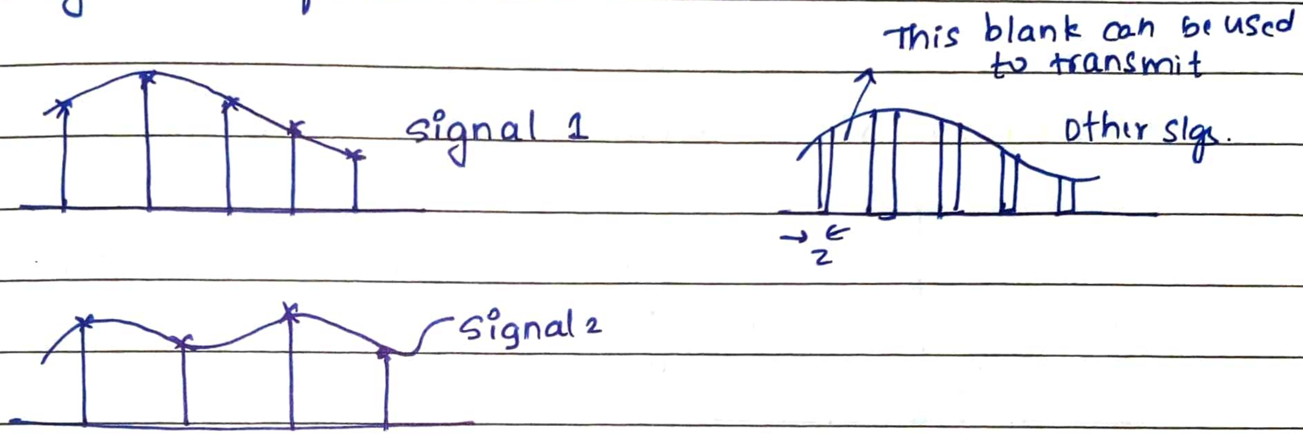


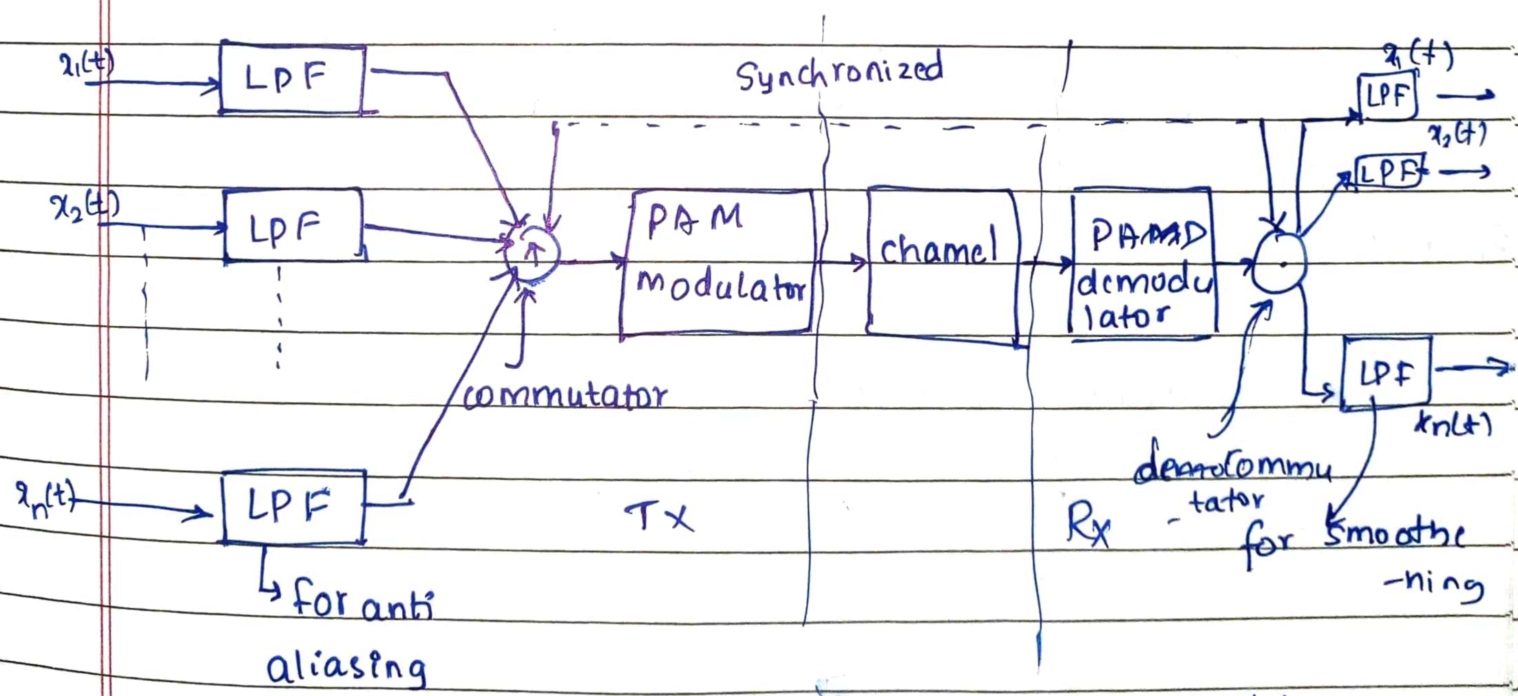
Amplifier is used to maintain ratio of $\frac{T_s}{n!}$

reconstructn phase :

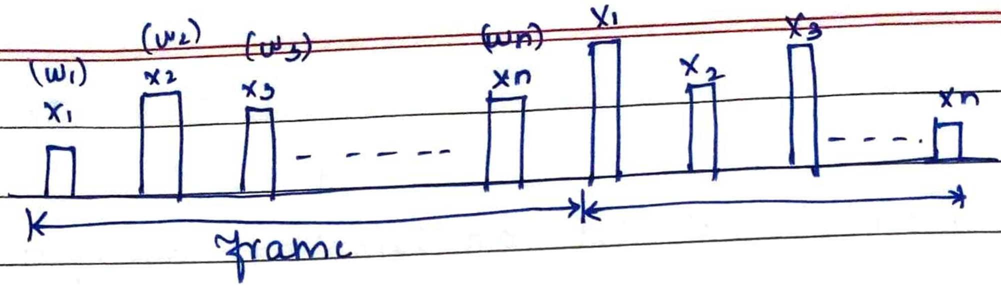
TIME DIVISION MULTIPLEXING: (TDM)

- The sampled PAM (flat top sampling) waveform is off for most of time.
- During the OFF period, the channel can be used to transmit samples of other waveforms
- The concept of inter leavings samples from serval signals into a single waveforms is called TDM.





