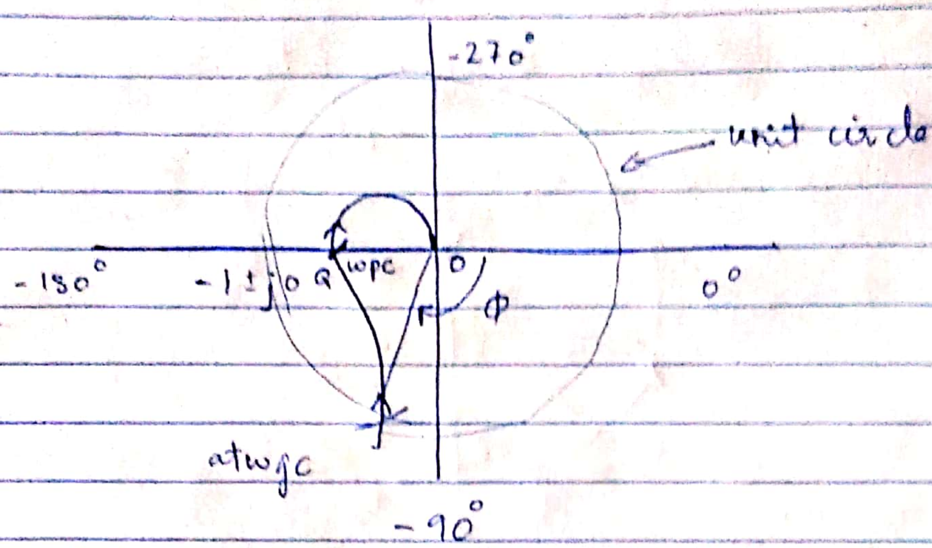
24/1/19

GM ; PM ; ω_{gc} & ω_{pc} from the polar plot



$$PM = 180° + \underset{\text{at } \omega = \omega_{gc}}{\angle G(j\omega)H(j\omega)} = 180° + \angle\phi$$

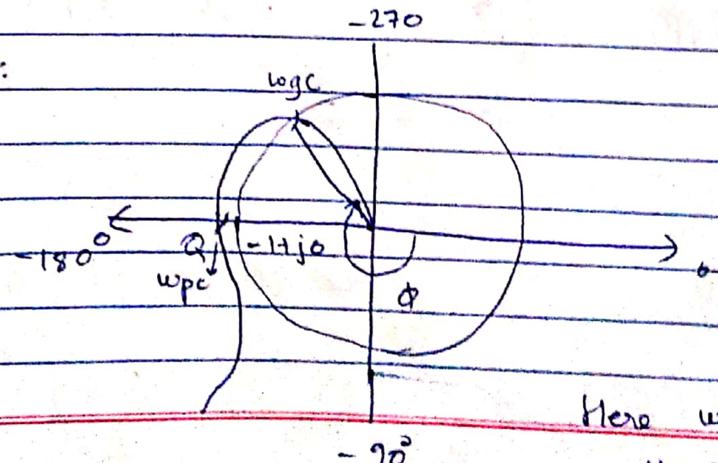
if $\phi = -135°$ (say)

$PM = 180 + (-135) = +45°$

$GM = ? = 20 \log_{10} \frac{1}{|OQ|}$

$\omega_{pc} > \omega_{gc} \rightarrow$ so stable system

Case 2:



Here $\omega_{gc} > \omega_{pc}$

unstable.

Nyquist Criteria

Z be the no. of zeros of the OLTF, in RH of s plane. and 'P' be no. of poles of OLTF on RH of s plane.

& N = No. of 'n' ~~suppliments~~ encirclements of critical points $-1+j0$ by Nyquist plot.

