

CHAPTER - 1



Principle of Amplitudes:

⇒ Purpose of communication system:-

- is to transmit information bearing signal through communication channel from one point to another through succession of process
- 1) The generation of message signal: voice, music, picture or computer data
- 2) The description of that message signal with certain measure of precision by a set of symbols: electrical, aural or visual
- 3) The encoding of these symbols in a form that is suitable for transmission over a physical medium
- 4) The transmission of the encoded symbols to the desired destinations
- 5) The decoding and reproduction of original symbols
- 6) The recreation of the original signal ~~with~~ ~~is~~ by and also eliminating the noise
Noise is added in during transmissions due to imperfections in the system.

Transmitted power:- Its the average power of the transmitted signal

Channel Bandwidth: Its defined as the band of frequencies allocated for the transmission of the message signal

Noise: refers to unwanted waves that tend to disturb the transmission and processing of message signal in a communication system.

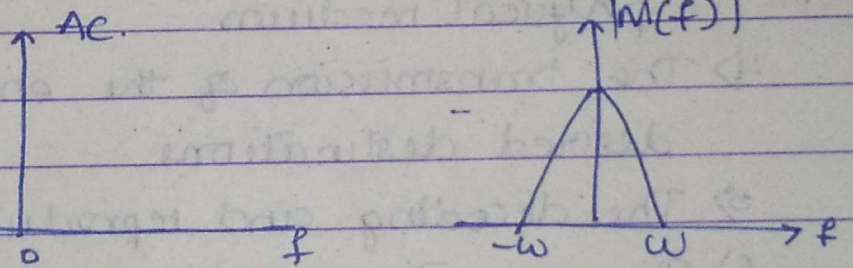
Sources may be internal or external to the system

Modulation:-

It is a process of frequency translation which imports the source information on to a Bandpass signal with carrier frequency f_c . By the introduction of amplitude or phase variations or both in accordance with modulating signal

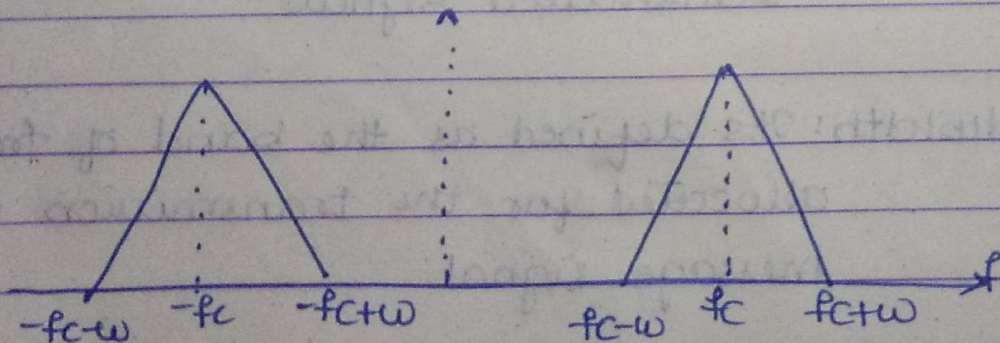
Base band:

Base band waveform has a spectral magnitude is non zero for frequencies in the vicinity of origin and negligible elsewhere



Band pass:

Band pass signal has a spectrum magnitude is non zero for a frequencies in some band concentrated above the frequencies $f = \pm f_c$ where $f_c > 0$. The spectral magnitude is 0 elsewhere



Modulation: $\rightarrow$ Amplitude Modulation
 $\rightarrow$ Angle Modulation $\rightarrow$ frequency Modⁿ
 $\rightarrow$ phase Modⁿ

Amplitude Modulation:

Its a process of changing amplitude of carrier signal in accordance with instantaneous values of message signal keeping frequency and phase constant.

Derivation

$m(t)$ - message signal / Modulating signal

$c(t)$ - carrier signal

$s(t)$ - Modulated signal

$$c(t) = A_c \cos 2\pi f_c t$$

A_c - carrier amplitude

f_c - carrier frequency.

$$s(t) = [A_c + m(t)] \cos 2\pi f_c t$$

$$= A_c [1 + \frac{1}{A_c} m(t)] \cos 2\pi f_c t$$

$$\left[s(t) = A_c [1 + k_a m(t)] \cos 2\pi f_c t \right] \quad \left\{ \frac{1}{A_c} = k_a \right.$$

general equation
for AM

k_a - amplitude sensitivity

$$\mu = |k_a m(t)|_{\max}$$

$$k_a = \frac{\mu}{m(t)_{\max}}$$

$$\mu = \frac{A_m}{A_c}$$

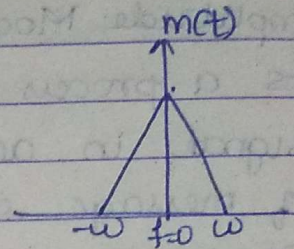
$$g \quad G(f) = \int_{-\infty}^{\infty} g(t) e^{-j2\pi f t} dt$$

μ is modulation index or depth of modulation or degree of modulation or % of modulation or modulation factor or modulation ratio

$$\% \mu = \frac{A_m}{A_c} \times 100$$

$$s(t) = A_c \{1 + k_a m(t)\} \cos 2\pi f_c t$$

Applying FT



$$s(t) = A_c \cos 2\pi f_c t + A_c k_a m(t) \cos 2\pi f_c t$$

$$S(f) = \int_{-\infty}^{\infty} s(t) e^{-j2\pi f t} dt$$

$$S(f) = \int_{-\infty}^{\infty} (A_c \cos 2\pi f_c t + A_c k_a m(t) \cos 2\pi f_c t) e^{-j2\pi f t} dt$$

$$S(f) = \int_{-\infty}^{\infty} A_c \cos 2\pi f_c t e^{-j2\pi f t} dt + A_c k_a \int_{-\infty}^{\infty} m(t) \cos 2\pi f_c t e^{-j2\pi f t} dt$$

$$S(f) = \int_{-\infty}^{\infty} A_c \left(\frac{e^{j2\pi f_c t} + e^{-j2\pi f_c t}}{2} \right) e^{-j2\pi f t} dt + A_c k_a \int_{-\infty}^{\infty} m(t) \left[\frac{e^{j2\pi f_c t} + e^{-j2\pi f_c t}}{2} \right] e^{-j2\pi f t} dt$$

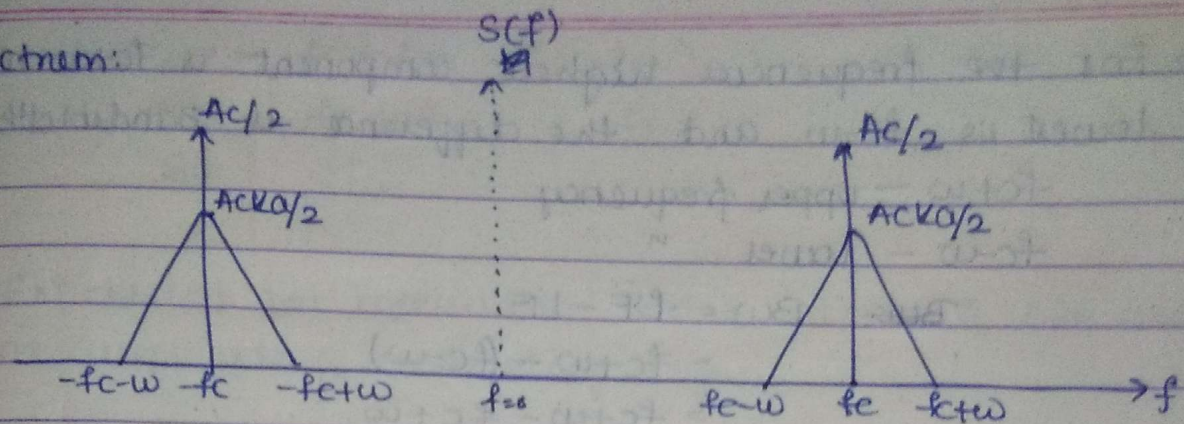
$$= \frac{A_c}{2} \int_{-\infty}^{\infty} e^{j2\pi f_c t} e^{-j2\pi f t} dt + \frac{A_c}{2} \int_{-\infty}^{\infty} e^{-j2\pi f_c t} e^{-j2\pi f t} dt + \frac{A_c k_a}{2} \int_{-\infty}^{\infty} m(t) e^{j2\pi f_c t} e^{-j2\pi f t} dt + \frac{A_c k_a}{2} \int_{-\infty}^{\infty} m(t) e^{-j2\pi f_c t} e^{-j2\pi f t} dt$$

$$= \frac{A_c}{2} \int_{-\infty}^{\infty} e^{-j2\pi (f-f_c) t} dt + \frac{A_c}{2} \int_{-\infty}^{\infty} e^{-j2\pi (f+f_c) t} dt + \frac{A_c k_a}{2} \int_{-\infty}^{\infty} m(t) e^{j2\pi (f-f_c) t} dt + \frac{A_c k_a}{2} \int_{-\infty}^{\infty} m(t) e^{-j2\pi (f+f_c) t} dt$$

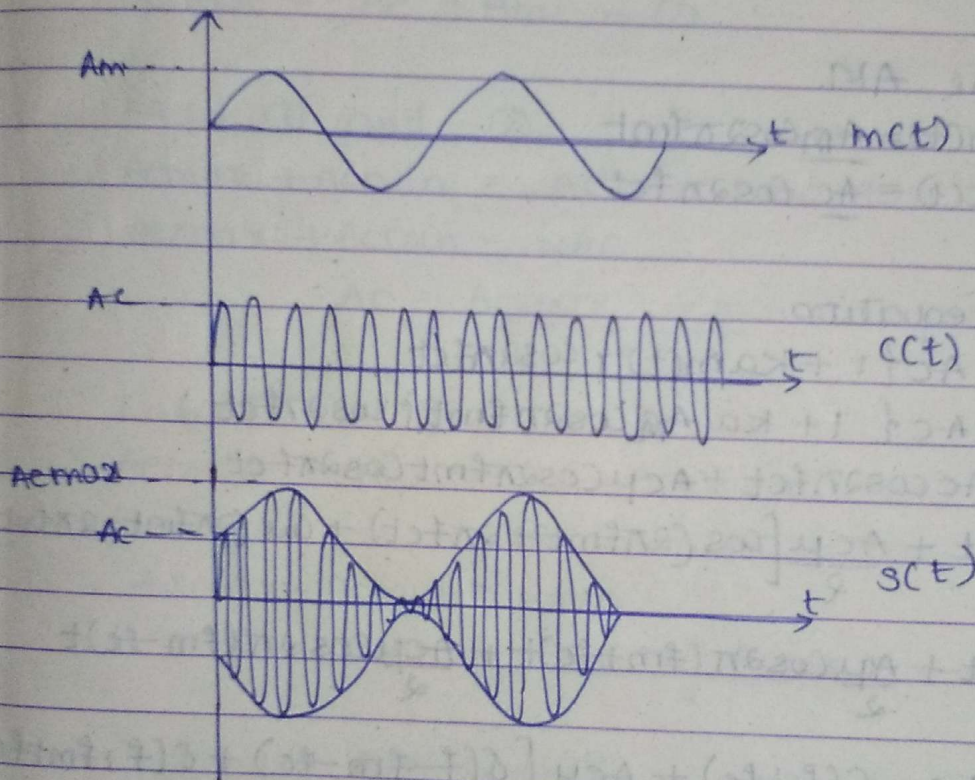
$$= \frac{A_c}{2} [\delta(f-f_c) + \delta(f+f_c)] + \frac{A_c k_a}{2} [m(f-f_c) + m(f+f_c)]$$

$$= \frac{A_c}{2} \delta(f-f_c) + \frac{A_c}{2} \delta(f+f_c) + \frac{A_c k_a}{2} M(f-f_c) + \frac{A_c k_a}{2} M(f+f_c)$$

spectrum:



waveform:



★

- Message signal centered @ $f=0$ extending from $-w$ to w is translated to $f_c = \pm f_c$
- For $f_c \geq w$ The spectrum is non overlapping and can be filtered out using Bandpass filter
- if $f_c < w$ The message spectrum is not same as the $m(t)$ and there is overlapping.
- Transmission Bandwidth is specified by the width of spectrum of the two frequencies.

→ For the frequencies highest component is $f_c + w$ and lowest is $f_c - w$ and the difference is Bandwidth

$f_c + w$ - upper frequency

$f_c - w$ - lower "

$$\text{BW} = \text{UF} - \text{LF}$$

$$= f_c + w - (f_c - w)$$

$$= f_c + w - f_c + w$$

$$\text{BW} = 2w$$

* single tone AM.

$$m(t) = A_m \cos 2\pi f_m t$$

$$c(t) = A_c \cos 2\pi f_c t$$

General AM equation

$$s(t) = A_c \{1 + k_a m(t)\} \cos 2\pi f_c t$$

$$= A_c \{1 + k_a A_m \cos 2\pi f_m t\} \cos 2\pi f_c t$$

$$= A_c \cos 2\pi f_c t + A_c \mu \cos 2\pi f_m t \cos 2\pi f_c t$$

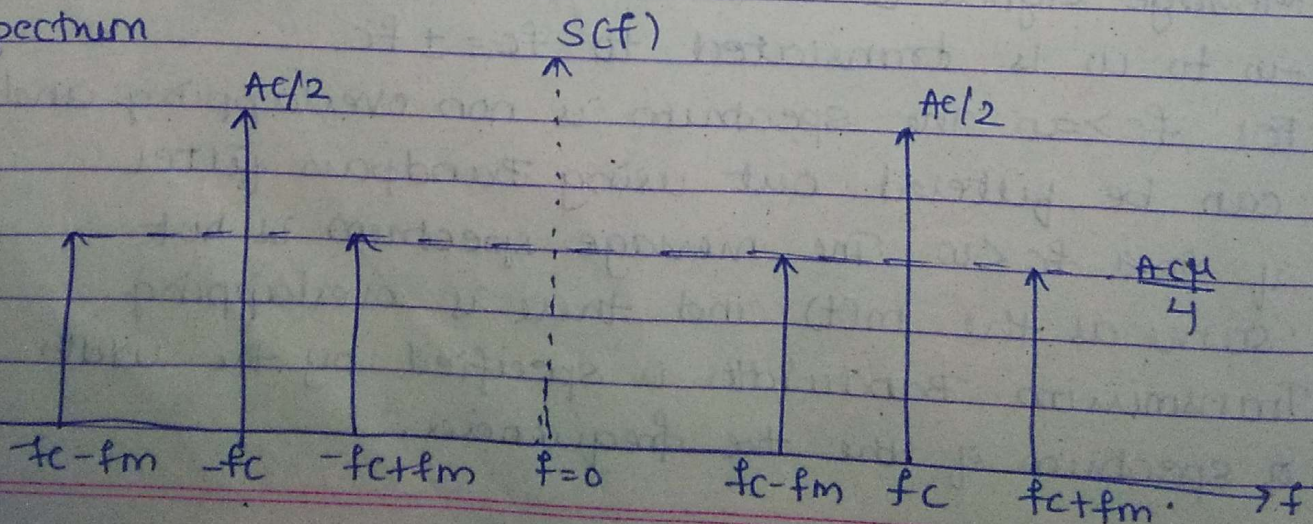
$$= A_c \cos 2\pi f_c t + \frac{A_c \mu}{2} [\cos (2\pi f_m t + 2\pi f_c t) + \cos (2\pi f_m t - 2\pi f_c t)]$$

$$= A_c \cos 2\pi f_c t + \frac{A_c \mu}{2} \cos 2\pi (f_m + f_c) t + \frac{A_c \mu}{2} \cos 2\pi (f_m - f_c) t$$

$$= \frac{A_c}{2} \delta(f - f_c) + \frac{A_c}{2} \delta(f + f_c) + \frac{A_c \mu}{4} [\delta(f - f_m - f_c) + \delta(f + f_m + f_c)$$

$$+ \delta(f - f_m + f_c) + \delta(f + f_m - f_c)]$$

Spectrum



$$\text{Bandwidth} = f_c + f_m - (f_c - f_m)$$

$$Bw = f_c + f_m - f_c + f_m$$

$$\underline{Bw = 2f_m}$$

* Expression for modulation index in terms of min and max amplitude of modulating signal

$$A_{c \max} = A_c + A_m \quad \text{--- (1)}$$

$$A_{c \min} = A_c - A_m \quad \text{--- (2)}$$

Add (1) and (2)

$$A_{c \max} + A_{c \min} = A_c + A_m + A_c - A_m$$

$$A_{c \max} + A_{c \min} = 2A_c$$

$$A_c = \frac{A_{c \max} + A_{c \min}}{2} \quad \text{--- (3)}$$

sub (1) and (2)