→ Commutator & decommutators works as switch.

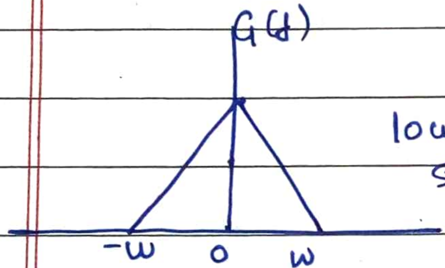


• The minimum bandwidth (TDM - PAM)

$W \Rightarrow W_1 + W_2 + W_3 + \dots + W_n$

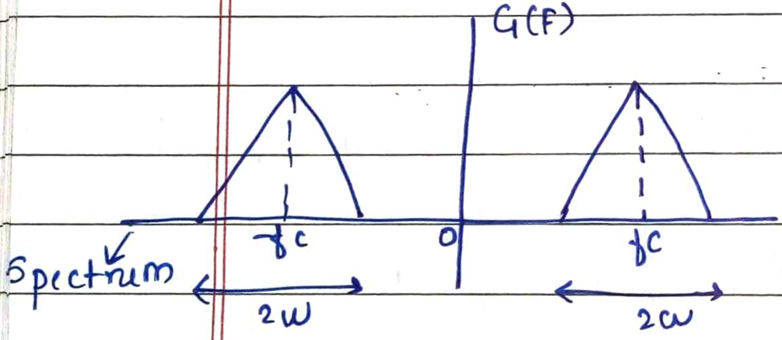
$$\boxed{W = \sum_{i=1}^{N} W_i}$$

* Quadrature Sampling of bandpass Signal:



low pass signal

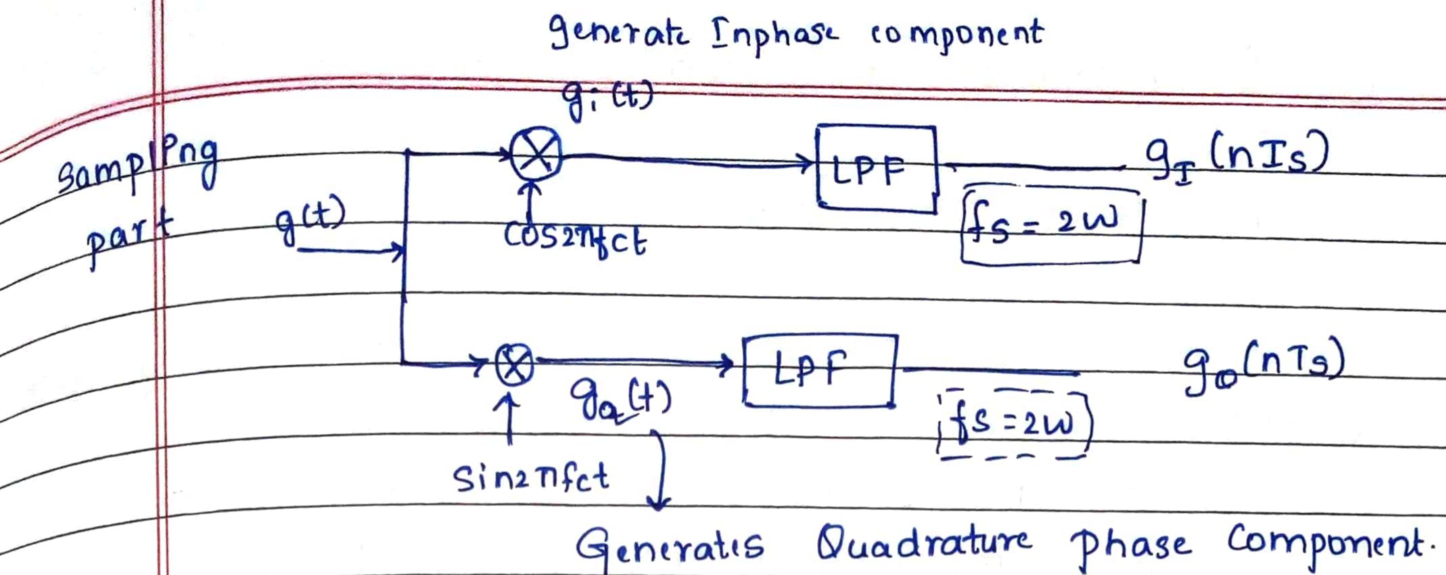
Signal is centred about zero Hz & BW = 2W