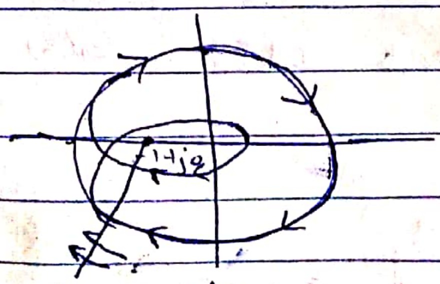
The Nyquist stability criteria states that for absolute stability of system, no. of n encirclements ~~supplements~~ of critical points $-1+j0$ by Nyquist plot must be equal to no. of poles of OLTF which are in the right half of s-plane & in clockwise direction.

It means that not a single zero should exist in right half of s-plane if system has to be stable.

i.e $N = Z - P$ but for stability $Z = 0$

hence $N = -P$ for absolute stability

Ex:



$N = -2$ (clkwise)

$\rightrightarrows$ anticlk

$N = +2$

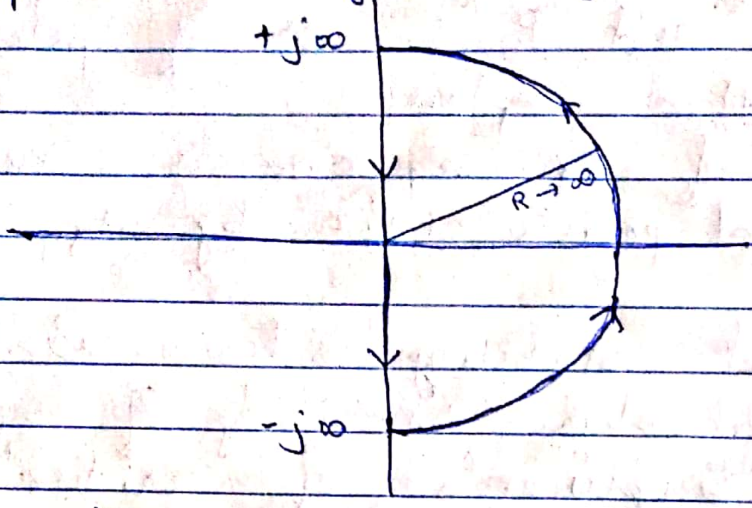
if $\rightleftarrows$

$N = 0$

$\rightarrow$

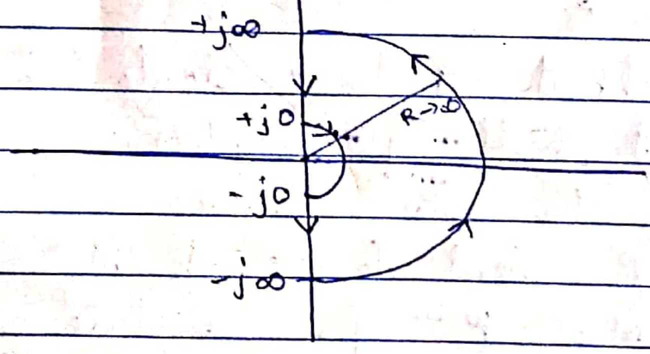
Nyquist path

i) No poles at origin & on jω axis



Nyquist path when no poles on origin & jω

ii) when pole at origin



Ex: Control System has OLTF $G(s)H(s) = \frac{K}{s(s+2)(s+10)}$

Sketch Nyquist plot & determine range of 'K' for stability.