$$A_{c \max} - A_{c \min} = A_c + A_m - (A_c - A_m)$$

$$= A_c + A_m - A_c + A_m$$

$$A_{c \max} - A_{c \min} = 2A_m$$

$$A_m = \frac{A_{c \max} - A_{c \min}}{2} \quad \text{--- (4)}$$

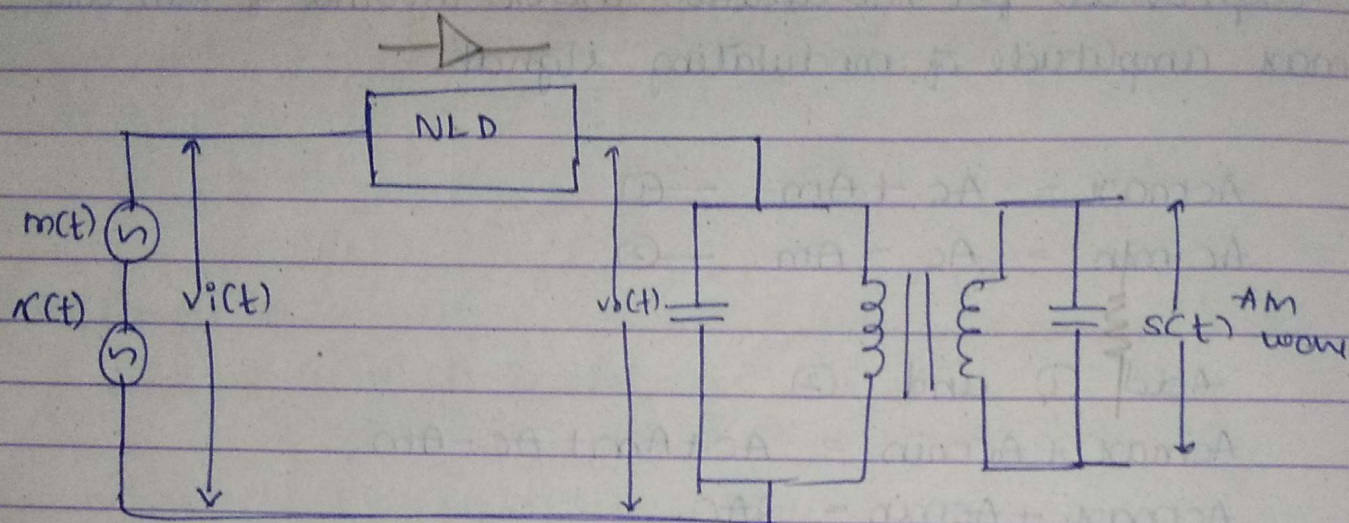
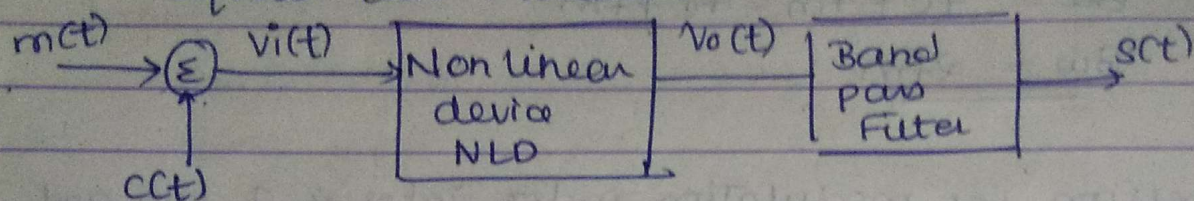
$$\mu = \frac{A_m}{A_c} \quad \text{--- (5)}$$

substitute (3) and (4) in (5)

$$\mu = \frac{A_{c \max} - A_{c \min} / 2}{A_{c \max} + A_{c \min} / 2}$$

$$\mu = \frac{A_{c \max} - A_{c \min}}{A_{c \max} + A_{c \min}}$$

* Generation of AM wave Square Law Modulator



Require 3 feature

- ~~A~~ means of carrier and modulating signal
- A non linear element
- a band pass filter to extract desired modulated signal

Semiconductor diodes and transistors are the most common non linear devices

filter required is usually satisfied using single or double tuned filter.

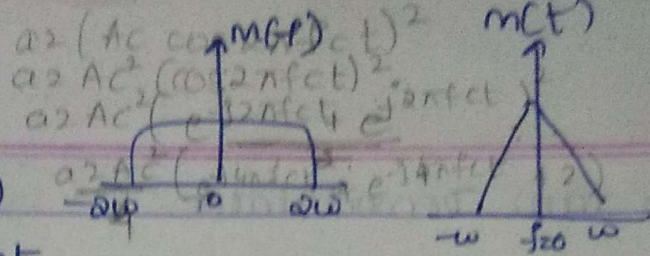
when a non linear element such as a diode is suitably biased and operated in its restricted ~~area~~ portion of its characteristics curve (ie relatively weak signal) it is observed that the characteristics of the diode load resistor combination is represented closely by square law.

$$V_o(t) = a_0 + a_1 v_i(t) + a_2 v_i(t)^2 + a_3 v_i(t)^3 + \dots$$

$$V_o(t) = a_0 + a_1 v_i(t) + a_2 v_i(t)^2$$

$$v(t) = m(t) + c(t)$$

$$c(t) = A_c \cos 2\pi f_c t$$



$$v(t) = a_0 + a_1 [m(t) + c(t)] + a_2 [m(t) + c(t)]^2$$

$$= a_0 + a_1 [m(t) + A_c \cos 2\pi f_c t] + a_2 [m(t)^2 + (A_c \cos 2\pi f_c t)^2 + 2m(t) A_c \cos 2\pi f_c t]$$

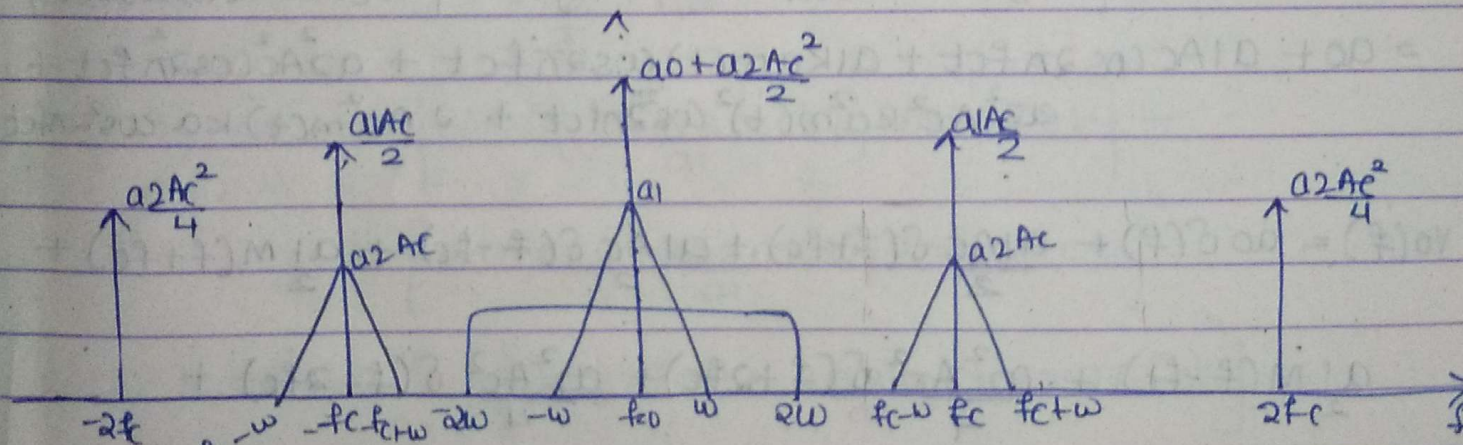
$$= a_0 + a_1 m(t) + a_1 A_c \cos 2\pi f_c t + a_2 m(t)^2 + a_2 A_c^2 \cos^2 2\pi f_c t + 2a_2 m(t) A_c \cos 2\pi f_c t$$

fourier transform.

$$= a_0 \delta(f) + a_1 M(f) + \frac{a_1 A_c}{2} \delta(f + f_c) + \frac{a_1 A_c}{2} \delta(f - f_c) +$$

$$a_2 M^2(f) + \frac{a_2 A_c^2}{2} \delta(f + f_c) + \frac{a_2 A_c^2}{2} \delta(f - f_c) + \frac{a_2 A_c^2}{2} \delta(f)$$

$$+ a_2 A_c M(f + f_c) + a_2 A_c M(f - f_c)$$

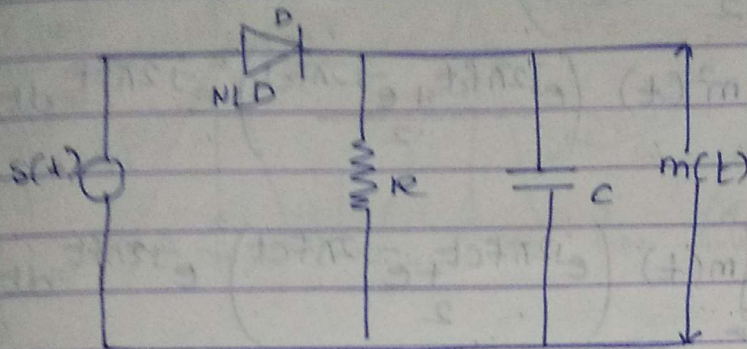


filter. $f_c - W > 2W$ $f_c > 3W$

USB. $f_c + W < 2f_c$ $f_c > W$

Square law demodulator

$s(t) \rightarrow$ [NLD] $\rightarrow v_o(t)$ [LPF] $\rightarrow m(t)$



$$v_i(t) = s(t)$$

$$v_o(t) = a_0 + a_1 v_i(t) + a_2 v_i(t)^2 + a_3 v_i(t)^3 + \dots$$

$$v_o(t) = a_0 + a_1 s(t) + a_2 s(t)^2$$

$$s(t) = a_1 A_c \left(1 + \frac{2a_2}{a_1} m(t) \right) \cos 2\pi f_c t$$

$$\frac{2a_2}{a_1} = k_a$$

$$v_o(t) = a_0 + a_1 \left[a_1 A_c \left(1 + \frac{2a_2}{a_1} m(t) \right) \cos 2\pi f_c t \right] + a_2 \left[a_1 A_c \left(1 + \frac{2a_2}{a_1} m(t) \right) \cos 2\pi f_c t \right]^2$$

$$v_o(t) = a_0 + a_1^2 A_c \left[\cos 2\pi f_c t + \frac{2a_2}{a_1} m(t) \cos 2\pi f_c t \right] +$$

$$a_2 a_1^2 A_c^2 \left[\cos 2\pi f_c t + \frac{2a_2}{a_1} m(t) \cos 2\pi f_c t \right]^2$$

$$v_o(t) = a_0 + a_1^2 A_c \left[\cos 2\pi f_c t + \frac{2a_2}{a_1} m(t) \cos 2\pi f_c t \right] +$$

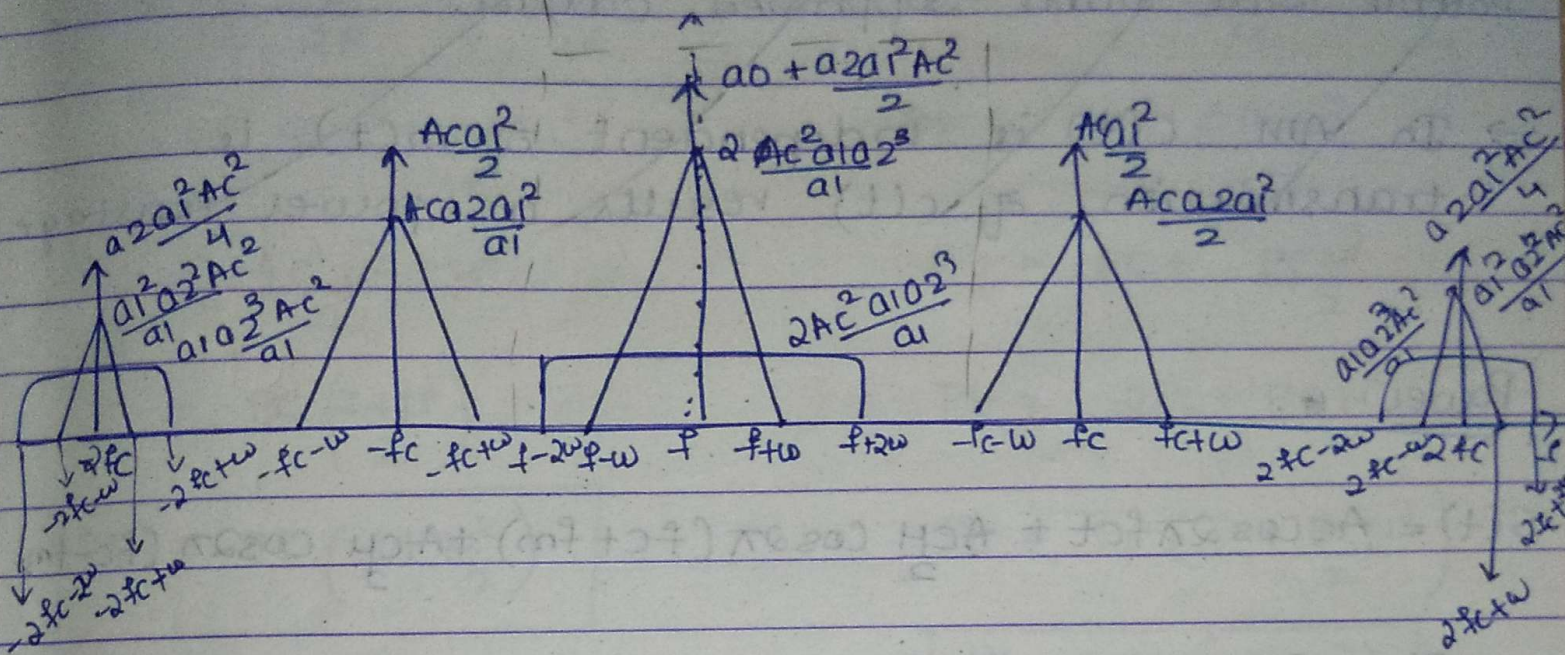
$$a_2 a_1^2 A_c^2 \left[\cos^2 2\pi f_c t + \frac{4a_2^2}{a_1^2} m^2(t) \cos^2 2\pi f_c t + \frac{4a_2}{a_1} m(t) \cos^2 2\pi f_c t \right]$$