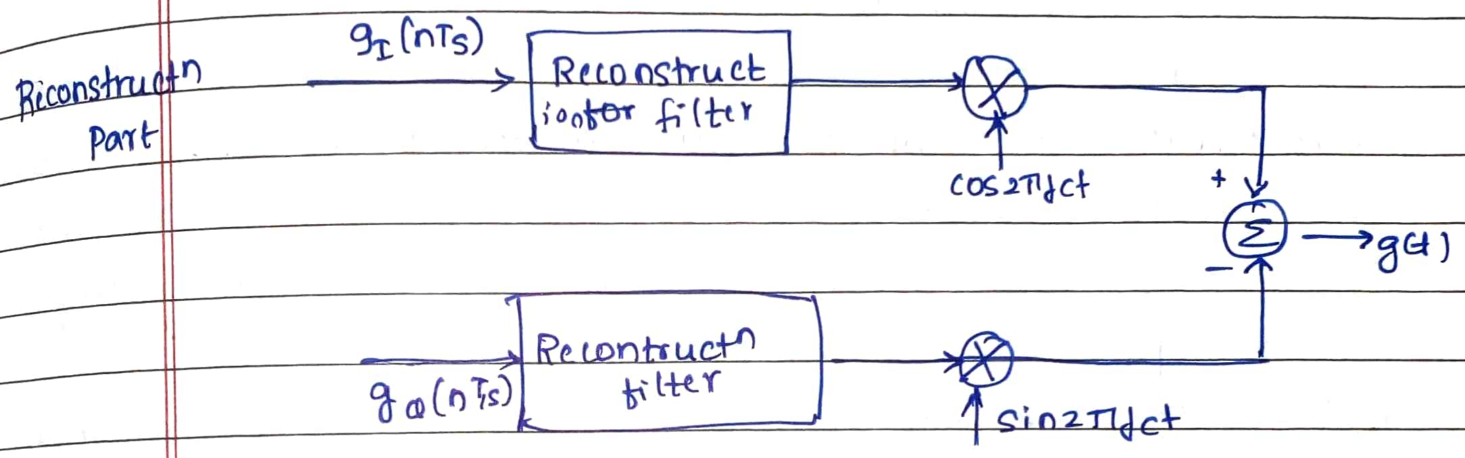


Bandpass signals

signals are centred @ fc & -fc & BW = 2W

* Quadrature samplings is used for low pass Signal → false (used for Bandpass Signal - True).





$$\boxed{g(t) = g_I(t)\cos 2\pi f_c t - g_q(t)\sin 2\pi f_c t}$$

quadrature signal $g(t) = g_{I}(t)\cos(2\pi f_c t) - g_q(t)\sin(2\pi f_c t)$

in phase component Quadrature component

Quantization and Coding techniques:

PULSE CODE MODULATION:

→ Pulse-code Modulation is a technique for rounding off the amplitudes of samples of an analog waveform.
This is the second of two operations required to change an analog signal into a digital signal. The rounding-off operation is known as quantization.

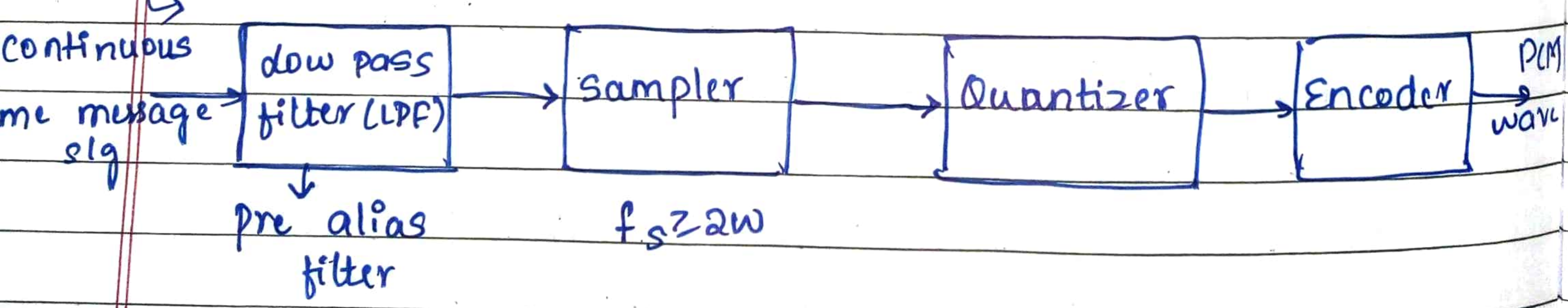
→ One each sample value is rounded to the appropriate quantizatn level, we need to transmit only enough info so that receiver knows which level is being sent. The various levels are binary coded & the binary code corresponding to the particular round-off level is sent.

→ Thus, the essential operations in the PCM transmitter are sampling, quantizing and encoding.

→ PCM systems are complex in that the message signal is scubjected to a large number of operations

Basic Elements in PCM System:

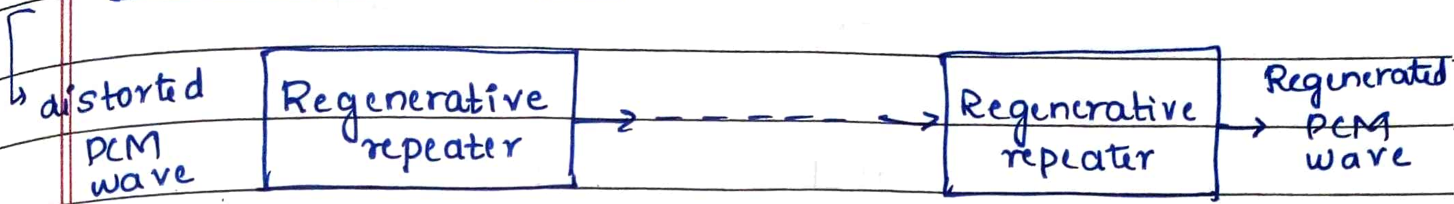
Transmitter:



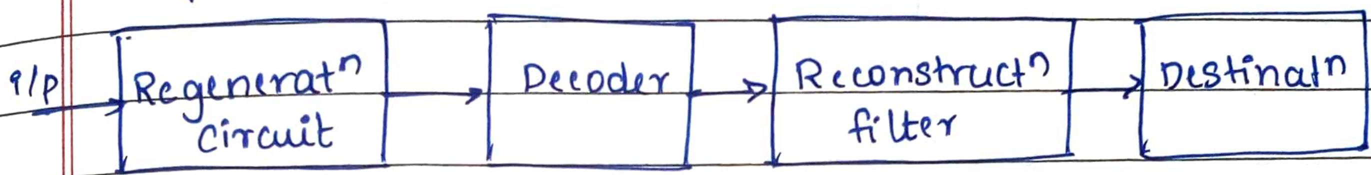
Quantization & coding Techniques.