→ Step 1: The no. of poles & zeros on RH

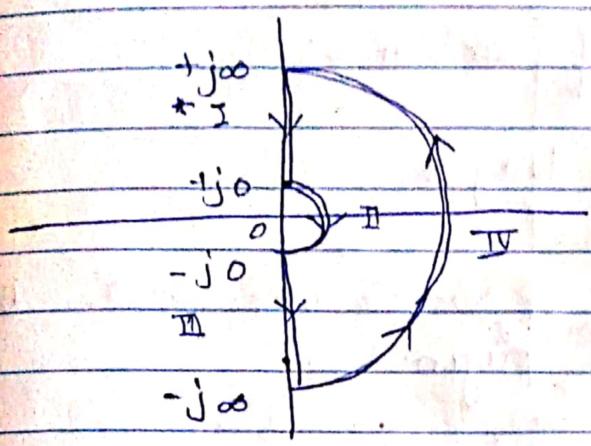
P = 0 no poles on RH

Z = 0

Step 2: $\boxed{N = 0} = -P$ for stability

↳ No encirclement of critical pt (-1+j0) by Nyquist plot

③ Nyquist plath



④ with $s = j\omega$

$$G(j\omega)H(j\omega) = \frac{K}{j\omega(j\omega+2)(j\omega+10)}$$

$$|M| = \frac{1}{\omega\sqrt{4+\omega^2}\cdot\sqrt{\omega^2+100}}$$

$$\phi = -90 - \tan^{-1}\frac{\omega}{2} - \tan^{-1}\frac{\omega}{10}$$

Analysis of sections :-

*Section I : $+j\infty$ to $+j0$

		M	ϕ	
Starting pt	$\omega = +\infty$	0	$-270°$	Rotation of plot :
Terminating pt	$\omega = +0$	∞	$-90°$	$-90-(-270)$
				$= +180$ (~~clk~~) (anti)

Section II : $+j0$ to $-j0$

		M	ϕ	
Starting pt	$\omega = +0$	∞	$-90°$	Rotation of plot :
Terminating pt	$\omega = -0$	∞	$+90°$	$90+90 = 180$ (anti)

Scanned by CamScanner

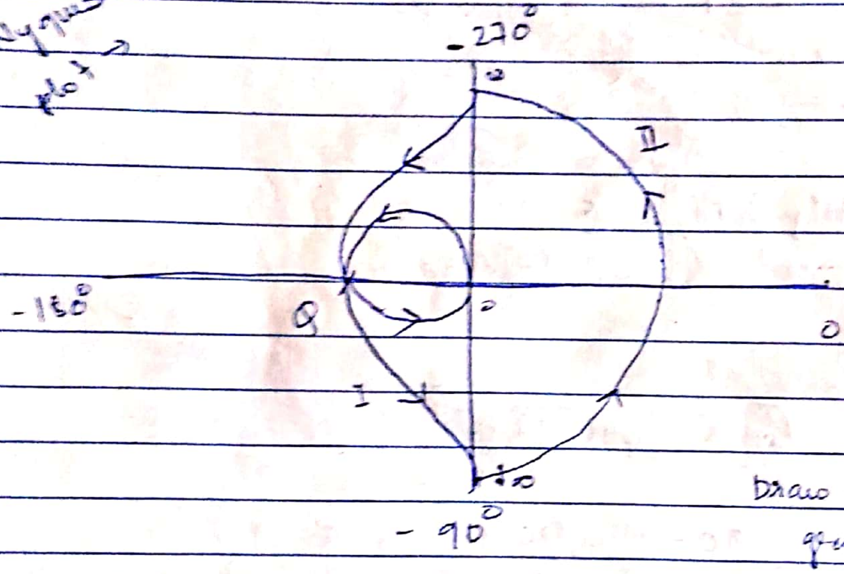
Section III : $-j0$ to $-j\infty$

Mirror img of section I

Section IV : $-j\infty$ to $+j\infty$

No need to analysis as mag. is '0' & it will be at origin.

Nyquist plot →



Draw on ordinary graph

Step 5 : Determine the length OQ

$$G(j\omega)H(j\omega) = \frac{K}{j\omega(j\omega+2)(j\omega+10)}$$

~~Rationalise~~

a

$$= \frac{K}{j\omega(j\omega+2)(j\omega+10)} \times \frac{-j\omega(-j\omega+2)(10-j\omega)}{-j\omega(2-j\omega)(10-j\omega)}$$

$$= \frac{-12K\omega^2}{\omega^2 \times (4+\omega^2)(100+\omega^2)} - j\frac{K\omega(20-\omega^2)}{\omega^2(4+\omega^2)(100+\omega^2)}$$

Equating img term to zero.
we get:

$$\omega(20-\omega^2) = 0$$

$$\boxed{\omega = \sqrt{20}} \text{ rad/sec} = \omega_{pc}$$

subt. ω_{pc} in real term to find OQ

$$OQ = \frac{-12k\omega^2}{\omega^2(4+3\omega^2)(100+\omega^2)} = \boxed{\frac{-k}{240}}$$

Step 6: Nyquist plot : Drawn.