Applying FT

$$v_o(t) = \int_{-\infty}^{\infty} a_0 e^{-j2\pi f t} + a_1^2 A_c \cos \left(\frac{e^{j2\pi f t} + e^{-j2\pi f t}}{2} \right) e^{-j2\pi f t} dt +$$

$$\begin{aligned}
 & \frac{a_1^2 A_c}{2} \frac{2a_2}{a_1} \int_{-\infty}^{\infty} m(t) \left(e^{j2\pi f_c t} + e^{-j2\pi f_c t} \right) dt + e^{j2\pi f t} dt + \\
 & a_2 a_1^2 A_c^2 \int_{-\infty}^{\infty} \left(\frac{e^{j2\pi f_c t} + e^{-j2\pi f_c t}}{2} \right)^2 e^{-j2\pi f t} dt + \\
 & a_2 a_1^2 A_c^2 \frac{4a_2^2}{a_1^2} \int_{-\infty}^{\infty} m^2(t) \left(\frac{e^{j2\pi f_c t} + e^{-j2\pi f_c t}}{2} \right)^2 e^{-j2\pi f t} dt \\
 & + a_2 a_1^2 A_c^2 \frac{4a_2}{a_1} \int_{-\infty}^{\infty} m(t) \left(\frac{e^{j2\pi f_c t} + e^{-j2\pi f_c t}}{2} \right)^2 e^{-j2\pi f t} dt
 \end{aligned}$$

$$\begin{aligned}
 & = a_0 \delta(f) + \frac{a_1^2 A_c}{2} \delta(f+f_c) + \frac{a_1^2 A_c}{2} \delta(f-f_c) + \\
 & \frac{a_1^2 a_2 A_c}{a_1} m(f+f_c) + \frac{a_1^2 a_2 A_c}{a_1} m(f-f_c) + \\
 & \frac{a_2 a_1^2 A_c^2}{4} \delta(f+2f_c) + \frac{a_2 a_1^2 A_c^2}{4} \delta(f-2f_c) + \frac{a_2 a_1^2 A_c^2}{2} \delta(f) \\
 & + \frac{4a_2^3 A_c^2}{4a_1^2} m^2(f+2f_c) + \frac{4a_2^3 A_c^2}{4a_1^2} m^2(f-2f_c) + \frac{4a_2^3 A_c^2 m^2}{4a_1} \\
 & + \frac{4a_1^2 a_2^2 A_c^2}{4a_1} m(f+2f_c) + \frac{4a_1^2 a_2^2 A_c^2}{4a_1} m(f-2f_c) + \frac{4a_1^2 a_2^2 A_c}{2a_1} m(f) \\
 & = a_0 \delta(f) + \frac{a_1^2 A_c}{2} \delta(f+f_c) + \frac{a_1^2 A_c}{2} \delta(f-f_c) + \\
 & \frac{a_1^2 a_2 A_c}{a_1} m(f+f_c) + \frac{a_1^2 a_2 A_c}{a_1} m(f-f_c) + \\
 & \frac{a_2 a_1^2 A_c^2}{4} \delta(f+2f_c) + \frac{a_2 a_1^2 A_c^2}{4} \delta(f-2f_c) + \frac{a_2 a_1^2 A_c^2}{2} \delta(f) \\
 & + \frac{a_1 a_2^3 A_c^2}{a_1^2} m^2(f+2f_c) + \frac{A_c^2 a_1 a_2^3}{a_1^2} m^2(f-2f_c) + \frac{2A_c^2 a_1 a_2^3}{a_1} m(f) \\
 & + \frac{a_1^2 a_2^2 A_c^2}{a_1} m(f+2f_c) + \frac{a_1^2 a_2^2 A_c^2}{a_1} m(f-2f_c) + \frac{2a_1^2 a_2^2 A_c}{a_1} m(f)
 \end{aligned}$$



$$w < f_c - w$$

$$f_c > 2w$$

$$-w < f_c + w$$

$$f_c > 2w$$

$$m'(t) = m(t) \cos 2\pi f_c t + \frac{2a_2}{a_1} a_1^2 A_c + a_1^2 A_c \cos 2\pi f_c t$$

$$= a_1^2 A_c \frac{2a_2}{a_1} m(t) \cos 2\pi f_c t + a_1^2 A_c \cos 2\pi f_c t$$

$$= a_1 A_c (1 + k_a m(t)) \cos 2\pi f_c t$$

$$\frac{2a_2}{a_1} = k_a$$

$$m'(t) = \underbrace{a_0 + \frac{a_1^2 A_c^2}{2}}_{\text{dc component blocked by cap}} + \underbrace{A_c^2 k_a a_2 m(t) + a_2 A_c a_1^2 m^2(t)}_{\text{neglected becoz low level modulating signal}}$$

$$m'(t) = A_c^2 k_a a_2 m(t)$$

Double Side Band Suppressed carrier

→ In AM $c(t)$ is independent of $m(t)$ i.e. transmission of $c(t)$ results in power wastage

Power

$$S(t) = A_c \cos 2\pi f_c t + \frac{A_c \mu}{2} \cos 2\pi (f_c + f_m) + \frac{A_c \mu}{2} \cos 2\pi (f_c - f_m)$$

$$P_t = P_c + P_{USB} + P_{LSB}$$

$$= \frac{\left(\frac{A_c}{\sqrt{2}}\right)^2}{R} + \frac{\left(\frac{A_c \mu / 2}{\sqrt{2}}\right)^2}{R} + \frac{\left(\frac{A_c \mu / 2}{\sqrt{2}}\right)^2}{R} \quad \left\{ \begin{array}{l} P = \frac{V_{rms}^2}{R} \\ V_{rms} = \frac{V}{\sqrt{2}} \end{array} \right.$$

$$= \frac{A_c^2}{2R} + \frac{A_c^2 \mu^2}{8R} + \frac{A_c^2 \mu^2}{8R}$$

$$= \frac{A_c^2}{2R} + \frac{2A_c^2 \mu^2}{8R}$$

$$P_t = \frac{A_c^2}{2R} + \frac{A_c^2 \mu^2}{4R}$$

$$P_c = \frac{A_c^2}{2R}$$

Side Band Power

$$P_s = \frac{A_c^2 \mu^2}{8R} + \frac{A_c^2 \mu^2}{8R}$$

$$= \frac{2A_c^2 \mu^2}{8R}$$

$$= \frac{A_c^2 \mu^2}{4R}$$

$$P_s = \frac{P_c \mu^2}{2}$$

$$P_T = \frac{A_c^2}{2R} + \frac{A_c^2 \mu^2}{4R}$$

$$= \frac{2A_c^2 + A_c^2 \mu^2}{4R}$$

$$= \frac{A_c^2 (2 + \mu^2)}{4R}$$

$$= P_C \left(\frac{2 + \mu^2}{2} \right)$$

$$P_T = P_C \left(1 + \frac{\mu^2}{2} \right)$$

$$P_T = P_C + P_{C\mu^2/4} + P_{C\mu^2/4}$$

$$P_T = P_C + \frac{2P_C\mu^2}{4}$$

$$P_T = P_C + \frac{P_C\mu^2}{2}$$

$$P_T = \frac{2P_C + P_C\mu^2}{2}$$

$$P_T = P_C \left(\frac{2 + \mu^2}{2} \right)$$

$$P_T = P_C \left(1 + \frac{\mu^2}{2} \right)$$

Average power in side band.

$$P_{SB} = P_{LSB} = P_{USB} = \frac{P_C \mu^2}{2}$$

Efficiency

$$\eta = \frac{P_S}{P_T}$$

$$= \frac{P_C \mu^2}{2}$$

$$P_C \left(\frac{2 + \mu^2}{2} \right)$$

$$\eta = \frac{\mu^2}{2 + \mu^2}$$

Conclusion :-

if 100% modulation ie $\mu=1$ $\eta = \frac{1}{3}$ 33% modu

$$\frac{P_s}{P_t} = \eta$$

$$\frac{P_s}{P_t} = \frac{1}{3}$$

$$3P_s = P_t$$

$$P_s = \frac{1}{3} P_t$$

$\frac{1}{3}^{\text{rd}}$ of total power is in modulated wave in AM

$$\textcircled{*} \quad s(t) = A_c(1 + k_a m(t)) \cos 2\pi f_c t$$

$$P_t = P_c \left(1 + \frac{\mu^2}{2}\right)$$

$$P_s = P_c \frac{\mu^2}{2}$$

$$P_t = P_{s1} + P_{s2} + P_c$$

{ if two modulating
signal is given

$$\mu_t = \sqrt{\mu_1^2 + \mu_2^2}$$

$$\mu = |k_a m(t)|_{\text{max}}$$

$$V_t = V_c \sqrt{1 + \frac{\mu^2}{2}}$$

$$I_t = I_c \sqrt{1 + \frac{\mu^2}{2}}$$

$$B_w = 2f_m$$

side band frequency

$$f_c + f_m = \text{USB}$$

$$f_c - f_m = \text{LSB}$$

avg amplitude of side band = $\frac{A_c \mu}{2}$

Amplitude of each side band = $\frac{A_c \mu}{4}$

$$\mu = \frac{A_{\text{max}} - A_{\text{min}}}{A_{\text{max}} + A_{\text{min}}}$$

DSBSC

Double side Band Suppressed Carrier

→ In AM $c(t)$ is independent of $m(t)$ this results in power wastage in transmission of $c(t)$

→ Suppressing the carrier component results in DSBSC

→ We obtain modulated wave is proportional to product of carrier wave & message signal.

Derivation

Let $m(t)$ - message signal (triangle wave)

$c(t)$ - carrier signal

$s(t)$ - modulated signal

Let $c(t) = A_c \cos 2\pi f_c t$

$$s(t) = A_c m(t) \cos 2\pi f_c t$$

$$S(f) = \int_{-\infty}^{\infty} s(t) e^{-j2\pi f t} dt$$

$$= \int_{-\infty}^{\infty} (A_c m(t) \cos 2\pi f_c t) e^{-j2\pi f t} dt$$

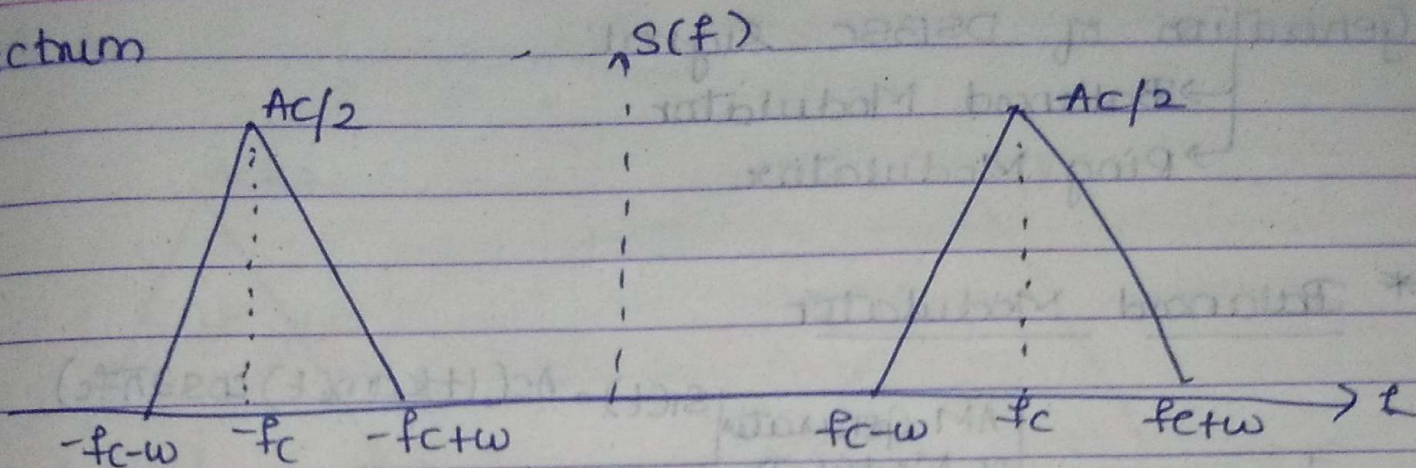
$$= \frac{A_c}{2} \int_{-\infty}^{\infty} m(t) \left[\frac{e^{j2\pi f_c t} + e^{-j2\pi f_c t}}{2} \right] e^{-j2\pi f t} dt$$

$$= \frac{A_c}{2} \int_{-\infty}^{\infty} m(t) e^{j2\pi f_c t} e^{-j2\pi f t} dt + \frac{A_c}{2} \int_{-\infty}^{\infty} m(t) e^{-j2\pi f_c t} e^{-j2\pi f t} dt$$

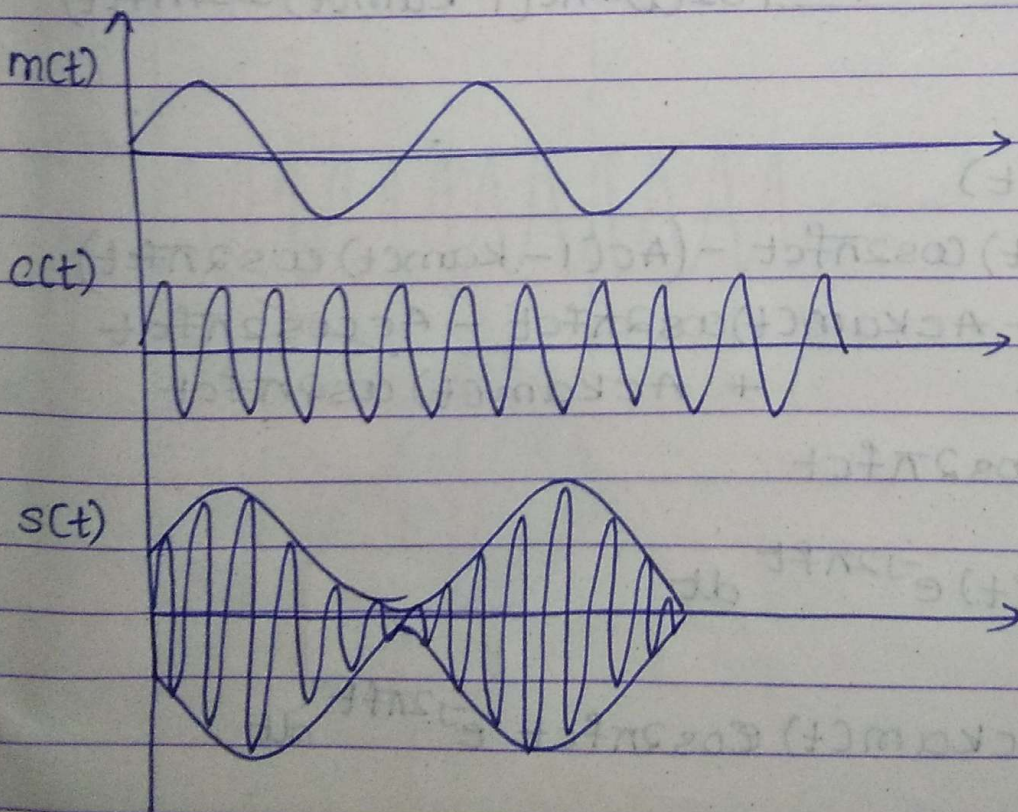
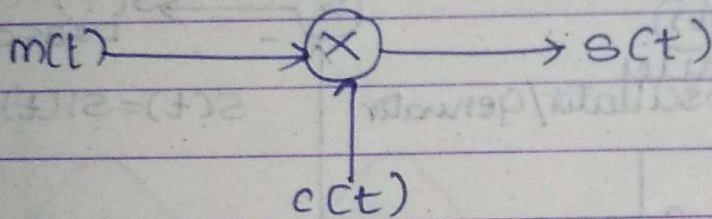
$$= \frac{A_c}{2} \int_{-\infty}^{\infty} m(t) e^{-j2\pi (f - f_c) t} dt + \frac{A_c}{2} \int_{-\infty}^{\infty} m(t) e^{-j2\pi (f + f_c) t} dt$$

$$= \frac{A_c}{2} M(f - f_c) + \frac{A_c}{2} M(f + f_c)$$

Spectrum



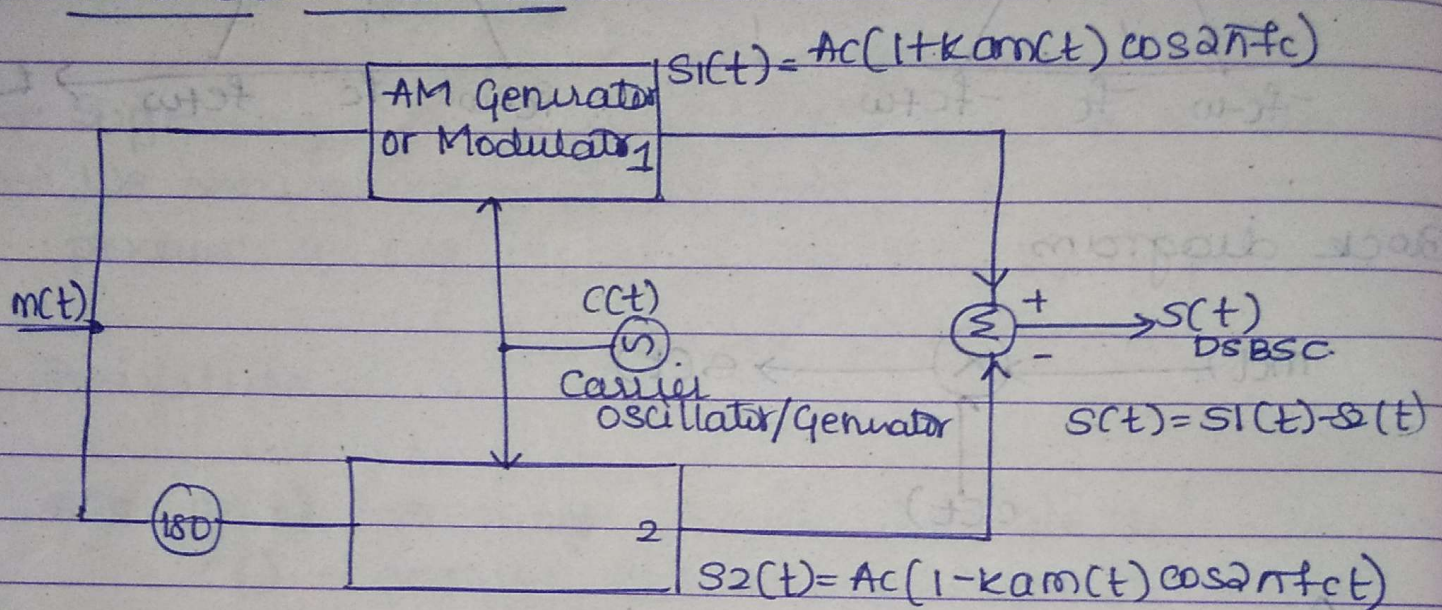
Block diagram



Generation of DSBSC signal

- Balanced Modulator
- Ring Modulator

* Balanced Modulator



$$s(t) = s_1(t) - s_2(t)$$

$$\begin{aligned}
 &= A_c(1 + k_a m(t)) \cos 2\pi f_c t - (A_c(1 - k_a m(t)) \cos 2\pi f_c t) \\
 &= A_c \cancel{\cos 2\pi f_c t} + A_c k_a m(t) \cos 2\pi f_c t - A_c \cancel{\cos 2\pi f_c t} + A_c k_a m(t) \cos 2\pi f_c t
 \end{aligned}$$