Pulse-code Modulation:

Transmission path



Receiver:



① **Sampling:**

The incoming message wave is sampled with a train of narrow rectangular pulses so as to closely approximate the instantaneous sampling process.

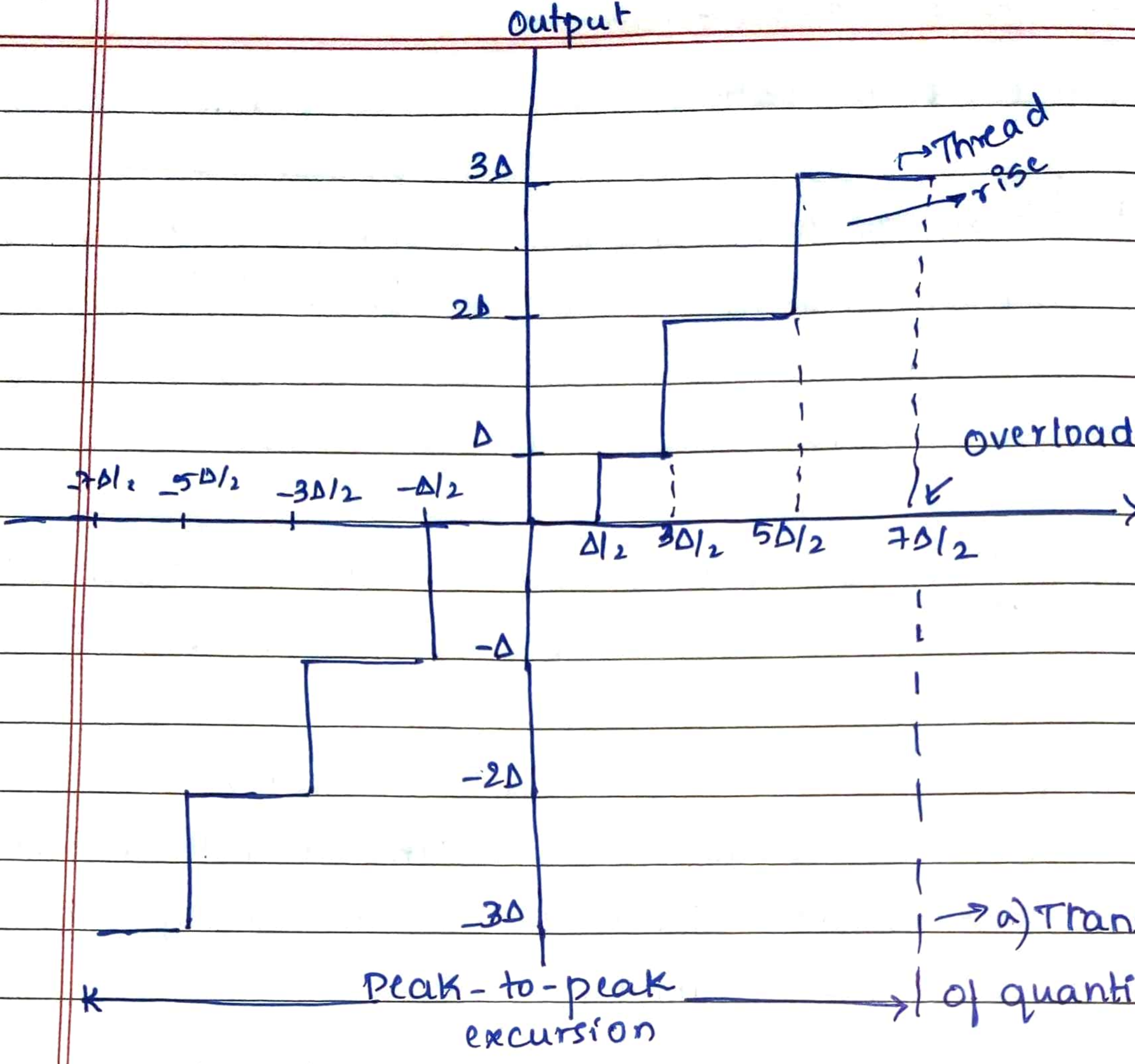
② **Quantizing:**

The conversion of an analog sample of the signal into a digital form is called as quantizing process.

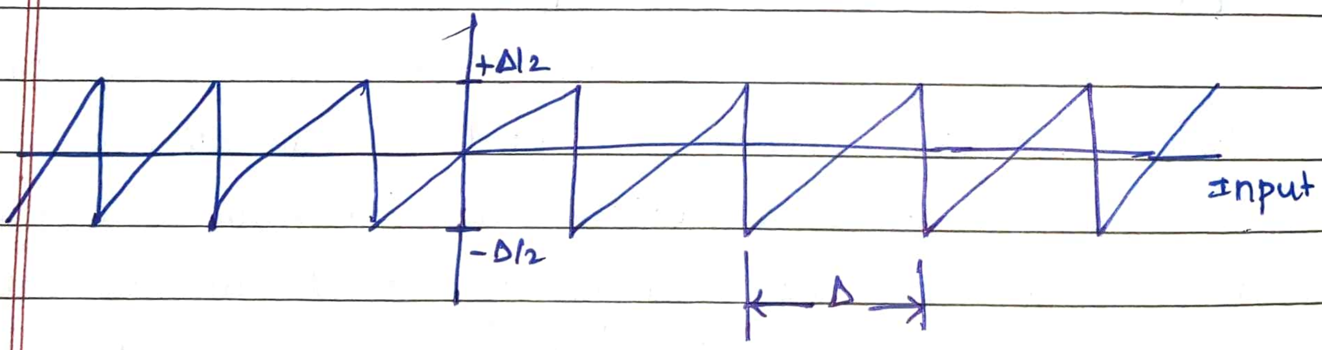
The quantizing process has a two fold effect

1) The peak to peak range of input sample values is subdivided into a finite set of decision levels or decision thresholds that are aligned with the "risers" of the staircase.