Step 7: $|OQ| = \frac{k}{240} < +1$ for stability

$$\boxed{k < 240}$$

range of k $\boxed{240 > k > 0}$ for stability.

Eg: $G(s)H(s) = \frac{40}{(s+4)(s^2+2s+2)}$, det the gain margin & stability of s/m from Nyquist plot.

→ ① No. of poles on RH of s plane
$P = 0$

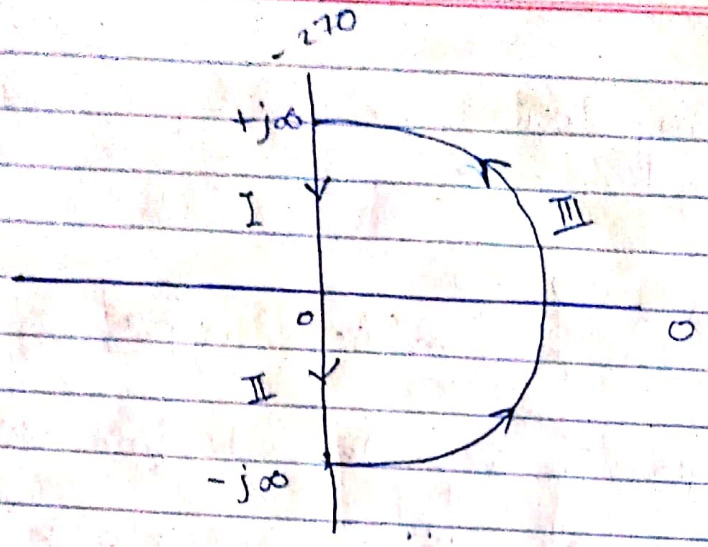
② For stability $N = -P = 0$ [i.e. the critical pt. $-1+j0$ should not be encircled by Nyquist plot.

③ Nyquist path.

110 rad/sec

$(2j\omega - j\omega^3 - 2\omega^2 + 8 - 6\omega^2 + 8j\omega)(8 - 6\omega^2 + j\omega^3 - 10j\omega)(8 - 6\omega^2 - j\omega^3 + 10j\omega$

$= 64 - 48\omega^2 - 8j\omega^3 + 80j\omega - 48\omega^2 + 36\omega^4$



(4) $s = j\omega$

$$G(j\omega)H(j\omega) = \frac{40}{(j\omega+4)(j\omega^2 + 2j\omega + 2)}$$

$$= \frac{40}{(j\omega+4)(2-\omega^2+2j\omega)}$$

$$|M| = \frac{40 + j0}{\sqrt{\omega^2+16}\,\sqrt{(2-\omega^2)^2 + 4\omega^2}}$$

Imp (*) Always, quadratic pole consitutes an angle of -180° as $\omega \to \infty$ and its angle is 0° when $\omega \to 0$

Hence $\angle\phi$ for OLTF will be

$$= 0 - \tan^{-1}\frac{\omega}{4} - \tan^{-1}\frac{2\omega}{2-\omega^2}$$

180° when $\omega \to \infty$

(4) Analysis of sections

Section I : $s = +j\infty$ to 0

		$\|M\|$	ϕ	Rotation
Starting pt	$\omega = +\infty$	0	-270	of polt
				$= 0-(-270)$
Terminating pt	$\omega = 0$	5.	0	$= 270°$

Section II : Mirror image of Section I

Section III : $-j\infty$ to $+j\infty$ No need to analyse

		$\|M\|$	ϕ
Starting pt	$\omega \Rightarrow \infty$	0	-270
Terminating	$\omega \Rightarrow -\infty$	0	-270