$$s(t) = 2A_c k_a m(t) \cos 2\pi f_c t$$

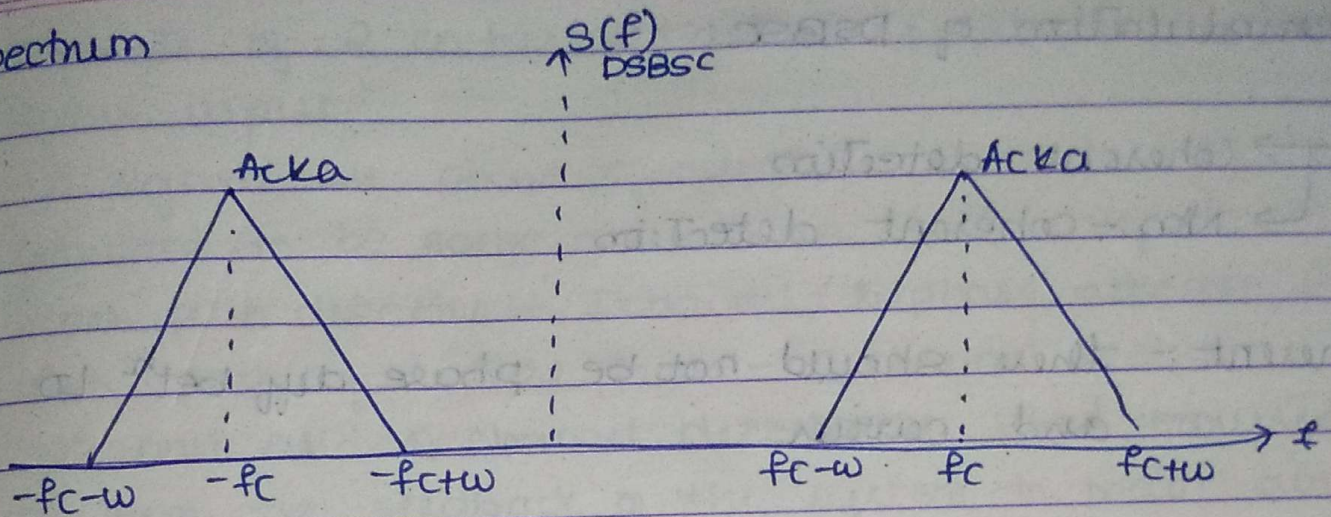
$$\begin{aligned}
 S(f) &= \int_{-\infty}^{\infty} s(t) e^{-j2\pi f t} dt \\
 &= \int_{-\infty}^{\infty} 2A_c k_a m(t) \cos 2\pi f_c t e^{-j2\pi f t} dt
 \end{aligned}$$

$$\begin{aligned}
 &= 2A_c k_a \int_{-\infty}^{\infty} m(t) \left(\frac{e^{j2\pi f_c t} + e^{-j2\pi f_c t}}{2} \right) e^{-j2\pi f t} dt \\
 &= A_c k_a \int_{-\infty}^{\infty} m(t) e^{-j2\pi (f - f_c) t} + m(t) e^{-j2\pi (f + f_c) t} dt
 \end{aligned}$$

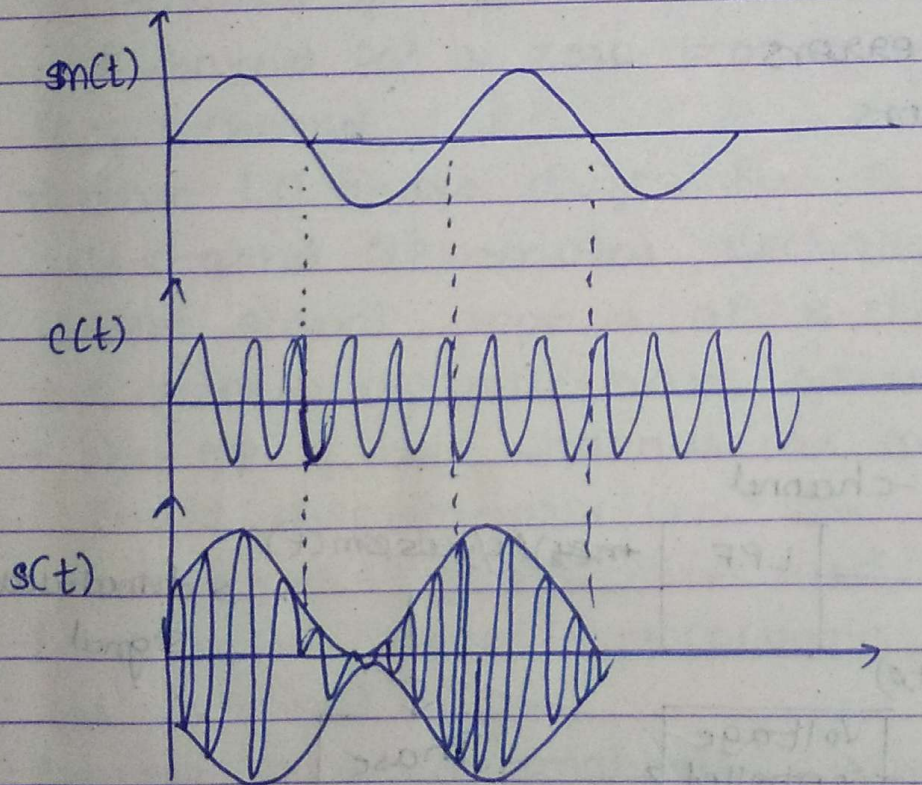
$$= A_c k_a M(f - f_c) + A_c k_a M(f + f_c)$$

Subcarrier

Spectrum



waveform



Demodulation of DSBSC

- Coherent detection
- Non-coherent detection

Coherent :- there should not be phase diff betⁿ LO and carrier

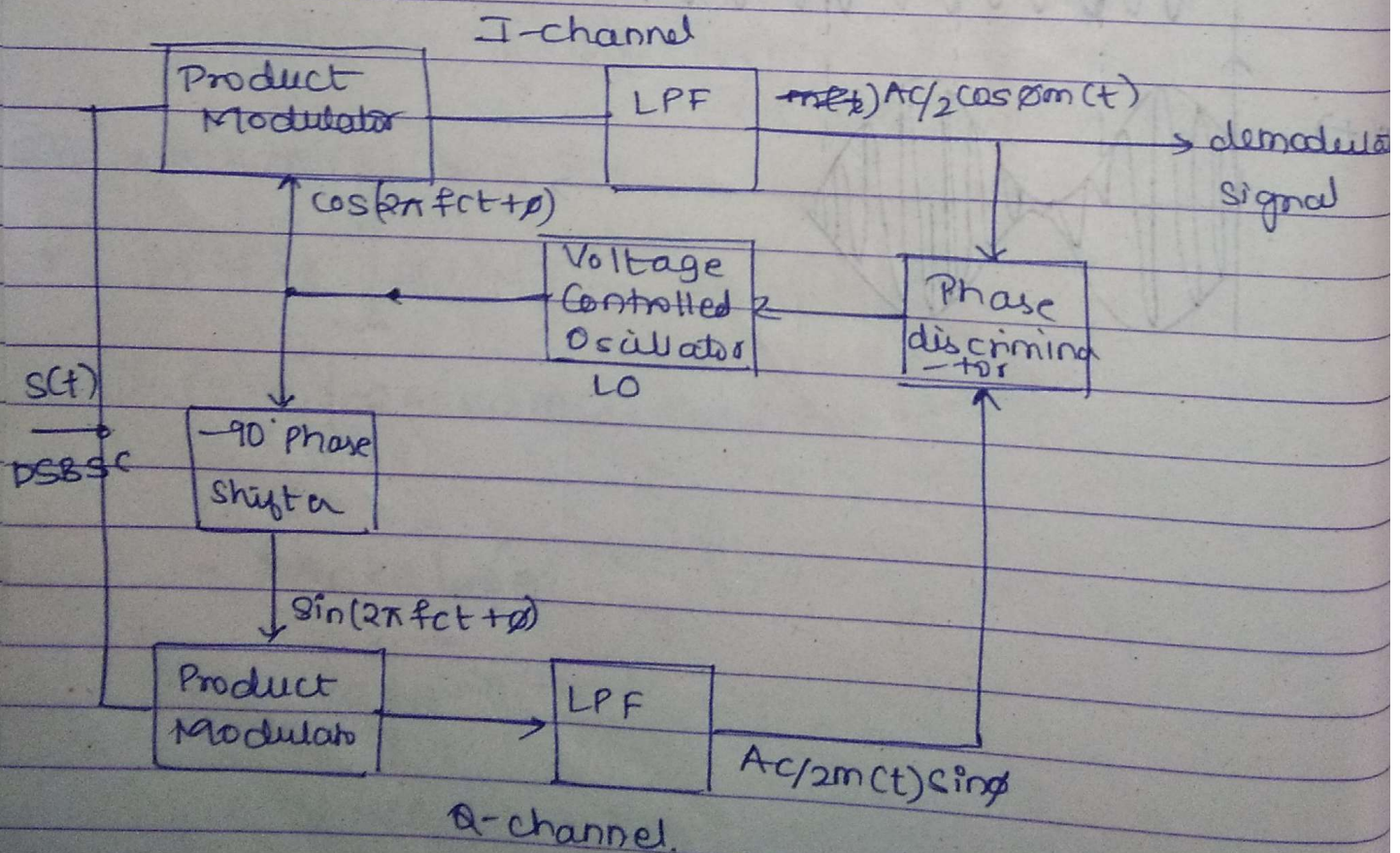
Non-coherent :- There can be.

→ OR there can be no oscillators.

Errors → frequency errors
→ phase errors

Coherent Receiver

Coherent Receiver:



- consists of 2 coherent detectors supplied with same input.
- LO signals are phase quadrature to each other.
- adjusted to be same as carrier frequency f_c .
- upper path detector - I channel / inphase detector
- lower path quadrature phase - Q channel detector
- I channel and Q channel detector are used coupled to form -ve feedback of the system to maintain LO synchronous with carrier.
- when LO is in phase with f_c
 - I-channel o/p contains desired demodulated signal
 - Q channel o/p is zero bcoz of quadrature null effect of Q channel
- when LO phase drifts from its proper value by θ rad.
 - I-channel o/p remains unchanged
 - some signal appears at Q-channel $\sin \theta \neq 0$
- Q channel o/p will have same polarity as I-channel
- The o/p of both channels are combined in Phase discriminator
 - ↳ consists of multiplier and LPF
- DC control signal - proportional to phase error ϕ is obtained as o/p.
- hence receiver automatically corrects for phase errors.
- Phase lock is ceased with modulation and has to be reestablished with the reappearance of modulation.

Quadrature Null effect:

- Caused due to phase error in DSBSC.
 - ↑
due to disturbance in the channel.
- phase error reduces o/p of demodulator.
- phase error 90° output is 0
- phase error 0° output is maximum.

Single Sideband Modulation

Single Sideband Suppressed Carrier SSBSC

- Standard AM and DSBSC require bandwidth twice of message bandwidth hence wastage of bandwidth
- One half of transmission bandwidth is occupied by USB and other half by LSB
- Both are uniquely related by virtue of symmetry property about f_c
- Given any one the other can be recovered
- In SSBSC only one side band is transmitted the other SB and carrier are suppressed thus the channel need to provide bandwidth same as that of the message bandwidth
elimination of high power carrier wave
- disadvantage: cost and complexity

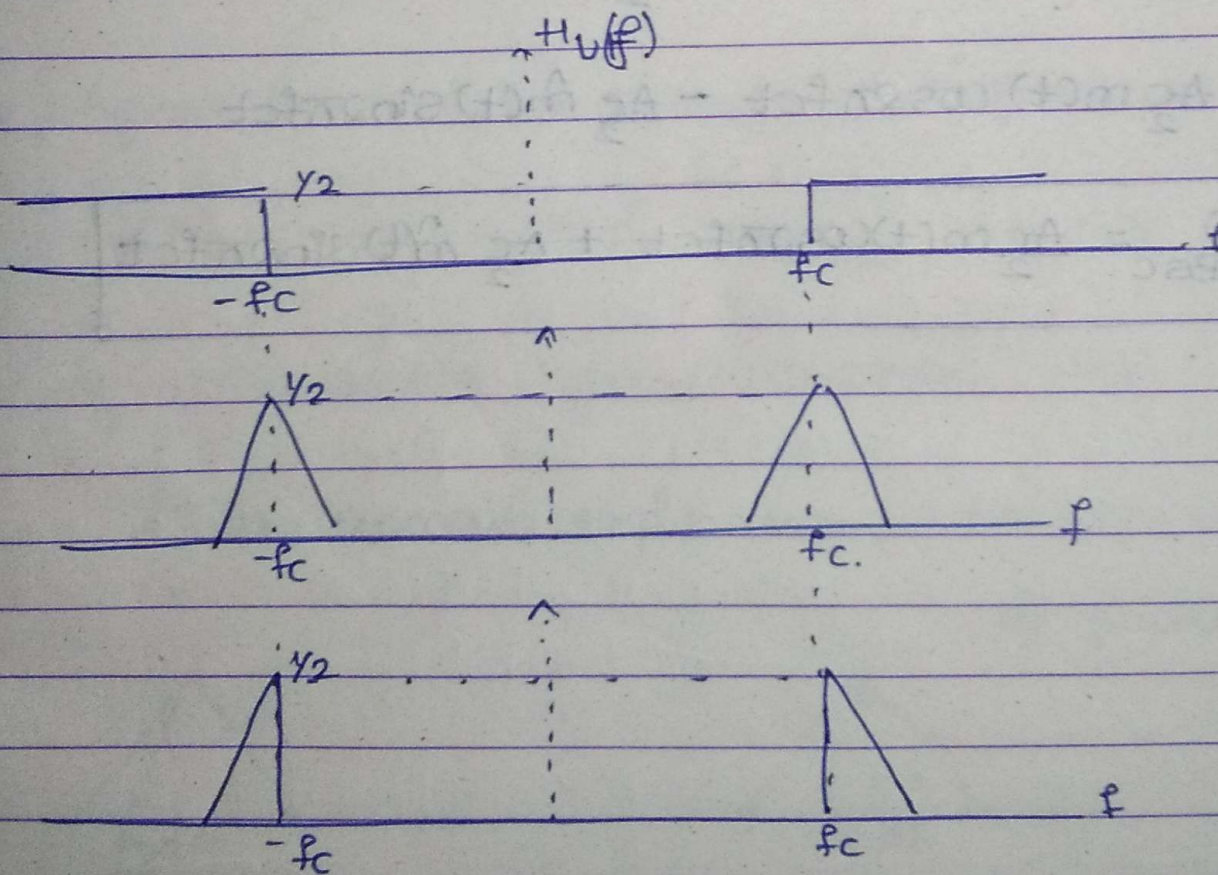
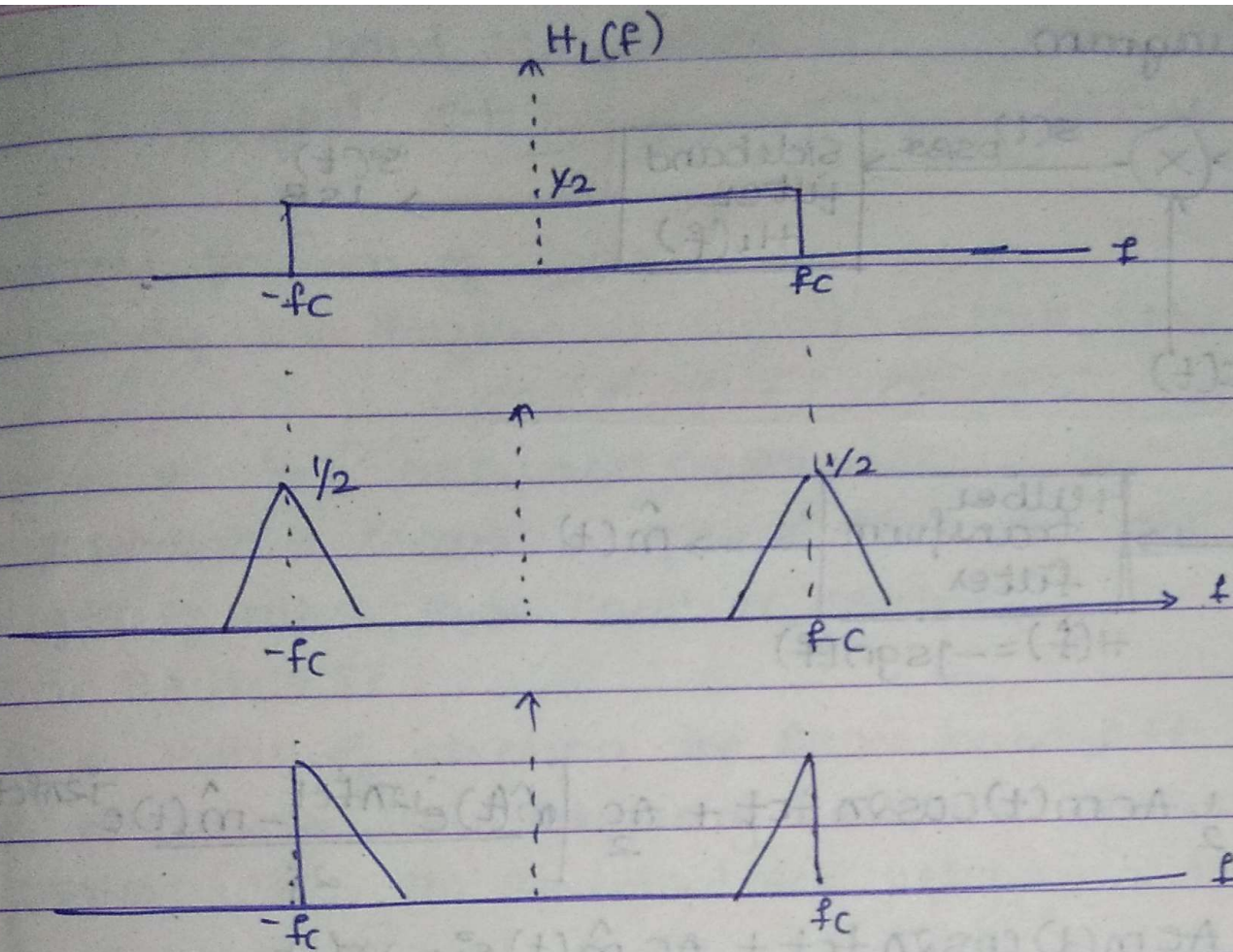
Generation of SSBSC