2) The o/p is assigned a discrete value selected from a finite set of representatⁿ levels or reconstructⁿ values that are aligned with "threads" of staircase



a) Transfer characteristic of quantiser of midtread type



b) variatn of quantizatn error with i/p

Acc to fig ②, the decision thresholds of quantizer are located @ $\pm \Delta/2, \pm 3\Delta/2, \pm 5\Delta/2, \ldots$ & representatⁿ levels are located @ $0, \pm \Delta, \pm 2\Delta, \ldots$ where Δ is step size.

→ A uniform quantiser characterized in this way is referred to as a symmetric quantizer of midtread type, because the origin lies in middle of tread of staircase

→ If decision threshold are located @ $0, \pm \Delta, \pm 2\Delta$ & representatⁿ levels are located @ $\pm \Delta/2, \pm 3\Delta/2 \ldots$ then it is called as symmetric quantizer of midriser type, becoz in this case origin lies in middle of a riser of staircase.

→ Overload level → absolute value of w is one half of peak-to-peak range of i/p sample values.

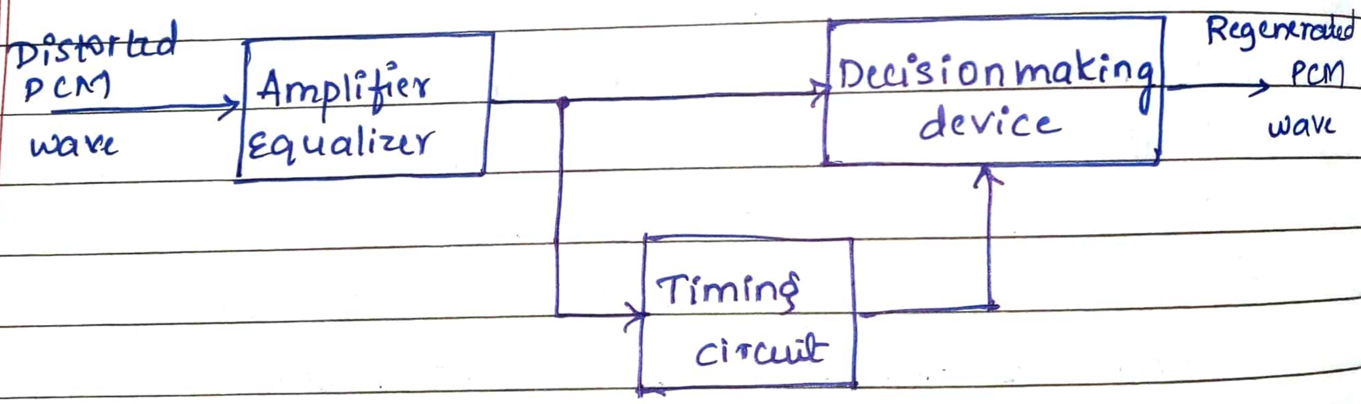
3) Encoding: translating the discrete set of sample values to a more appropriate form of signal.

4) Regeneration:

The most important feature of PCM systems lies in the ability to control the effects of distortion & noise produced by transmitting a PCM wave through a channel.

This capability is accomplished by reconstructing PCM wave by means of chain of regenerative repeaters located @ sufficiently close spacing along transmission route.

Regenerative Repeater : performs equalizatn, timing & decision making



* Equilizer shapes received pulses so as to compensate for effect of amplitude & phase distortn's produced by imperfectn's in transmission characteristics of channel.
* Timing ckt provides a periodic pulse train, derived from received pulses, for sampling the equalised pulses @ instants of time where SNR is max.

* Decision device is enabled when, @ sampling time determined by the timing ckt, the amplitude of equalized pulse plus noise exceeds a predetermined voltage level

QUANTIZATION NOISE & SIGNAL-TO-Quantizatⁿ-NOISE RATIO

→ Quantization noise is produced in the transmitter end of PCM system by rounding off sample values of an analog baseband signal to nearest permissible representatⁿ levels of quantizer.

→ Let 'L' be total representatⁿ levels of memoryless quantizer,

$x \rightarrow$ be quantizer input,

y be quantizer output

By Transfer characteristics of quantizer

$$y = Q(x)$$

Q be uniform distributed over sig $-\Delta$ to $\Delta/2$

W is staircase func.

Suppose input x lies inside interval

$$\mathcal{I}_k = \{x_k < x \leq x_{k+1}\}, \quad k = 1, 2, \ldots L$$

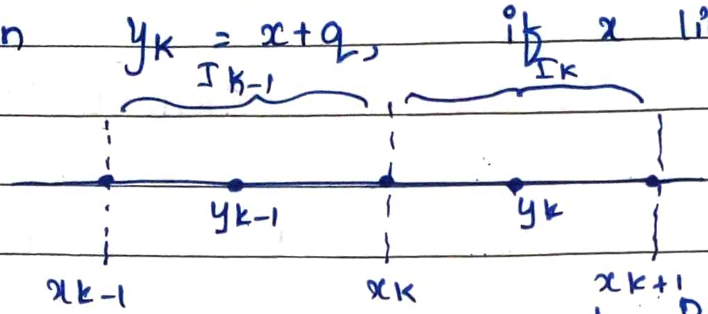
where x_k and x_{k+1} are decision thresholds of interval $\mathcal{I}_k$ as depected in figure

correspondingly o/p y takes discrete values y_k, $K = 1, 2 \ldots L$ That is,

$y = y_K$, if x lies in the interval $\mathcal{I}_K$

Let q denotes quantizatn error, with values in range $-\Delta/2 \leq q \leq \Delta/2$.

then $y_K = x + q$, if x lies in $\mathcal{I}_K$ interval



↳ Decision thresholds of quantizer

We assume that quantizer i/p x is sample value of a random variable X of zero mean & variance σ_X^2.