(5) ~~Nyquist~~ ~~plot~~ Rationalise

$$\frac{40}{(j\omega+4)(2-\omega^2+2j\omega)} \times \frac{(4-j\omega)(2-\omega^2-2j\omega)}{(4-j\omega)(2-\omega^2-2j\omega)}$$

$$= \frac{40(\cancel{8}-4\cancel{\omega^2}-8j\omega-2j\omega+j\omega^3 4-2\cancel{\omega^2})}{(16+\omega^2)((2-\omega^2)^2+4\omega^2)}$$

$(16+\omega^2)[\omega^4+4+\cancel{\ldots}]$

$(16+\omega^2)[4+\omega^4-\cancel{4\omega^2}+\cancel{4\omega^2}]$

$$= \frac{40(8-6\omega^2)}{\text{deno}} + 40(j\omega^3-10j\omega)$$

Eq. img part to zero

$$40(j\omega^3-10j\omega) = 0$$

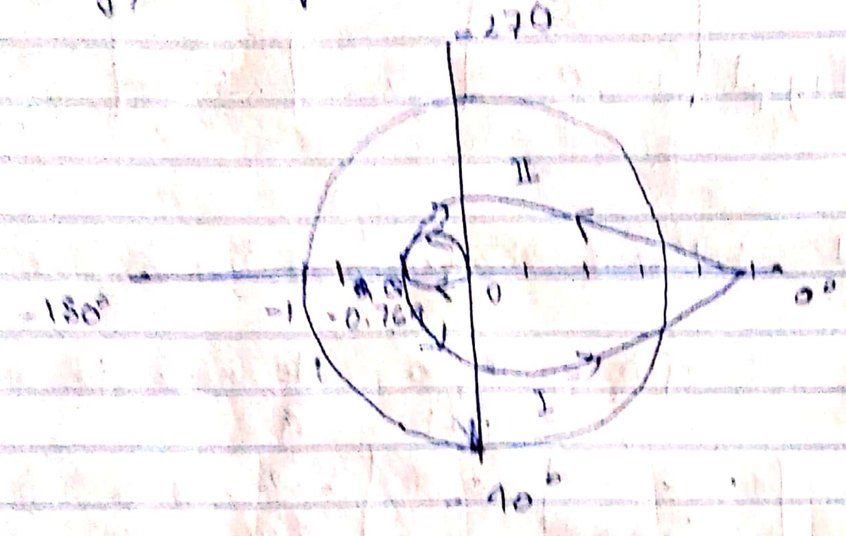
$$j\omega^3 = 10j\omega$$

$$\omega pc = \boxed{\omega = \sqrt{10}\ \text{rad/sec}}$$

Subst. ω in real part

$\boxed{OR = -0.769}$

⑥ Nyquist plot:



⑦ As per step ② for stability

$N = -P = 0$

i.e. critical point $-1+j0$ must be outside Nyquist plot

⑧ Comment on stability

$|OR| = 0.769 < 1$

Hence as shown on Nyquist plot the critical point is not encircled by Nyquist plot.

& this is req. cond. for stability

Hence given system is stable.

$$GM = 20 \log \frac{1}{|OQ|} = 2.28$$

Ex: The OLTF of s/m is given by

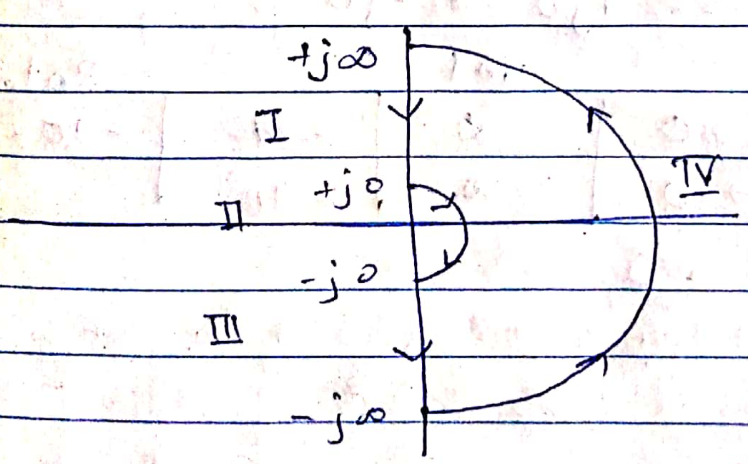
$$G(s)H(s) = \frac{K(s+1)(s+3)}{s(s+2)(s-4)}$$

draw Nyquist plot and det. range of 'K' for which CLS is stable. Verify ans. using RH criteria.