LSB

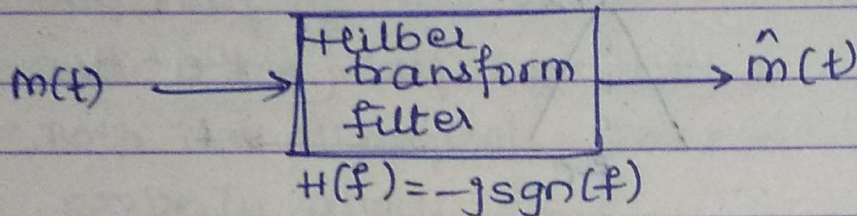
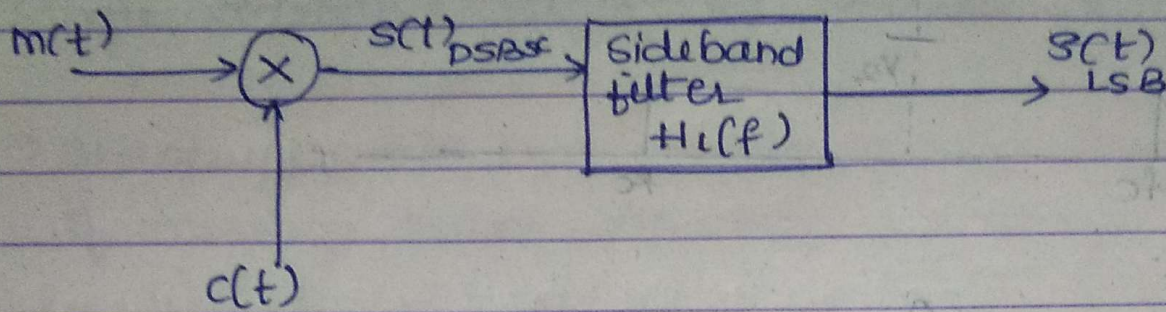
Side band filtering frequency:

Hilbert filter

$$H_L(f) = \frac{1}{2} \left[\text{Sgn}(f+f_c) - \text{Sgn}(f-f_c) \right]$$



Block diagram



$$S(t)_{\text{LSB}} = \frac{1}{2} A_c m(t) \cos 2\pi f_c t + \frac{A_c}{2} \left[\frac{\hat{m}(t) e^{j2\pi f_c t} - \hat{m}(t) e^{-j2\pi f_c t}}{2j} \right]$$

$$S(t)_{\text{LSB}} = \frac{A_c}{2} m(t) \cos 2\pi f_c t + \frac{A_c}{2} \hat{m}(t) \sin 2\pi f_c t$$

$$S(t)_{\text{USB}} = \frac{A_c}{2} m(t) \cos 2\pi f_c t - \frac{A_c}{2} \hat{m}(t) \sin 2\pi f_c t$$

$$S(t)_{\text{SSBSC}} = \frac{A_c}{2} m(t) \cos 2\pi f_c t \pm \frac{A_c}{2} \hat{m}(t) \sin 2\pi f_c t$$

Vestigial side band Modulation

VSBSC: Vestigial side band suppressed carrier

→ overcome problem of filter.

→ extremely low frequencies signal is lost (tv, wideband)

compromise betⁿ SSB and DSBSC

one sideband is completely passed whereas the traces or vestige of other side band is retained.

$$B = W + \delta f$$

It has virtue of covering the same bandwidth as SSB while retaining low frequency baseband characteristic of DSBSC. Thus its standard for television transmission and similar ex.

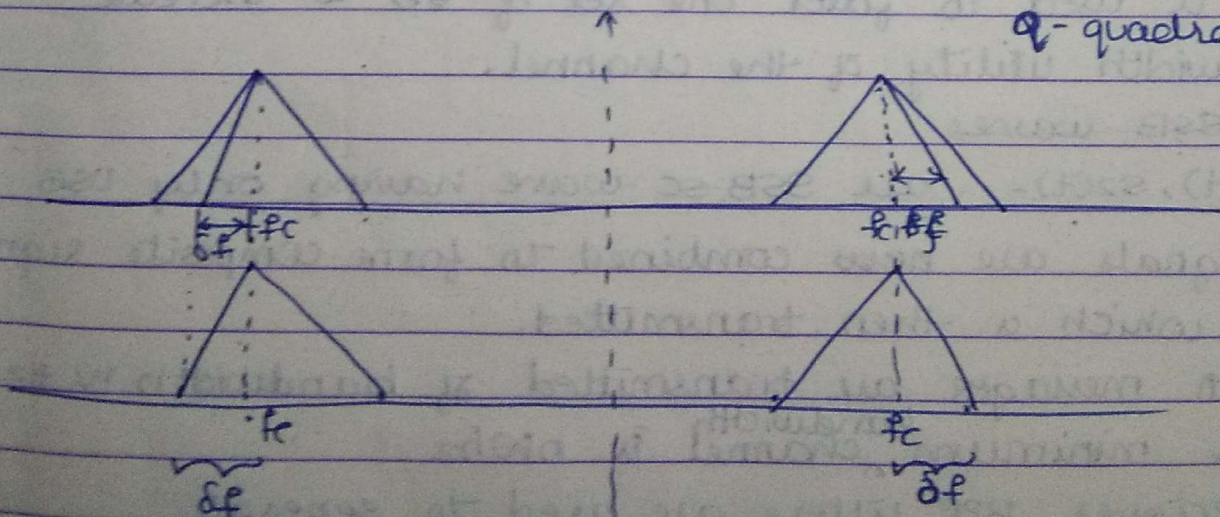
$$S(t)_{VSB} = \frac{A_c m(t) \cos 2\pi f_c t + A_c m_q(t) \sin 2\pi f_c t}{2} \quad \text{--- (1)}$$

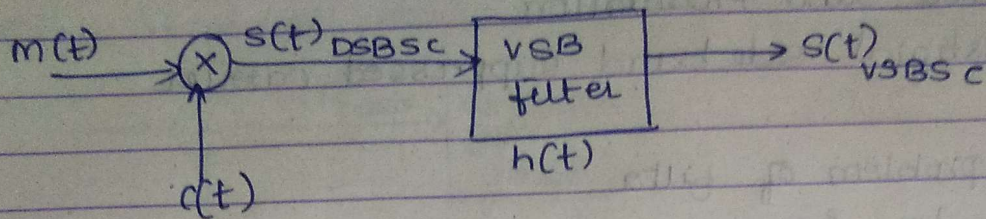
→ Time domain expⁿ having LSB and USB vestig

$$S(t)_{VSB} = \frac{A_c m(t) \cos 2\pi f_c t - A_c m_q(t) \sin 2\pi f_c t}{2} \quad \text{--- (2)}$$

→ Time domain expⁿ having USB and LSB vestig

q - quadrature phase





Frequency Division Multiplexing:

Multiplexing is a technique where a number of independent signals can be combined into a composite signal suitable for transmission over a single communication channel.

The technique of separating signal spectrum in frequency domain is called FDM.

→ Incoming message signals are assumed to be of LPF type.
 → LPF designed to remove high frequency components that do not contribute to message spectrum but is capable of disturbing other msg signals in the channel.

→ The filtered signals are modulated to carrier frequency. The resulting waves are DSBSC and do not overlap.

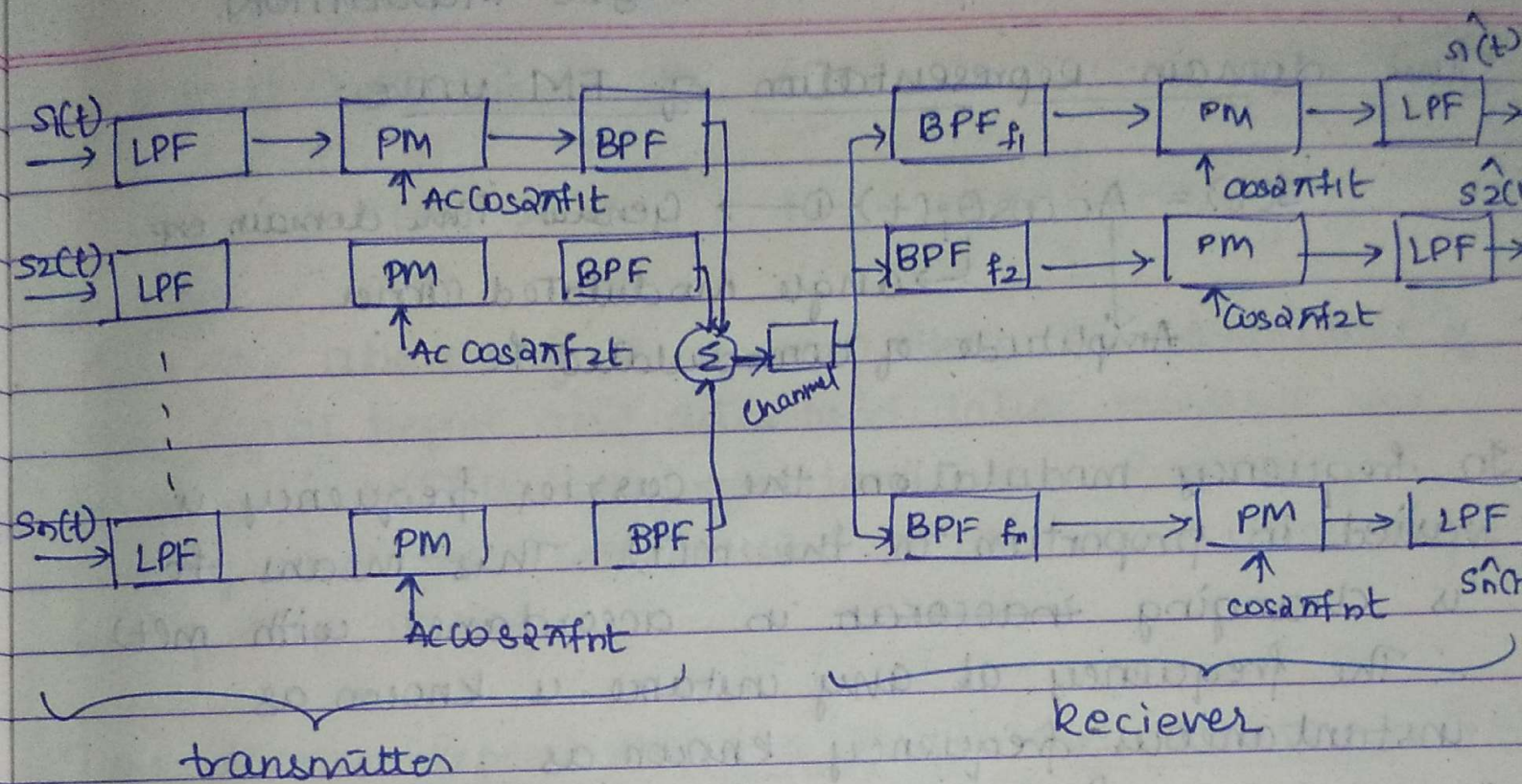
→ BPF is used to filter one set of SB to increase bandwidth utility of the channel.

→ If SSB waves

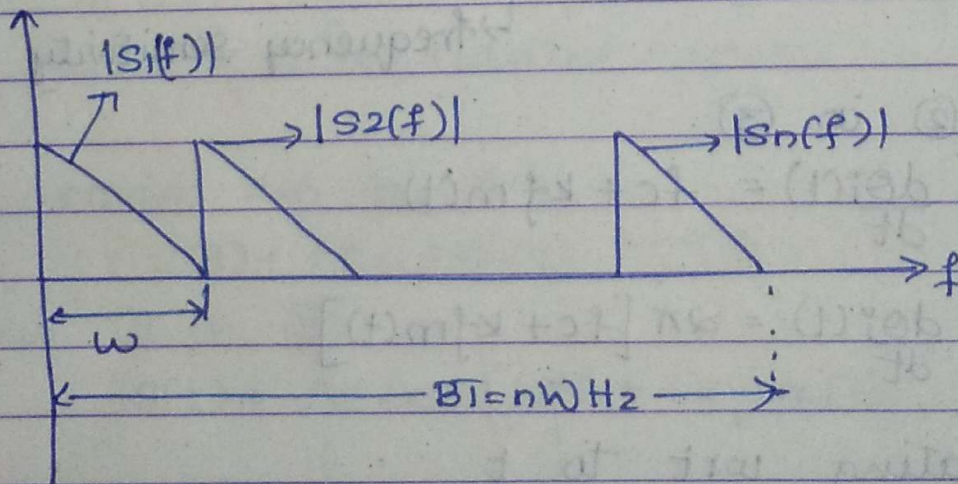
if $s_1(t), s_2(t), \dots$ are SSB-SC waves having only USB all signals are ~~not~~ combined to form composite signal $s(t)$ which is then transmitted.

→ If n messages are transmitted of bandwidth w Hz then minimum ^{bandwidth} channel is nw Hz.

→ @ receiver BPF filters are used to separate modulated signals. Then by ^{using} detectors it is possible to recover original signal.



Magnitude spectrum of composite waveform
 $s(t) = s_1(t) + s_2(t) + \dots + s_n(t)$
 for the freq



Chapter-II

PRINCIPLES OF ANGLE MODULATION.

Time domain representation of FM wave:

$$s(t) = A_c \cos \theta_i(t) \quad \text{--- (1) --- General Time domain exp}$$

↓
Amplitude of unmodulated f_c

↘
angle modulated carrier

In frequency modulation the carrier frequency is varied in proportion to the $m(t)$. This means f_c is changing in accordance with $m(t)$.
∴ The frequency at any instance is known as instantaneous frequency known as

$$f_i(t) = \frac{1}{2\pi} \frac{d\theta_i(t)}{dt} \quad \text{--- (2) ---}$$

$f_i(t)$ is linearly varied with $m(t)$ ∴

$$\therefore f_i(t) = f_c + k_f m(t) \quad \text{--- (3) ---}$$

↓
frequency sensitivity h_z/v

Sub (2) in (3)

$$\frac{1}{2\pi} \frac{d\theta_i(t)}{dt} = f_c + k_f m(t)$$

$$\frac{d\theta_i(t)}{dt} = 2\pi [f_c + k_f m(t)]$$

Integrating wrt to t

$$\int \frac{d\theta_i(t)}{dt} dt = \int_0^t 2\pi [f_c + k_f m(t)] dt$$

$$\theta_i(t) = 2\pi \int_0^t [f_c + k_f m(t)] dt$$

$$\theta_i(t) = \int_0^t 2\pi f_c + 2\pi k_f m(t) dt$$

$$\theta_i(t) = 2\pi f_c t + 2\pi k_f \int_0^t m(t) dt \quad \text{--- (4) ---}$$

substituting ④ in ①

$$s(t) = A_c \cos \left[2\pi f_c t + 2\pi k_f \int_0^t m(t) dt \right]$$

→ Non linear function

Conclusion

from the above eqⁿ it is seen that the Amplitude of the signal before and after modulation remains the same. If $m(t)$ is varying instantaneous frequency is varying and the zero crossing of FM wave is no longer having same spacing.