Let random variable Q denote quantizatn error, & q denote its sample value.

We assume Q is uniformly distributed over range $-\Delta/2$ to $\Delta/2$,

$$f_Q(q) = \begin{cases} 1/\Delta & -\Delta/2 \leq q \leq \Delta/2 \\ 0 & \text{otherwise.} \end{cases}$$

$f_Q(q)$ is probability density func of quantizatn error.

Variance

$$\sigma_Q^2 = E[Q^2] \longrightarrow \text{mean square value}$$

$$= \int_{-\infty}^{\infty} q^2 f_Q(q)\, dq$$

$$\therefore \sigma_Q^2 = \frac{1}{\Delta} \int_{-\Delta/2}^{\Delta/2} q^2 \, dq$$

$$= \frac{\Delta^2}{12}$$

When baseband slg is reconstructed at receiver o/p, we obtain the original slg plus quantizatn noise.

$\therefore$ we may define an o/p slg to quantizatn noise ration (SNR) as

$$(SNR)_o = \frac{\sigma_x^2}{\sigma_Q^2} = \frac{\sigma_x^2}{\Delta^2/12}$$

* Demerits of PCM:

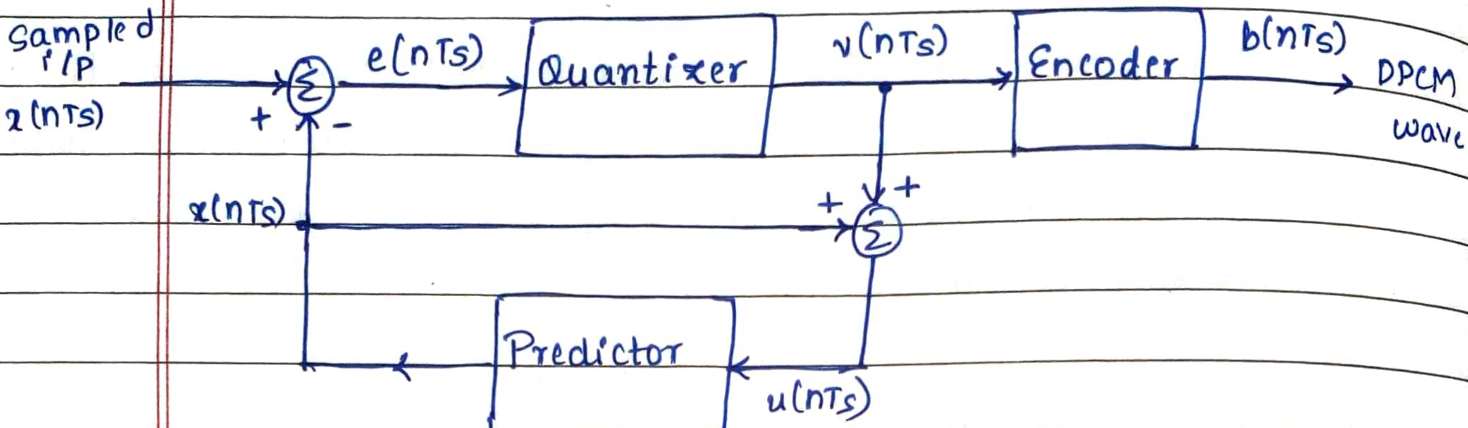
① Increase no. of levels, no. of bits increase ↑ Bandwidth

② Step size ↑, Qerror ↑, SNR ↓

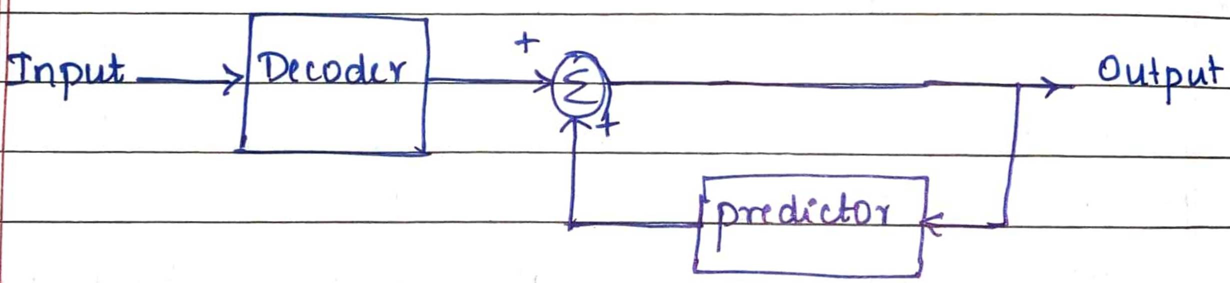
③ High degree of correlatn bt any two adjacent samples.

DIFFERENTIAL PULSE CODE MODULATON [DPCM].

DPCM Transmitter



DPCM Receiver



→ As we know we have High degree of correlat^n in PCM. To overcome this we use DPCM

→ Suppose a baseband Sig $x(t)$ is sampled @ $f_s = 1/T_s$ to produce a sequence of correlated samples T_s seconds apart.

Let the sequence be denoted by $\{x(nT_s)\}$, where 'n' takes on integer values. In this scheme the input to the quantizer is a signal

$$e(nT_s) = x(nT_s) - \hat{x}(nT_s)$$

w is a diff bt the unquantised i/p sample $x(nT_s)$ and a predictⁿ of it, denoted by $\hat{x}(nT_s)$.

$e(nT_s)$ is called as predictⁿ error, since it is the amount by w the predictor fails to predict input exactly. By encoding quantizer o/p we obtain an imp variatⁿ of PCM, w is DPCM.