→ ① $P = 1$; $N = -P = -1$

② For abs. stability $N = -P = -1$ [i.e critical pt $-1+j0$ shld be encircled by Nyquist plot, once in clockwise dir)

③ Nyquist path



④ $$G(j\omega)H(j\omega) = \frac{K(1+j\omega)(3+j\omega)}{j\omega(2+j\omega)(-4+j\omega)}$$

$$|M| = \frac{\sqrt{1+\omega^2}\sqrt{9+\omega^2}}{\omega\sqrt{4+\omega^2}\sqrt{16+\omega^2}}$$

Note : $(s-4) = j\omega - 4 = -4 + j\omega$ $\quad \phi_1 = \angle 180 - \tan^{-1}\omega/4$

then when $\omega \to \infty$ $\quad \phi_p = \angle +90^\circ$

11^{ly} for $\omega \to 0$ or -0
$\phi_p = 180^\circ$

$= 0 + \tan^{-1}\omega + \tan^{-1}\frac{\omega}{3} - 90 - \tan^{-1}\frac{\omega}{2} - \angle 180 - \tan^{-1}\frac{\omega}{4}$

Analysis of Sections

Section I : $s = j\infty$ to $+j0$

		IM	ϕ	Rotation of plot
Starting pt	$\omega \to +\infty$	0	-90	
Terminating pt	$\omega \to +0$	∞	-270	$-270-(-90) = -180^\circ$ (clk)

Section II : $s = +j0$ to $-j0$

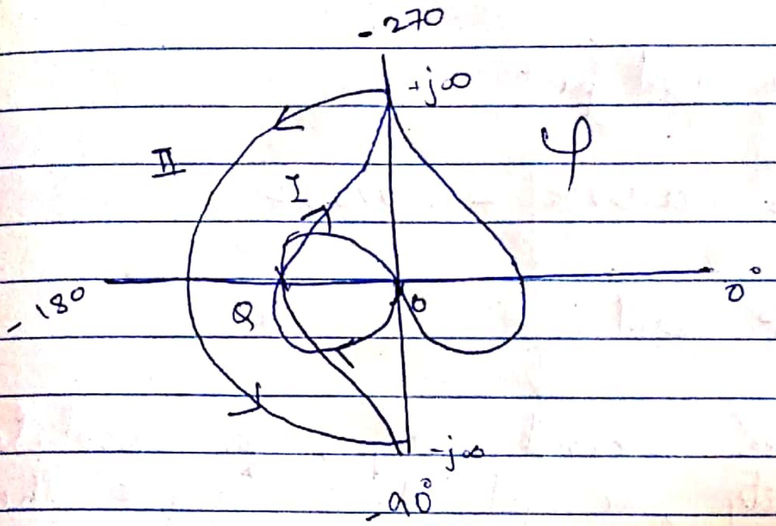
		IM		Rotation
Starting pt	$\omega \to +0$	∞	-270	$-90-(-270)$
Terminating	$\omega \to -0$	∞	-90°	$= +180$ (anti)

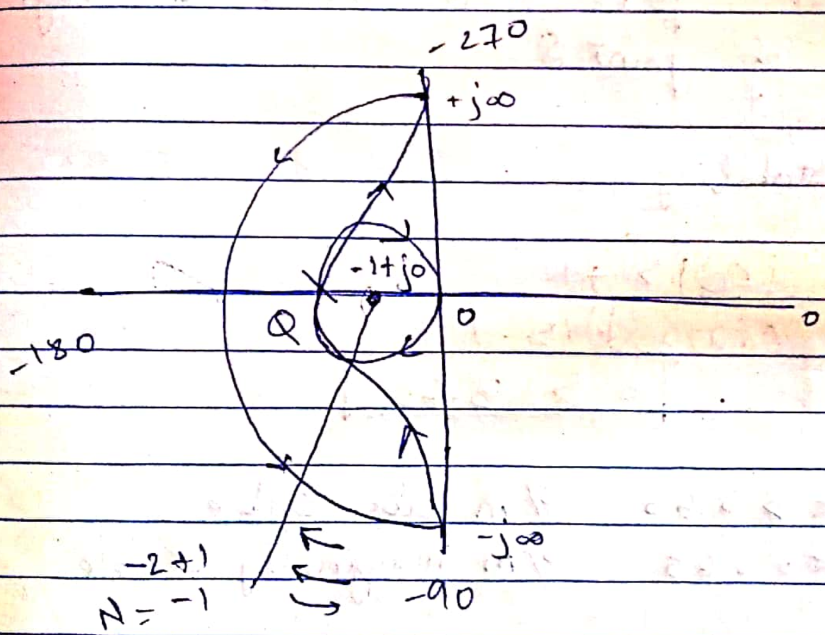
$= 0 + \tan^{-1}\omega + \tan^{-1}\frac{\omega}{3} + 90 - \tan^{-1}\frac{\omega}{2} - \angle 180^\circ$

Section III : Mirror image of I

" IV : Analysis not req.

(4)





(5) Rationalize

$$\frac{K(1+j\omega)(3+j\omega)}{j\omega(2+j\omega)(-4+j\omega)} \times j\omega($$

$$= \frac{-K(26+6\omega^2)}{(4+\omega^2)(16+\omega^2)} - j\frac{(K)(\omega^4-3\omega^2-24)}{\omega(4+\omega^2)(16+\omega^2)}$$

Eq. img to zero

$\omega_{pc} = \omega = \underline{2.57 \text{ rad/sec}}$

Sub ω in real part

$OQ = \cancel{-0.2757K} - 0.2735\,k$

⑥ Nyquist plot :

⑦ As per second step, for absolute stability, we got $N = -1$, hence the critical point $-1+j0$ should be to right sides of point Q.

For stability

$$|OQ| > +1$$
$$0.2735\,k > 1$$
$$\boxed{k > 3.65}$$

For $k > 3.65$ s/m abs. stable

$k = 3.65$ s/m marginally stable

Verification using RH

$$1 + G(s)H(s) = 0$$
$$1 + \frac{k(1+s)(3+s)}{s(2+s)(-4+s)} = 0$$

$$(2s + s^2)(-4 + s) + (k + ks)(3 + s) = 0$$
$$-8s + 2s^2 - 4s^2 + s^3 + 3k + ks + 3ks + ks^2 = 0$$
$$s^3 + s^2(2 - 4 + k) + s(-8 + k + 3k) + 3k = 0$$

$$\begin{array}{c|cc} s^3 & 1 & -8+4k \\ s^2 & -2+k & 3k \\ s^1 & -k-8/-2+k & 0 \\ s^0 & 3k & \end{array}$$

$$\frac{3k-8-4k}{-2+k} = \frac{-k-8}{-2+k}$$

$$A(\omega) = \frac{-k-8}{-2+k} = 0$$

$$\frac{dA(\omega)}{dk} = 0 = \frac{-1,-1}{(k-2)^2} = 0$$

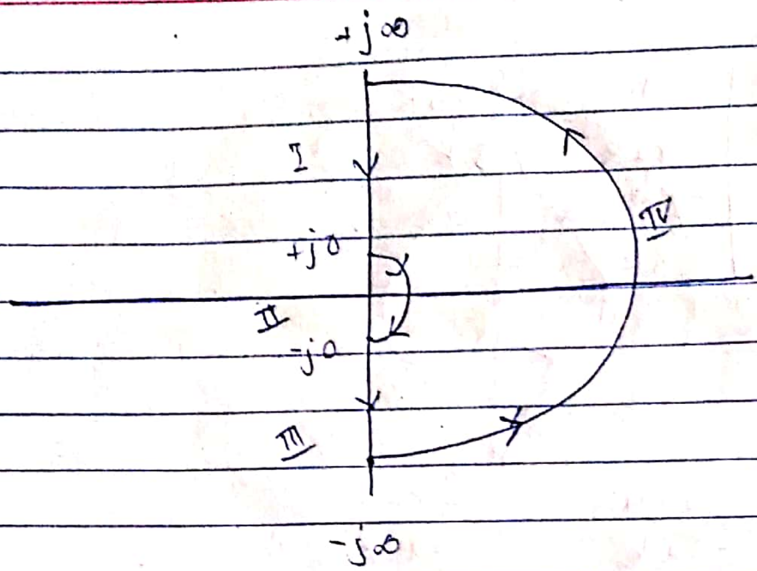
Qn: Draw Nyquist plot for s/m with

$$G(s)H(s) = \frac{10(s+3)}{s(s-1)}$$ comment on CL stability

→ ① Poles = 1

N = - P = -1

② Nyquist path



③ $G(j\omega)H(j\omega) = \dfrac{10(j\omega+3)}{j\omega(j\omega-1)}$

$$|M| = \frac{10\sqrt{9+\omega^2}}{\omega\sqrt{1+\omega^2}}$$

$$\phi = 0 + \tan^{-1}\frac{\omega}{3} - \tan^{-1}\omega + \tan^{-1}\omega - 90^\circ$$

$$= \tan^{-1}\frac{\omega}{3} - 90^\circ - (180^\circ - \tan^{-1}\omega)$$

Section 1 :

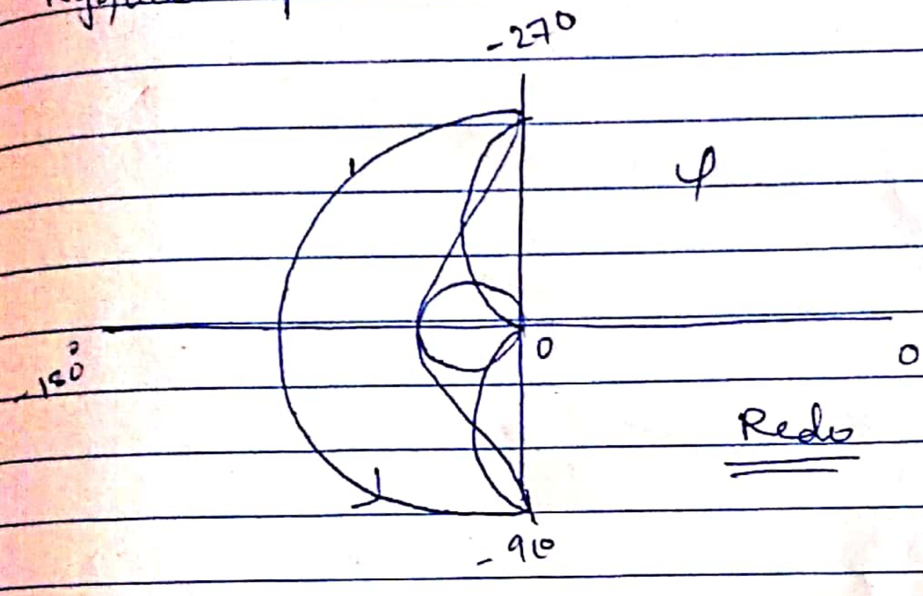
		$\lvert M \rvert$	ϕ	Rotation of plot
Starting pt	$\omega \to \infty$	0	-180	-90
Terminal pt	$\omega \to 0$	∞	-270°	

Section II : $s = j0$ to $-j0$

		$\lvert M \rvert$	ϕ	Rotation of plot
Start pt	$\omega \to +0$	∞	-270°	
Ter pt	$\omega \to -0$	∞	-90°	$+180^\circ$

section II : Mirror image

(5) Nyquist plot



Note : as one plot is on RH so OLTF is unstable but CLS is stable as per Nyquist plot.