Time domain Representation of Phase Modulated Waves (PM)

Here the phase of the modulated carrier $\theta_i(t)$ is varied in accordance with $m(t)$

$$\theta_i(t) = 2\pi f_c t + k_p m(t) \quad \text{--- ①}$$

↓
phase of unmodulated carrier

→ phase sensitivity - radians/volt

time domain exp for PM

$$s(t) = A_c \cos \theta_i(t) \quad \text{--- ②}$$

Sub ① in ②

$$s(t) = A_c \cos [2\pi f_c t + k_p m(t)]$$

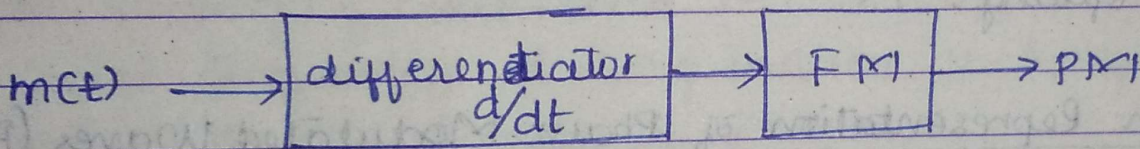
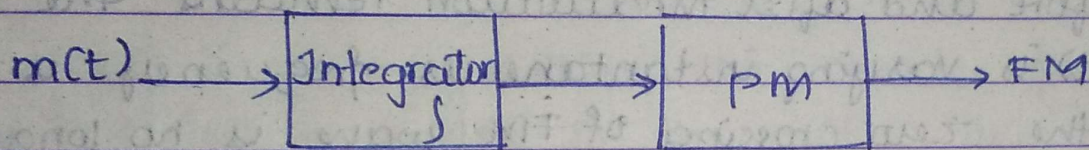
Conclusion:

→ The envelope of PM wave is a constant equal to the amp of unmodulated carrier

→ The zero crossing of PM is no longer in their regular spacing. bcoz the instantaneous freq of PM is proportional to the time derivative of $m(t)$

- ★ In FM and PM $\theta_i(t)$ changes but in diff manner
- ★ In FM $\theta_i(t)$ is directly proportional to Integral of $m(t)$
- ★ " PM $\theta_i(t)$ " " " " " derivative " "

Block diagram



Frequency deviation and Modulation index (β)

Consider single tone modulating signal

$$m(t) = A_m \cos 2\pi f_m t \quad \text{--- (1)}$$

$$s(t) = A_c \left[\cos 2\pi f_c t + 2\pi k_f \int_0^t m(t) dt \right] \quad \text{--- (2)}$$

Sub (1) in (2)

$$s(t) = A_c \left[\cos 2\pi f_c t + 2\pi k_f \int_0^t A_m \cos 2\pi f_m t dt \right]$$

$$= A_c \left[\cos 2\pi f_c t + 2\pi k_f \cdot A_m \cdot \frac{\sin 2\pi f_m t}{2\pi f_m} \right]_0^t$$

$$= A_c \left[\cos 2\pi f_c t + k_f \frac{A_m}{f_m} \sin 2\pi f_m t \right]_0^t$$

$$= A_c \left[\cos 2\pi f_c t + k_f \frac{A_m}{f_m} \sin 2\pi f_m t - \sin 0^\circ \right]$$

$$= A_c \left[\cos 2\pi f_c t + k_f \frac{A_m}{f_m} \sin 2\pi f_m t \right]$$

$k_f A_m = \Delta f$ — frequency deviation

$$= A_c \left[\cos 2\pi f_c t + \frac{\Delta f}{f_m} \sin 2\pi f_m t \right]$$

$\frac{\Delta f}{f_m} = \beta$ — modulation index.

$$= A_c \left[\cos 2\pi f_c t + \beta \sin 2\pi f_m t \right]$$

$\Delta f \Rightarrow$ frequency deviation is the maximum departure of instantaneous freq of the resulting fm signal from f_c

$$\Delta f = |f_i(t) - f_c|_{\max}$$

$$= k_f A_m$$

Modulation Index $\beta = \frac{k_f A_m}{f_m}$

$$\beta = \frac{\Delta f}{f_m}$$

* $\beta < 1 \text{ rad}$ NBFM

$\beta > 1 \text{ rad}$ WBFM.

Narrow band frequency Modulation

has narrow bandwidth which is equal to twice msg signal.

Time domain exp of FM

$$s(t) = A_c \cos \theta_i(t) \quad \text{--- (1)}$$

For single tone sinusoidal Modulation

$$\theta_i(t) = 2\pi f_c t + \beta \sin 2\pi f_m t \quad \text{--- (2)}$$

$$\therefore s(t)_{FM} = A_c \cos[2\pi f_c t + \beta \sin 2\pi f_m t]$$

$$\cos A+B = \cos A \cos B - \sin A \sin B$$

$$s(t)_{FM} = A_c [\cos 2\pi f_c t \cos(\beta \sin 2\pi f_m t) - \sin 2\pi f_c t \sin(\beta \sin 2\pi f_m t)]$$

$$\text{since } \sin 0 = 0$$

$$\cos 0 = 1$$

$$\beta \ll 1$$

$$f_c + f_m - f = 0$$

$$f_c + f_m = f$$

$$\cos(\beta \sin 2\pi f_m t) \approx 1$$

$$\sin(\beta \sin 2\pi f_m t) \approx \beta \sin 2\pi f_m t$$

$$s(t)_{FM} = A_c [\cos 2\pi f_c t - \sin 2\pi f_c t \beta \sin 2\pi f_m t]$$

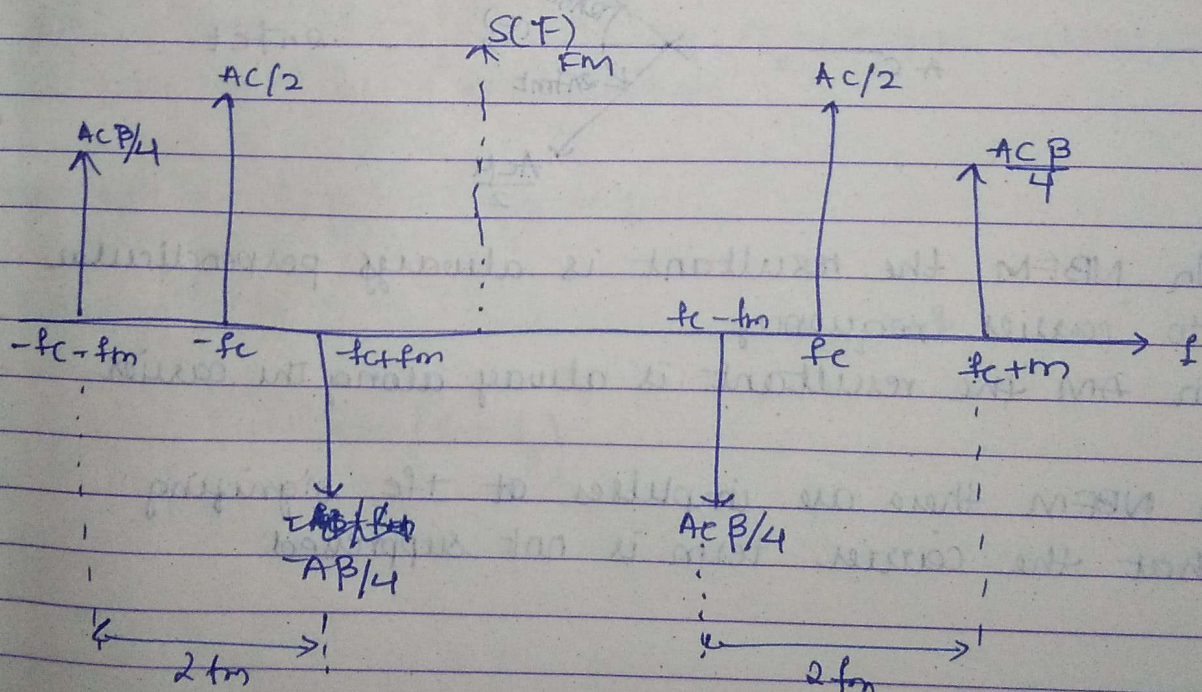
$$= A_c \cos 2\pi f_c t - A_c \beta \sin 2\pi f_c t \sin 2\pi f_m t$$

$$\sin A \sin B = -\frac{1}{2} (\cos(A+B) - \cos(A-B))$$

$$s(t)_{FM} = A_c \cos 2\pi f_c t + \frac{A_c \beta}{2} \cos(2\pi f_c t + 2\pi f_m t) - \frac{A_c \beta}{2} \cos\left(\frac{2\pi f_c t}{2\pi f_m t}\right)$$

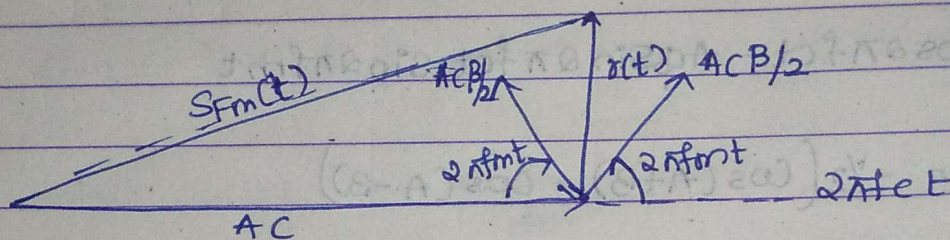
$$S(F)_{FM} = \frac{A_c}{2} [\delta(f+f_c) + \delta(f-f_c)] + \frac{A_c \beta}{4} [\delta(f_c + f_m - f) + \delta(f_c + f_m + f)]$$

$$- \frac{A_c \beta}{4} [(f_c - f_m - f) + (f_c - f_m + f)]$$



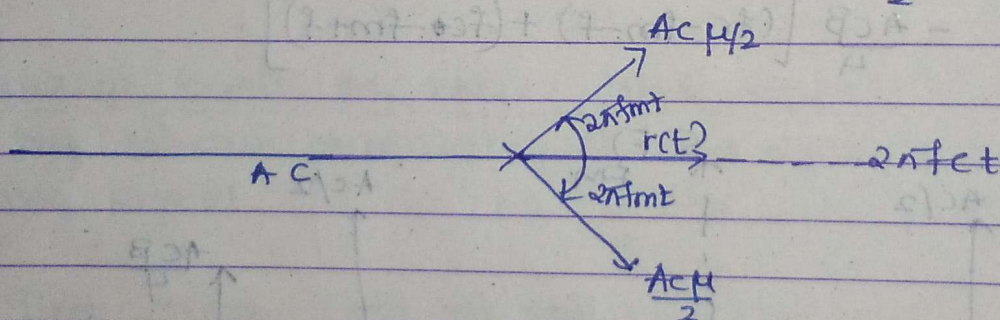
Phasor for NBFM

$$S_{FM}(t) = A_c \cos 2\pi f_c t + \frac{B A_c}{2} \cos 2\pi (f_c + f_m) t - \frac{B A_c}{2} \cos 2\pi (f_c - f_m) t$$



Phasor for Conventional AM

$$s(t) = A_c \cos 2\pi f_c t + A_c \mu/2 \cos 2\pi (f_c + f_m) t + \frac{A_c \mu}{2} \cos 2\pi (f_c - f_m) t$$



In NBFM the resultant is always perpendicular to carrier frequency.

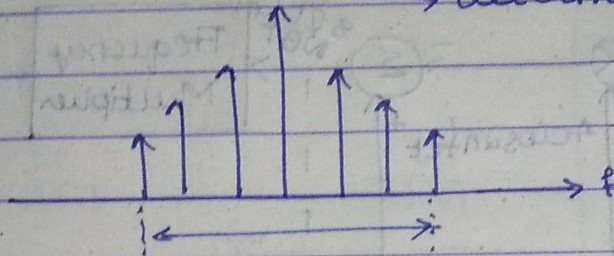
In AM the resultant is always along the carrier.

In NBFM there are impulses at $\pm f_c$ signifying that the carrier term is not suppressed.

WBFM - wide band FM

$$s(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos[2\pi(f_c + n f_m)t]$$

→ determine the amplitude of n^{th} sideband



side band power is increasing @ the rate of carrier components hence the power is same

→ Amplitude spectrum of FM signals contains a carrier component and an infinite set of side frequencies symmetrical on both sides

→ $\beta < 1$ $J_0(\beta)$ has significant value. and the FM signal comprises of f_c , $f_c + f_m$, $f_c - f_m$

→ $\beta > 1$, $J_0(\beta)$, $J_1(\beta)$ --- $J_n(\beta)$ has significant values has ~~deep~~ infinite sidebands

Transmission Bandwidth of FM

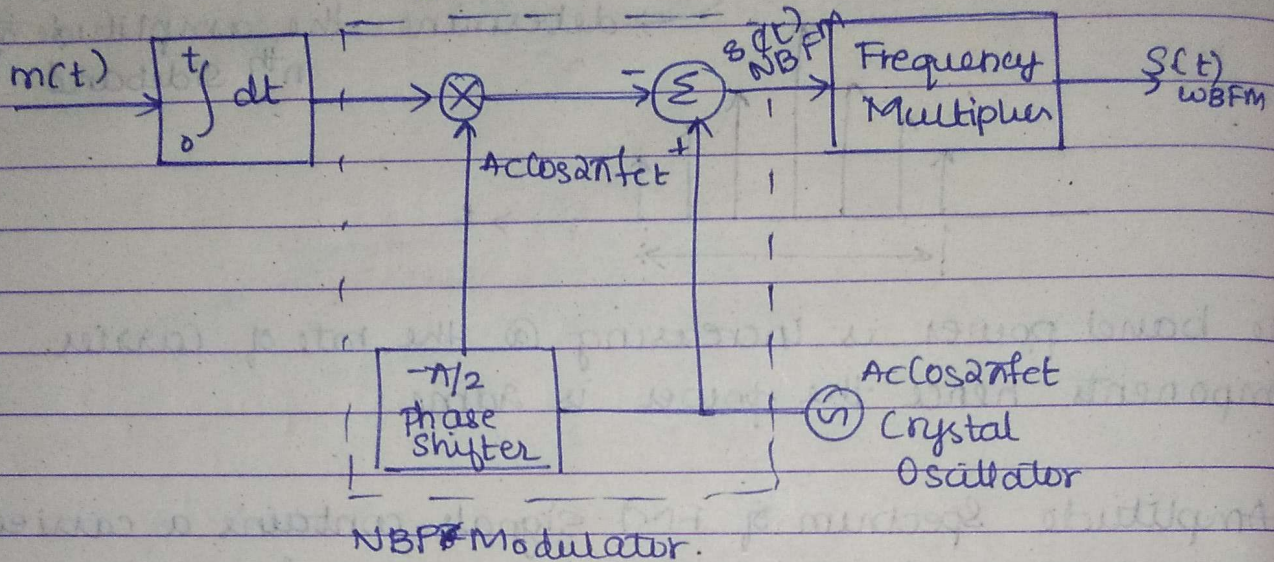
Carson's Rule:

$$BT = 2(\Delta f + f_m)$$

$$BT = 2\Delta f \left(1 + \frac{1}{\beta}\right)$$

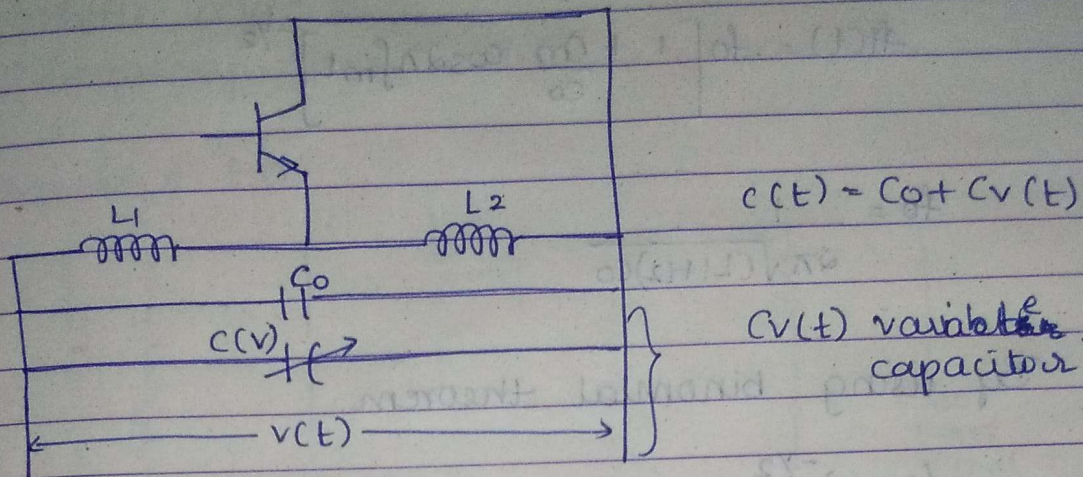
Generation of FM waves

Indirect Method of Generating FM waves



- A narrow band FM signal is generated using by integrating $m(t)$ and then using $\int m(t)$ to phase modulate the carrier generated by crystal oscillator
- $k_p m(t) \ll 1$ in order to minimize distortion in PM
- The NBFM thus generated is applied to frequency multiplier to increase the frequency deviation to desired level
- Frequency Multiplier should ensure that it provides the desired frequency deviation and carrier simultaneously