Let non linear func $Q(\cdot)$ define i/p - o/p characteristic of the quantizer. The quantizer o/p may be represented as

$$v(nT_s) = Q[e(nT_s)]$$
$$= e(nT_s) + q(nT_s)$$

where $q(nT_s)$ is quantizatⁿ error

predictor input $u(nT_s) = \hat{x}(nT_s) + v(nT_s)$

$\therefore$ $u(nT_s) = \hat{x}(nT_s) + e(nT_s) + q(nT_s)$

$$\boxed{\therefore u(nT_s) = x(nT_s) + q(nT_s)}$$

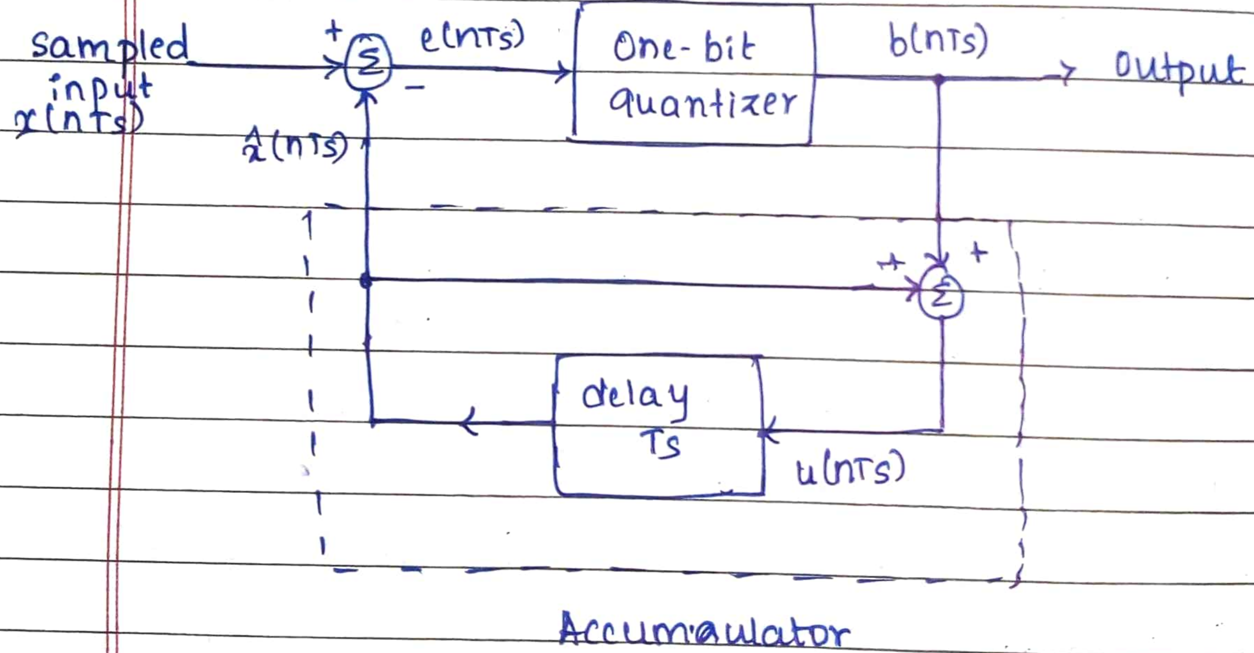
w represents a quantized version of i/p slg $x(nT_s)$

DELTA MODULATION:

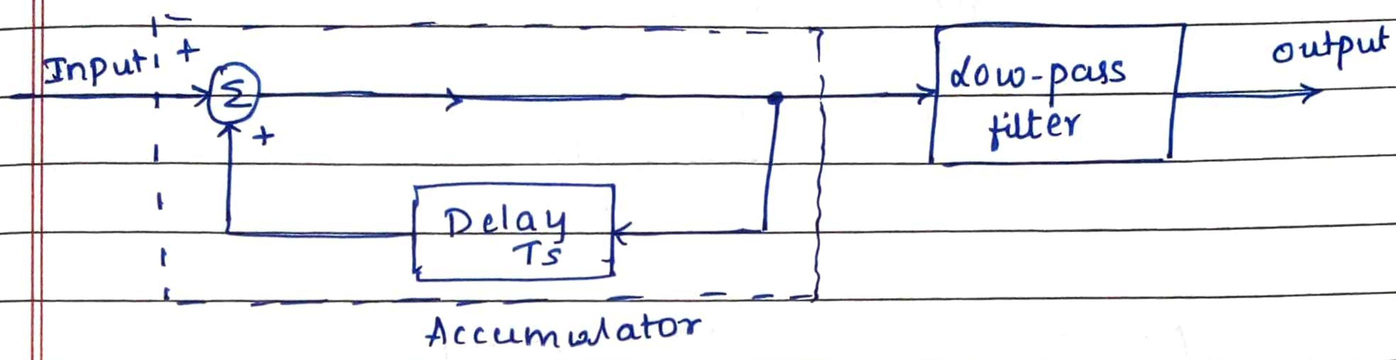
→ one bit version of DPCM.
→ Transmits only one bit per sample
→ generates another a staircase kind of sig.
→ Step size is fixed.