Direct Method of generating FM waves



- The instantaneous freq of the carrier is varied in accordance with $m(t)$ this is achieved by VCO
- VCO may be constructed using sinusoidal oscillator with a highly resonant n/w
- A fixed capacitor in parallel with variable voltage known as varactor can be used in frequency selective n/w.
- Capacitance of RB varactor diode depends on the voltage applied at the pn junction
larger v_{tq} → ~~less~~ small is its transition capacitance

→ ~~On fig~~ total capacitance

$$C(t) = C_0 + C_v(t)$$

$$C(t) = C_0 + C_m \cos 2\pi f_m t \quad \text{--- (1)}$$

$\downarrow$ $\rightarrow$ Max change in total capacitance
 total capacitance
 in absence of Modulation

$$f_c(t) = \frac{1}{2\pi \sqrt{(L_1 + L_2) C(t)}} \quad \text{--- (2)}$$

sub ① in ②

$$f_i(t) = f_0 \left[1 + \frac{C_m}{C_0} \cos 2\pi f_m t \right]^{-1/2}$$

$$f_0 = \frac{1}{2\pi \sqrt{(C_1 + C_2) C_0}}$$

By using binomial theorem.

$$(1+x)^{-1/2} \approx 1 - \frac{1}{2}x \quad \text{if } |x| \ll 1$$

$$\text{if } \left| \frac{C_m}{C_0} \right| \ll 1$$

$$f_i(t) = f_0 \left[1 - \frac{C_m}{2C_0} \cos 2\pi f_m t \right]$$

$$\text{let } \frac{\Delta f}{f_0} = -\frac{C_m}{2C_0}$$

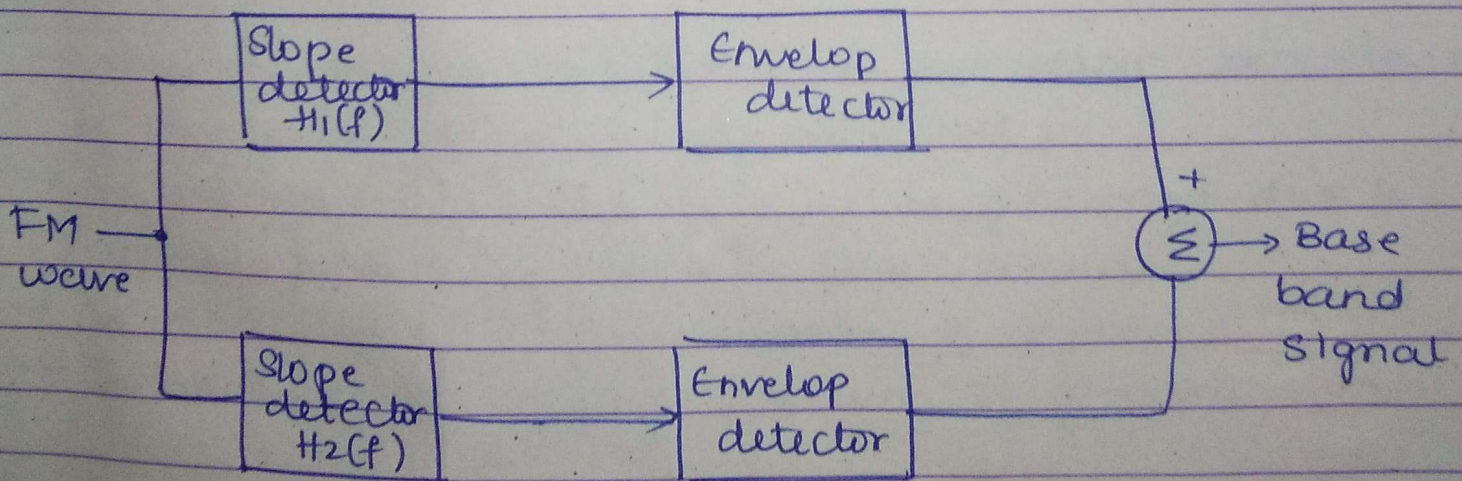
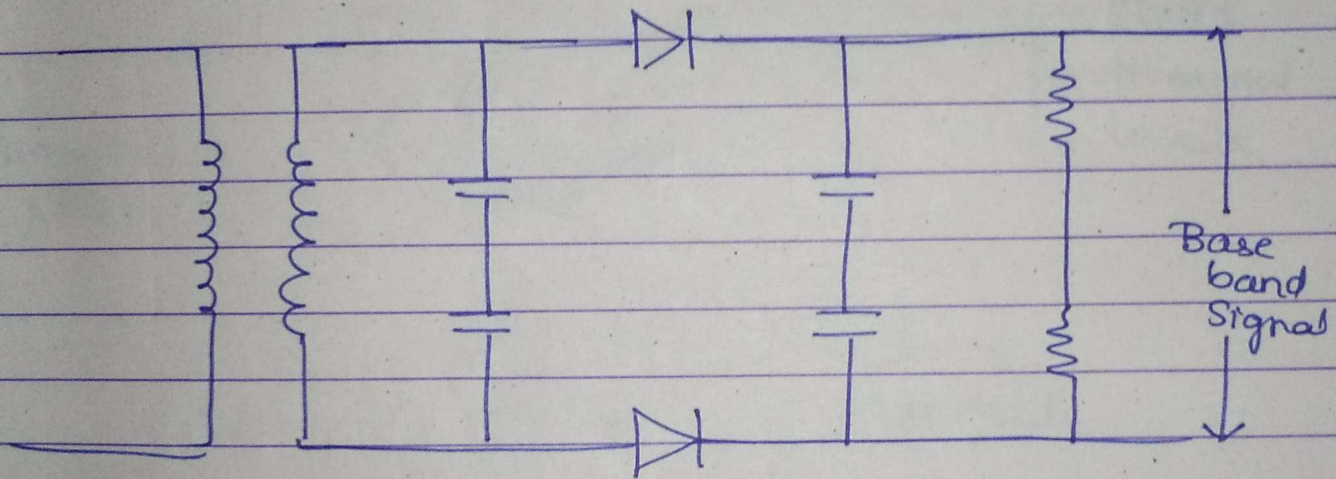
$$\therefore f_i(t) = f_0 - \Delta f \cos 2\pi f_m t$$

Demodulation of FM

→ Phase locked loop

→ Frequency discriminator → Balance FD

~~But~~ Balanced FD



(1)

Demodulation of FM waves

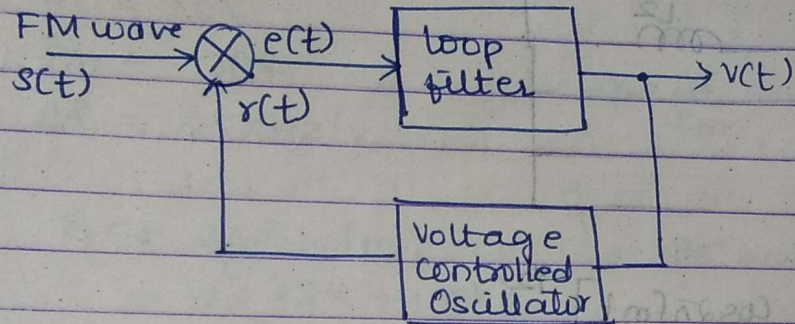
→ Phase locked loop (PLL)

→ Frequency discrimination

→ Balanced frequency discriminator

* Phase locked loop (PLL)

Block diagram:



Its a -ve feedback system

consists of 3 major components

→ Multiplier

→ loop filter

→ Voltage controlled oscillator

All the 3 components are connected together to form a feedback network

VCO is a sine wave generator → whose frequency is determined by the voltage applied external to it

Initially its assumed that VCO is adjusted and control voltage is zero

∴ two conditions are satisfied i) VCO frequency and f_c (unmodulated) are precisely set 2) VCO o/p has 90° phase shift with respect to unmodulated carrier f_c

$$s(t) = A \cos(2\pi f_c t + \phi_1(t)) \quad \text{--- ①}$$

be the input signal

$$VCO \text{ o/p is defined as } r(t) = A_r \cos(2\pi f_c t + \phi_2(t)) \quad \text{--- ②}$$

both these signals are applied to multiplier producing the following two components

→ high frequency component

$$k_m A_c A_v \cos [4\pi f_c t + \phi_1(t) + \phi_2(t)]$$

→ low frequency component

$$k_m A_c A_v \cos [\phi_1(t) - \phi_2(t)]$$

k_m - multiplier gain

high frequency component is eliminated by low pass action of the filter and VCO. ∴ Input to the loop filter is

$$e(t) = k_m A_c A_v \sin \cos [\phi_e(t)] \quad \text{--- (3)}$$

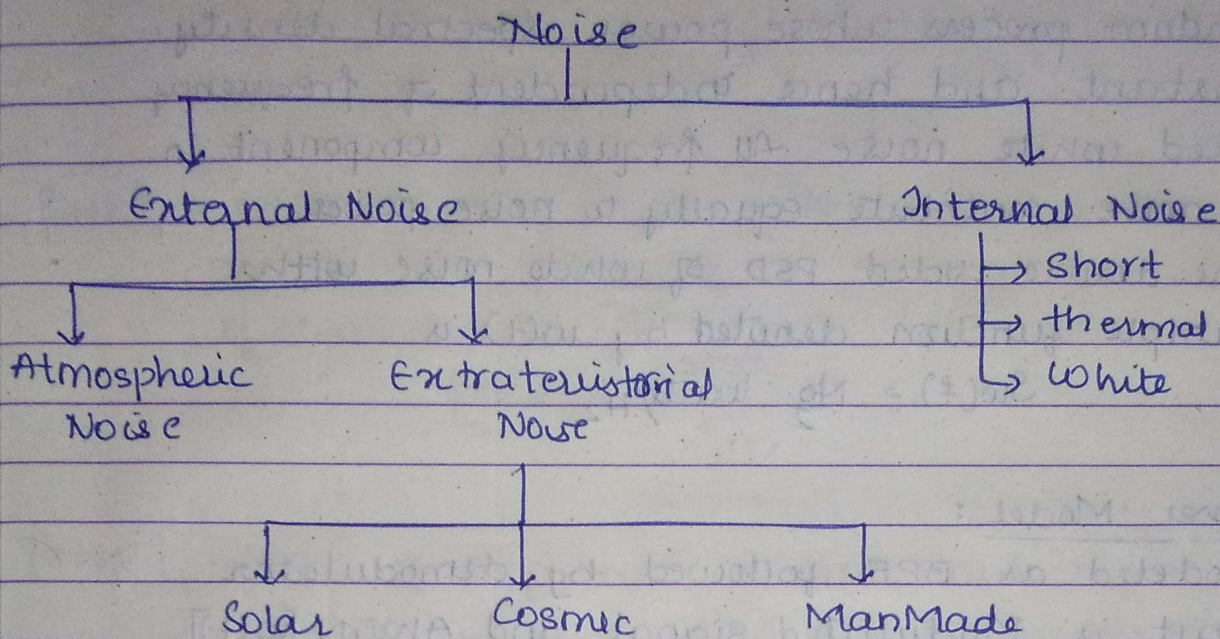
$\phi_e(t) \rightarrow$ phase error

The loop filter operates on its input to produce the output

$$v(t) = A_m \cos 2\pi f_m t$$

Chapter 3

NOISE & EFFECT OF NOISE ON RECEIVER



Short Noise:

- appears in active device due to random behaviour of change in charge carriers and holes
- In vacuum tubes short noise is generated due to random emission of electrons from cathode
- In SCD due to random diffusion of electrons
- In photodiode random emission of photons

Thermal Noise:

- generated due to random movement of electrons inside the volume of conductors or resistor even in the absence of electric field or potential diff
- The motion of free electron is due to thermal energy path of individual electron is random or zigzag
Bcoz of random movement of electrons there will be random fluctuations of currents & voltages

white noise:

A random process whose power spectral density is constant and hence independent of frequency is called white noise. All frequency component in white noise contribute equally to noise power.

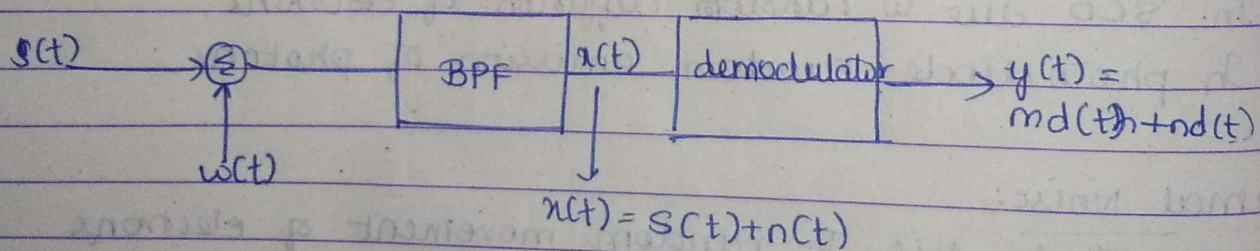
The double-sided PSD of white noise with a sample function denoted by $w(t)$ is

$$S_w(f) = \frac{N_0}{2} \text{ Watts/Hz}$$

Receiver Model:

- Modeled as BPF followed by demodulator
- input is Modulated signal and AWGN $w(t)$
- $w(t)$ accounts for channel noise & receiver noise called front-end noise.