Adv: Transmits only one bit sample
Signaling rate & BW is reduced
Tx & Rx implementatⁿ is simple
→ ADC is not involved

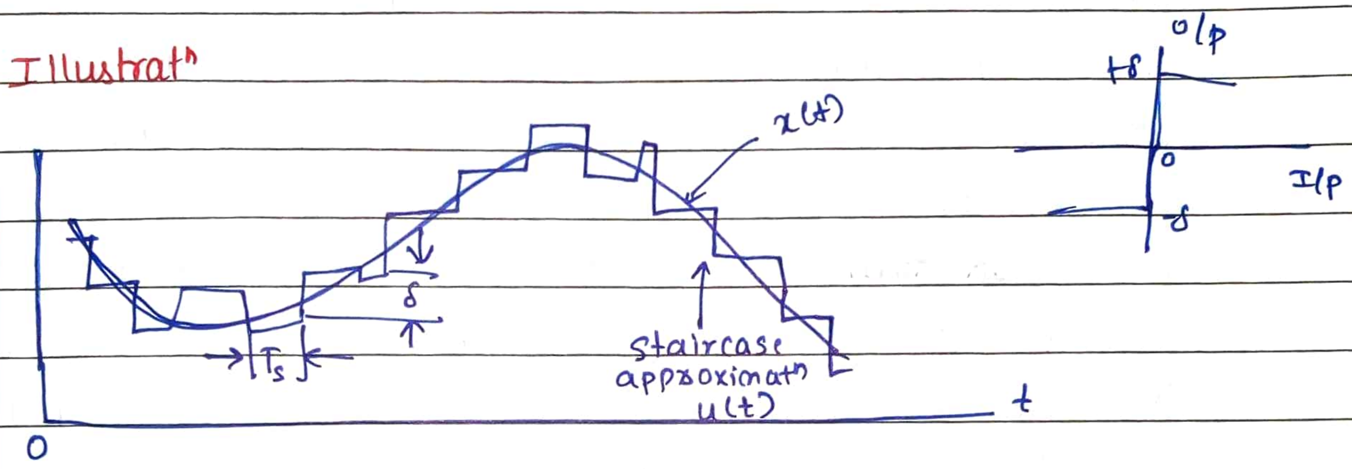
DM transmitter



DM receiver.



Illustratn



Basic principle of Δ modulatn may be formalised as following set of discrete - time rlns:

$$e(nT_s) = x(nT_s) - \hat{x}(nT_s)$$

$$= x(nT_s) - u(nT_s - T_s)$$

$$b(nT_s) = \delta \operatorname{sgn}(e(nT_s))$$

$$u(nT_s) = u(nT_s - T_s) + b(nT_s)$$

T_s = Sampling period

$e(nT_s)$ = predictn error

$\hat{x}(nT_s) = u(nT_s - T_s)$

→ The binary quantity $b(nT_s)$ is algebraic sign of error $e(nT_s)$, except for scaling factor δ.

$b(nT_s)$ is one-bit word transmitted by the DM System.

$$u(nT_s) = \delta \sum_{i=1}^{n} sgn\left[e(iT_s)\right]$$

$$= \sum_{i=1}^{n} b(iT_s)$$

ADAPTIVE DELTA MODULATION:

The performance of a delta modulator can be improved significantly by making the step size of the modulator assume a time-varying form.

In particular, during a steep segment of the input signal the step size is increased. Conversely, when the i/p sig is varying slowly, the step size is reduced. In this way the step size is adapted to the level of the input signal. The resulting method is called as adaptive delta modulatn (ADM).

In practical implementatns of the system, the step size $\Delta(nT_s)$ or $2\delta(nT_s)$ is constrained to lie between min & max values. In particular, we write

$$\delta_{min} \leq \delta(nT_s) \leq \delta_{max}$$

upper limit δ_{max}, controls amnt of slope-overload distortn.
lower " δ_{min}, controls amount of idle channel noise.
Inside these limits, adaptn rule for $\delta(nT_s)$, is expressed in general form

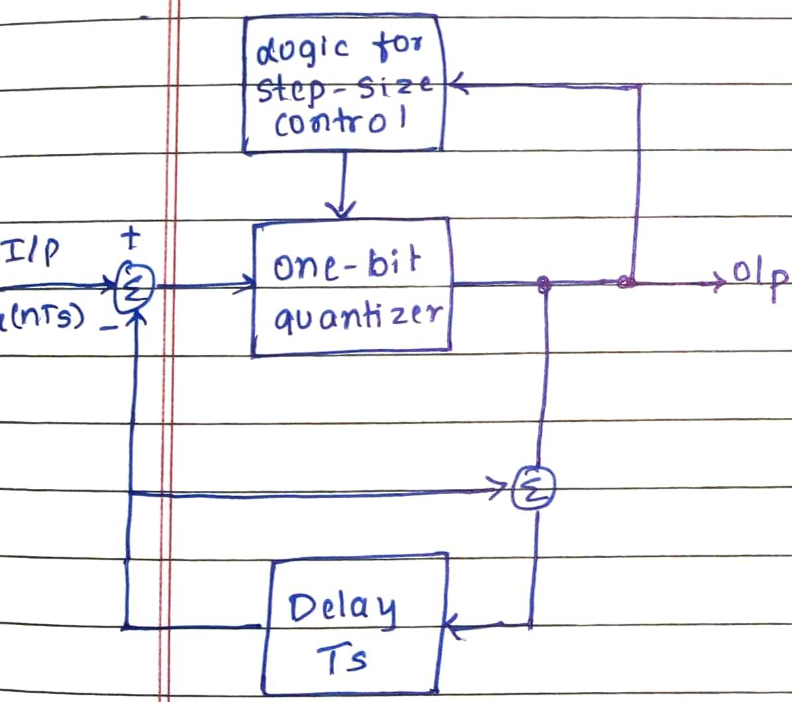
$$\delta(nT_s) = g(nT_s)\,\delta(nT_s - T_s)$$

where time-varying multiplier $g(nTs)$ depends on present binary o/p $b(nTs)$ of delta modulator & M previous values $b(nTs - Ts), \ldots b(nTs - MTs)$.

$$g(nTs) = \begin{cases} K & \text{if } b(nTs) = b(nTs - Ts) \\ K^{-1} & \text{if } b(nTs) \neq b(nTs - Ts) \end{cases}$$

This adaptn is called const factor ADM with one bit memory, where the term "one-bit memory" refers to the explicit utilizatn of single previous bit $b(nTs - Ts)$.

→ ADM Transmitter.



→ ADM receiver