- Bandwidth of BPF is equal to transmission bandwidth of modulated signal $S(t)$
- Hence modulated signal is passed without distortion while AWGN is within the Bandwidth of BPF



$$x(t) = s(t) + n(t)$$

$$n(t) = n_c(t) \cos 2\pi f_c t - n_s(t) \sin 2\pi f_c t$$

$n_c(t)$ & $n_s(t)$ are in phase quadrature component wrt $\cos 2\pi f_c t$

SNR = ratio of average power of the modulated signal $s(t)$ to the average power of the filtered noise $n(t)$

$[SNR]_o$ = ratio of avg power of the demodulated signal to the avg power of the noise at the output of demodulator

$$[SNR]_o = \frac{\overline{m_d^2(t)}}{\overline{n_d^2(t)}}$$

$[SNR]_c$ = ratio of avg power of modulated signal to the average power of noise in a Bandwidth of W Hz

$$\begin{aligned} [SNR]_c &= \frac{\overline{s^2(t)}}{\overline{n^2(t)}} \\ &= \frac{\overline{s^2(t)}}{N_b W} \end{aligned}$$

Figure of merit $\gamma = \frac{[SNR]_o}{[SNR]_c}$

$\gamma > 1$	$[SNR]_o > [SNR]_c$	γ is desirable
$\gamma = 1$	$[SNR]_o = [SNR]_c$	" " permissible
$\gamma < 1$	$[SNR]_o < [SNR]_c$	" " not "

* Avg power of $m(t)$

$$P_m = \overline{m^2(t)}$$

* Avg power of $km(t)$

$$\begin{aligned}\overline{[km(t)]^2} &= \overline{k^2 m^2(t)} \\ &= k^2 P_m\end{aligned}$$

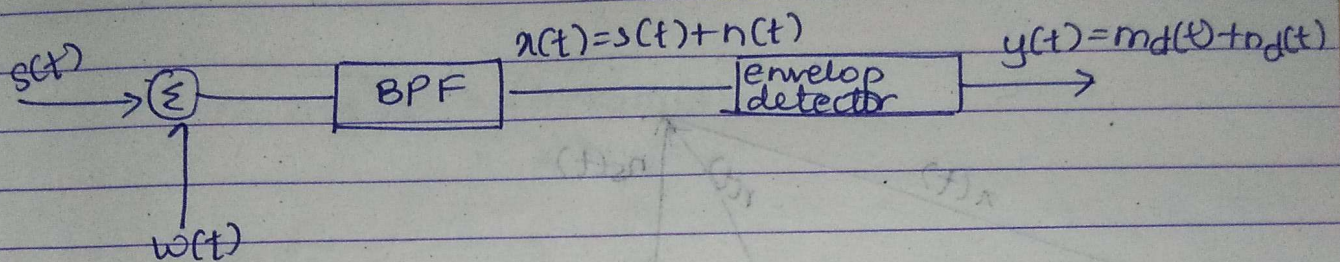
* Avg power of $A \cos 2\pi f_m t$

$$\begin{aligned}&= \overline{(A \cos 2\pi f_m t)^2} \\ &= \left(\frac{A_m}{\sqrt{2}}\right)^2 \\ &= \frac{A_m^2}{2}\end{aligned}$$

* Avg power of $km(t) \cos 2\pi f_m t$

$$\begin{aligned}&= \overline{(km(t) \cos 2\pi f_m t)^2} \\ &= \overline{k^2 m^2(t) \cos^2 2\pi f_m t} \\ &= k^2 P_m \left(\frac{1}{\sqrt{2}}\right)^2 \\ &= \frac{k^2 P_m}{2}\end{aligned}$$

Noise in AM Receivers



$$s(t) = A[1 + k_a m(t)] \cos 2\pi f_c t$$

$$[SNR]_c = \frac{\overline{s(t)^2}}{N_{0W}}$$

$$\begin{aligned} \overline{s(t)^2} &= \overline{(A_c \cos 2\pi f_c t)^2 + (A_c k_a m(t) \cos 2\pi f_c t)^2} \\ &= \frac{A_c^2}{2} + \frac{A_c^2 k_a^2 P_m}{2} \\ &= \frac{A_c^2}{2} [1 + k_a^2 P_m] \end{aligned}$$

$$[SNR]_c = \frac{A_c^2}{2N_{0W}} [1 + k_a^2 P_m]$$

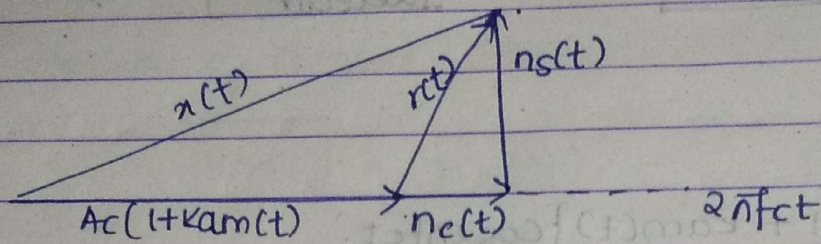
$n(t)$ appearing @ BPF

$$n(t) = n_c(t) \cos 2\pi f_c t - n_s(t) \sin 2\pi f_c t$$

Input to demodulator of AM receiver.

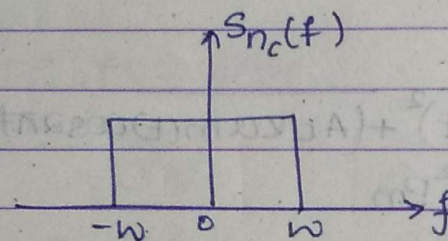
$$\begin{aligned} x(t) &= s(t) + n(t) \\ &= A_c [1 + k_a m(t)] \cos 2\pi f_c t + n_c(t) \cos 2\pi f_c t - n_s(t) \sin 2\pi f_c t \end{aligned}$$

Phasor diagram
for $x(t)$



$$A_c \gg n_s(t) \text{ \& } n_c(t)$$

PSD of Noise / Noise spectrum



Output of envelope detector

$$y(t) = |x(t)|$$

$$= \sqrt{\{A_c(1+k_a m(t)) + n_c(t)\}^2 + n_s^2(t)}$$

$$A_c \gg n_c(t) \text{ and } n_s(t)$$

$$= \sqrt{\{A_c(1+k_a m(t)) + n_c(t)\}^2}$$

$$= A_c(1+k_a m(t)) + n_c(t)$$

$$= A_c + A_c k_a m(t) + n_c(t)$$

passing through filter dc component is removed

$$y(t) = A_c k_a m(t) + n_c(t) = m_d(t) + n_d(t)$$

$$[SNR]_o = \frac{\overline{m_d(t)^2}}{n_d^2(t)}$$

$$\Rightarrow \overline{m_d^2(t)} = (\overline{A_{ckam}(t)})^2 = A_c^2 k a^2 P_m$$

$$n_d^2(t) = n_c(t)^2 = N_0 2W$$

$$[SNR]_o = \frac{A_c^2 k a^2 P_m}{N_0 2W}$$

$$FOM = \gamma = \frac{[SNR]_o}{[SNR]_c}$$

$$= \frac{A_c^2 k a^2 P_m}{2 N_0 W}$$

$$\frac{A_c^2 [1 + k a^2 P_m]}{2 N_0 W}$$

$$= \frac{k a^2 P_m}{1 + k a^2 P_m}$$

let $m(t)$ be single tone modulated signal
 $m(t) = A_m \cos a \pi f_m t$

$$\overline{m^2(t)} = \frac{A_m^2}{2} = P_m$$

$$FOM = \gamma = \frac{k a^2 \frac{A_m^2}{2}}{1 + k a^2 \frac{A_m^2}{2}}$$

$$\Rightarrow \mu = k a A_m$$

$$= \frac{\mu^2/2}{1 + \mu^2/2} = \frac{\mu^2/2}{2 + \mu^2/2}$$

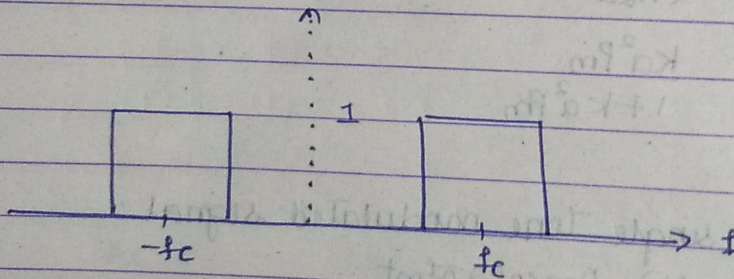
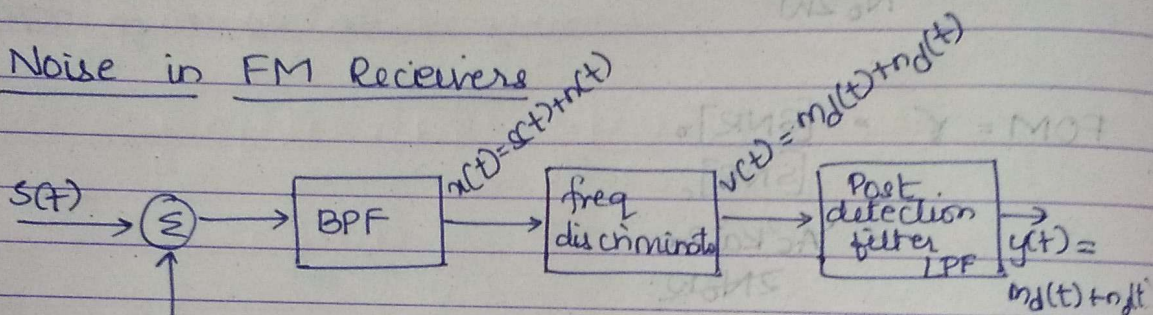
$$= \frac{\mu^2}{2 + \mu^2}$$

considering 100% modulation
 $\mu = 1$

$$r = \frac{1}{2+1} = \frac{1}{3}$$

Am system must transmit 3 times as much avg power as suppressed carrier system to achieve same quality of performance.

Noise in FM Receivers



$$S(t) = A_c \cos[2\pi f_c t + \phi(t)]$$

$$\phi(t) = 2\pi k_f \int_0^t m(t) dt$$

$$[SNR]_c = \frac{\text{Avg power of } S(t)}{\text{Avg power of } n(t) \text{ in bandwidth}}$$

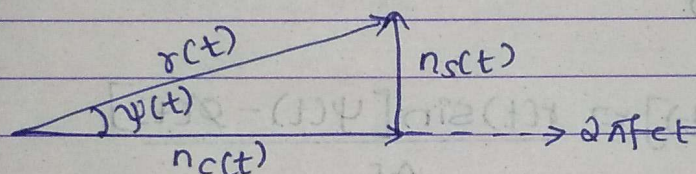
$$\text{Avg power of } s(t) = \overline{s^2(t)} = \frac{A_c^2}{2}$$

$$n(t) = \overline{n^2(t)} = N_0 W$$

$$[SNR]_c = \frac{A_c^2}{2 N_0 W}$$

$$n(t) = n_c(t) \cos 2\pi f_c t - n_s(t) \sin 2\pi f_c t$$

Phasor diagram for $n(t)$



$n(t)$ rewritten

$$n(t) = r(t) \cos[2\pi f_c t + \psi(t)]$$

↓ envelope of $n(t)$

$$r(t) = \sqrt{n_c^2(t) + n_s^2(t)}$$

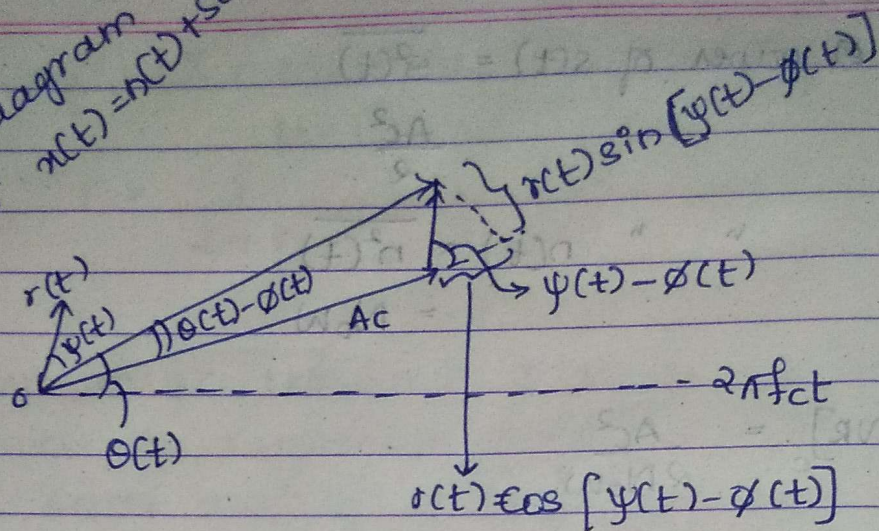
$$\psi(t) = \tan^{-1} \left[\frac{n_s(t)}{n_c(t)} \right]$$

Input of discriminator.

$$x(t) = s(t) + n(t)$$

$$= A_c \cos[2\pi f_c t + \phi(t)] + r(t) \cos[2\pi f_c t + \psi(t)]$$

phasor diagram
for $x(t) = n(t) + s(t)$



from phasor diagram

$$\tan[\theta(t) - \phi(t)] = \frac{r(t) \sin[\psi(t) - \phi(t)]}{A_c + r(t) \cos[\psi(t) - \phi(t)]}$$

$$A_c \gg r(t)$$

$$\tan[\theta(t) - \phi(t)] \approx \frac{r(t) \sin[\psi(t) - \phi(t)]}{A_c}$$

$$\psi(t) > \phi(t)$$

$$\tan[\theta(t) - \phi(t)] \approx \frac{r(t) \sin \psi(t)}{A_c}$$

$$r(t) \sin \psi(t) = n_s(t)$$

$$\tan[\theta(t) - \phi(t)] \approx \frac{n_s(t)}{A_c}$$

$$A_c \gg n_s(t)$$

$$\theta(t) - \phi(t) \approx \frac{n_s(t)}{A_c}$$

$$\theta(t) \approx \frac{n_s(t)}{A_c} + \phi(t)$$

frequency of $\omega(t)$.

$$f(t) = \frac{1}{2\pi} \frac{d\phi(t)}{dt}$$

$$f(t) = \frac{1}{2\pi} \frac{d}{dt} \left[\phi(t) + \frac{n_s(t)}{A_c} \right]$$

$$f(t) = \frac{1}{2\pi} \frac{d\phi(t)}{dt} + \frac{1}{2\pi A_c} \frac{dn_s(t)}{dt}$$

$$= \frac{1}{2\pi} \frac{d}{dt} \left[2\pi k_f \int_0^t m(t) dt \right] + \frac{1}{2\pi A_c} \frac{dn_s(t)}{dt}$$

output of frequency discriminator is proportional to $f(t)$

$$v(t) = k' f(t)$$

$$k' \geq 1$$

$$v(t) = f(t)$$

$$v(t) = k_f m(t) + \frac{1}{2\pi A_c} \frac{dn_s(t)}{dt}$$

$$= m_d(t) + n_d(t)$$

$$\begin{aligned} \text{Avg power of } m_d(t) &= \overline{m_d(t)^2} \\ &= [k_f m(t)]^2 \\ &= k_f^2 P_m \end{aligned}$$

$$n_d(t) = \frac{1}{2\pi A_c} \frac{dn_s(t)}{dt}$$

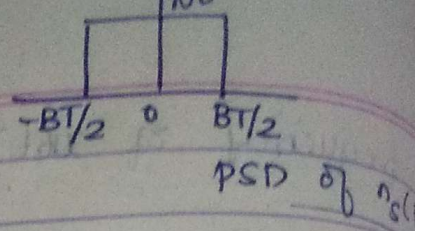
Taking transform

$$N_D(f) = \frac{1}{2\pi A_c} j2\pi f N_S(f)$$

$$|N_D(f)|^2 = \frac{f^2}{A_c^2} |N_S(f)|^2$$

$$S_{nd}(f) = \frac{f^2}{A_c^2} S_{ns}^2(f)$$

$$= \begin{cases} f^2/A_c^2 N_0 & |f| \leq BT \\ 0 & \text{otherwise} \end{cases}$$

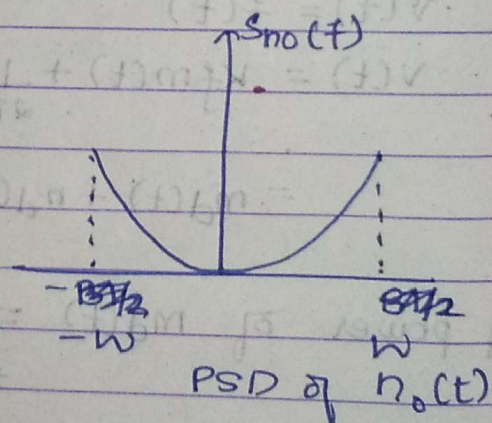
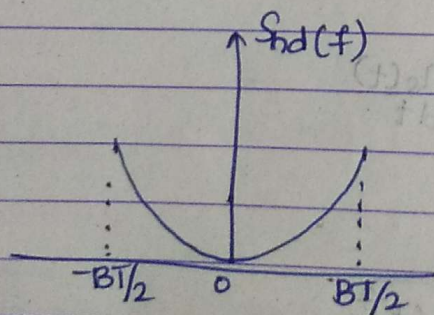


$$\text{Avg power of } \overline{n_o(t)^2} = \frac{N_0}{A_c^2} \int_{-W}^W f^2 df$$

$$= \frac{2N_0 W^3}{3A_c^2}$$

$$[SNR]_o = \frac{\overline{m^2(t)}}{\overline{n_o^2(t)}} = \frac{k_f^2 P_m}{2N_0 W^3 / 3A_c^2}$$

$$= \frac{3A_c^2 k_f^2 P_m}{2N_0 W^3}$$



$$FOM = \gamma = \frac{[SNR]_o}{[SNR]_c}$$

$$= \frac{3A_c^2 k_f^2 P_m}{2N_0 W^3}$$

$$\frac{A_c^2}{2N_0 W}$$

$$= \frac{3k_f^2 P_m}{W^2}$$

2.
D 57 $s(t)$

Let $m(t)$ be single tone modulating signal

$$m(t) = A_m \cos 2\pi f_m t$$

$$P_m = m^2(t) = \left(\frac{A_m}{\sqrt{2}}\right)^2$$

$$\Rightarrow \frac{A_m^2}{2} = P_m$$

$$FOM = \frac{3kf^2 A_m^2}{2f_m^2}$$

$$= \frac{3}{2} \left[\frac{kf^2 A_m}{f_m} \right]^2$$

$$= \frac{3}{2} \beta^2$$

$$\left\{ \frac{kf A_m}{f_m} = \beta \right.$$

$\mu=1$ ie 100% modulation

$$\frac{\mu^2}{2+\mu^2} = \frac{3}{2} \beta^2$$

$$\frac{1}{1+2} = \frac{3}{2} \beta^2$$

$$\frac{1}{3} = \frac{3}{2} \beta^2$$

$$\underline{\underline{\beta = 0.5}}$$

Capture effect: