Stability analysis in S.

i) Absolutely stable system
ii) $\pm j\omega$ (critically or marginally stable)
iii) Repeated roots on $j\omega$ axis $\rightarrow$ unstable system

or

Roots on RHS

iv) conditionally stable system
v) asymptotically stable.

Criteria to determine stability of system:

1) Hurwitz's criteria.
2) Routh-hurwitz's criteria (RH criteria)

$$CLTF = \frac{b_0 s^m + b_1 s^{m-1} + b_2 \dots b_n}{a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} \dots a_n} = \frac{P(s)}{Q(s)}$$

Let $q(s) = a_0 s^n + a_1 s^{n-1} + \dots a_n$ is the characteristic eqn.

Hurwitz's criterion

$$H_n = \begin{vmatrix} a_1 & a_3 & a_5 & \dots \\ a_0 & a_2 & a_4 & \dots \\ 0 & a_1 & a_3 & a_5 \dots \\ 0 & a_0 & a_2 & a_4 \dots \\ 0 & 0 & a_1 & a_3 \dots \\ 0 & 0 & a_0 & a_2 \dots \end{vmatrix}_{n \times n}$$

$D_1 = |a_1|$

$D_2 = \begin{vmatrix} a_1 & a_3 \\ a_0 & a_2 \end{vmatrix}$

$\vdots$

$D_n = |H|$

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For the system to be stable values of all sub determinants should be positive. Even if any one subdeterminant is -ve, the system is unstable.

Ex. Let us consider example given below. The system has a characteristic eqn:

$q(s) = s^3 + s^2 + s + 4 = 0$. Determine the stability of the system using Hurwitz's criterium

$\rightarrow$ $q(s) = s^3 + s^2 + s + 4 = 0$

$a_0 = 1 \quad a_1 = 1 \quad a_2 = 1 \quad a_3 = 4$

$$H = \begin{vmatrix} 1 & 4 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 4 \end{vmatrix}$$

$D_1 = |1| = 1$

$$D_2 = \begin{vmatrix} 1 & 4 \\ 1 & 1 \end{vmatrix} = -3$$

$$D_3 = |H| = \begin{vmatrix} 1 & 4 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 4 \end{vmatrix} = -12.$$

Disadvantages of Hurwitz criteria
- As order reaches a higher number, finding determinant becomes complex
- No. of roots poles present on RHS will not be known
- Marginal or critical stability cannot be determined.

Routh's array

$$\begin{array}{c|ccc} s^n & a_0 & a_2 & a_4 \\ s^{n-1} & a_1 & a_3 & a_5 \\ s^{n-2} & b_1 & b_2 & b_3 \\ \vdots & c_1 & c_2 & c_3 \\ & \vdots & & \\ s^0 & a_n & & \end{array}$$

$$b_1 = \frac{a_1 a_2 - a_0 a_3}{a_1} \qquad b_2 = \frac{a_1 a_4 - a_0 a_5}{a_1}$$

$$c_1 = \frac{b_1 a_3 - a_1 b_2}{b_1} \qquad c_2 =$$

Ex. $q(s) = s^3 + s^2 + s + 4 = 0$. Determine stability using RH criterion.

$a_0 = 1 \quad a_1 = 1 \quad a_2 = 1 \quad a_3 = 4$.

Routh's array:

s^3	1	1	0
s^2	1	4	0
s^1	-3	0	
s^0	4		

There is a change in sign so the system is unstable.
The sign changes twice, so there are 2 roots on the RHS

Q Determine the stability of the system whose characteristic eqn is given by $q(s) = s^3 + 6s^2 + 11s + 6 = 0$ using RH criteria.

$a_0 = 1 \quad a_1 = 6 \quad a_2 = 11 \quad a_3 = 6$.

Routh's array

s^3	1	11	0
s^2	6	6	0
s	~~4~~ 10	0	
s^0	6		

There is no change in sign, hence system is stable

Q $s^3 + 4s^2 + s + 16 = 0$ determine stability using RH criteria.

$a_0 = 1 \quad a_1 = 4 \quad a_2 = 1 \quad a_3 = 16$

Routh's array

s^3	1	1	0
s^2	4	16	0
s	-3	0	0
s^0	16	0	0

System is unstable as there is change in sign. there are 2 roots in RHS. of s plane.

Special case 1:

When first element of any of the rows of the Rouths array is zero but having atleast one non zero element in that row, consider the zero as a small constant ϵ

Method 1: consider zero as a small constant ϵ.

Ex - $s^5 + 2s^4 + 3s^3 + 6s^2 + 2s + 1 = 0$

Routh's array.

s^5	1	3	2
s^4	2	6	1
s^3	$\cancel{0}\ \epsilon$	3/2	0
s^2	$\frac{6\epsilon-3}{\epsilon}$	1	
s^1	$\frac{9\epsilon-\epsilon^2-4.5}{6\epsilon-3}$		
s^0	1		

=

s^5	1	3	2
s^4	2	6	1
s^3	ϵ	3/2	0
s^2	$-\infty$	1	
s^1	1.5		
s^0	1		

$$\frac{\frac{6\epsilon-3}{\epsilon}\times\frac{3}{2} - \epsilon}{\frac{6\epsilon-3}{\epsilon}} = \frac{\frac{9\epsilon-4.5}{\epsilon} - \frac{\epsilon}{1}}{\frac{6\epsilon-3}{\epsilon}} = \frac{9\epsilon-\epsilon^2-4.5}{6\epsilon-3}$$

$$\lim_{\epsilon\to 0} \frac{6\epsilon-3}{\epsilon} = \lim_{\epsilon\to 0} \frac{6\cancel{\epsilon}}{\cancel{\epsilon}} - \frac{3}{\epsilon} = -\infty.$$

$$\lim_{\epsilon\to 0} \frac{9\epsilon-\epsilon^2-4.5}{6\epsilon-3} = \frac{-4.5}{-3} \Rightarrow 3/2 = 1.5$$

Since there is sign change system is unstable.

Method 2: $s = 1/z$, Replace s by $1/z$ in main equation. Take LCM and rewrite the eqn by considering ascending powers of z using this new eqn. form Routh array & determine stability of the system

Ex. $s^5 + 2s^4 + 3s^3 + 6s^2 + 2s + 1 = 0$

$$\frac{1}{z^5} + 2\frac{1}{z^4} + 3\frac{1}{z^3} + 6\frac{1}{z^2} + 2\frac{1}{z} + 1 = 0$$

$$\frac{1 + 2z + 3z^2 + 6z^3 + 2z^4 + z^5}{z^5} = 0$$

$$z^5 + 2z^4 + 6z^3 + 3z^2 + 2z + 1 = 0$$

z^5	1	6	2
z^4	2	3	1
z^3	4.5	1.5	0
z^2	7/3	~~1~~	0
z^1	−3/7	0	0
z^0	1	0	0

System is unstable & has 2 roots ~~on~~ in the RHS.

Special case 2:

s^5	a	b	c
s^4	d	e	f
s^3	0	0	0

$A(s) = ds^4 + es^2 + 1$

$\frac{dA(s)}{ds} = 4ds^3 + 2es + 0$

Problems:

1) Consider the characteristic eqn. of a system given by $s^6 + 4s^5 + 3s^4 - 16s^2 - 64s - 48 = 0$. Find the number of roots with +ve real part, zero real part & −ve real part.

→ $g(s) = \underset{a_0}{s^6} + \underset{a_1}{4s^5} + \underset{a_2}{3s^4} + \underset{a_3}{0s^3} - \underset{a_4}{16s^2} - \underset{a_5}{64s} - \underset{a_6}{48} = 0$

s^6	1	3	−16	−48
s^5	4	0	−64	0
s^4	~~12~~ 3	0	−48	0
s^3	0 [12]	0	0	
s^2	0 [ε]	−48	0	
s^1	~~576~~ [ε] / ε	0	0	
s^0	−48	0		

s^3 → special case 2.

s^1 → special case 1.

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$A(s) = 3s^4 + 0 - 48$

$\frac{dA(s)}{ds} = 12s^3$

For s^1 row $\rightarrow \lim_{\epsilon \to 0} \frac{576}{\epsilon} = \infty$.

Looking at the sign change in the first column of Routh's array.

Hence the system is unstable. The sign is changing once & hence the system has 1 root in the RHS.

$A(s) = 3s^4 - 48 = 0$

$\cancel{3}s^4 = \cancel{48}\ 16$

$s^2 = \pm 4.$

$s_{1,2} = \sqrt{4} = \pm 2.$

$s_{3,4} = \sqrt{-4} = \pm j2.$

No. of roots with +ve real part = 1.

No. " " " 0 real part = $\cancel{4}$ 2 $(2j, -2j)$

" " " " -ve real part = $\cancel{4}$ 6 - 2 - 1

= 3 //

2) $F(s) = s^5 + s^4 + 2s^3 + 2s^2 + 3s + 15 = 0$

$a_0 \quad a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5$

s^5	1	2	3
s^4	1	2	15
s^3	0 [ϵ]	-12	0
s^2	$\frac{2\epsilon+12}{\epsilon}$ [∞]	15	0
s^1	[-12]	0	0
s^0	15		

→ Special case 1

For s^1: $\frac{\left(\frac{2\epsilon+12}{\epsilon}\right)(-12) - 15\epsilon}{\frac{2\epsilon+12}{\epsilon}} = \frac{-24\epsilon - 144 - 15\epsilon^2}{2\epsilon+12}$

$$\lim_{\epsilon \to 0} \frac{-24\epsilon - 144 - 15\epsilon^2}{2\epsilon + 12} = \frac{-144}{12} = -12$$

$$\lim_{\epsilon \to 0} \frac{2\epsilon + 12}{\epsilon} = \lim_{\epsilon \to 0} 2 - \frac{12}{\epsilon} = \infty.$$

s^5	1	2	3
s^4	1	2	15
s^3	ϵ	-12	0
s^2	∞	15	0
s^1	-12	0	0
s^0	15		

There is a change in the sign $\therefore$ system is unstable.
There are 2 roots in the RHS

3) $$TF = \frac{1000}{s^3 + 10s^2 + 31s + 1030}$$

$$q(s) = \underset{a_0}{s^3} + \underset{a_1}{10s^2} + \underset{a_2}{31s} + \underset{a_3}{1030} = 0$$

s^3	1	31	0
s^2	10	1030	0
s^1	~~-72/10~~	0	
s^0	1030	0	

System is unstable with 2 roots in the RHS

4) $P(s) = 3s^7 + 9s^6 + 6s^5 + 4s^4 + 7s^3 + 8s^2 + 2s + 6 = 0$. Determine the no. of roots in RHS & LHS.

4 in RHS & 3 in LHS.

5) Determine the LHS, RHS and no. of poles on the $j\omega$ axis for the systemic characteristic equation. And also determine the stability of the system.

$F(s) = s^8 + s^7 + 12s^6 + 22s^5 + 39s^4 + 59s^3 + 48s^2 + 38s + 20 = 0$

s^8	1	12	39	48	20
s^7	1	22	59	38	0
s^6	-10	-20	10	20	0
s^5	20	60	40	0	
s^4	10	30	20	0	
s^3	0 [40]	0 [60]	0 [40]		
s^2	$\frac{30\epsilon - 200}{\epsilon}$ [$-\infty$]	2			
s^1	2/3	0			
s^0	0	0			

s^3 → Special case 2

$$\cancel{\lim_{\epsilon \to 0} \frac{30\epsilon}{\epsilon} - \frac{200}{\epsilon} = -\infty}$$

$$\cancel{\frac{\left(\frac{30\epsilon - 200}{\epsilon}\right) 20}{\frac{30\epsilon - 200}{\epsilon}}}$$

$$A(s) = 10s^4 + 30s^2 + 20 \qquad = 10s^4 + 30s^2 + 20$$

$$\frac{dA(s)}{ds} = 40s^3 + 60s + \cancel{20} \qquad \frac{dA(s)}{ds} = 40s^3 + 60s$$

$$s^4 + 3s^2 + 2 = 0$$

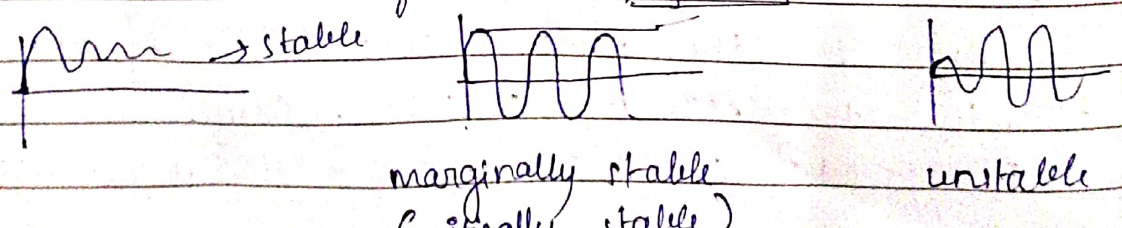
$$s^2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = -2, -1 \qquad s = \pm\sqrt{-2} \quad \pm\sqrt{-1}$$

$$\pm j\sqrt{2} \quad \& \pm j$$

The system is unstable & has 2 roots in the RHS.

BIBO Stable:

In a bounded system → [system] →



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Application of Routh's criteria: to determine the marginal value of K & frequency of sustained oscillations [where K is system gain]

When value of K makes any of the of the routh's array as zeroes other than s^0 row, then this value of K is called marginal value of K denoted by K_{mar}; with this value of K the system will be marginally stable oscillating at the freq. ω which can be determined by using the auxillary eqn of routh's array.

Let us consider an example,

For unity feedback system,

$$G(s) = \frac{K}{s(1+0.4s)(1+0.25s)}$$

. Determine the range of K for the system to be stable and also determine the marginal value of K & the frequency of sustained oscillations.

$1 + G(s)H(s) = 0 \;=\; 0.1s^3 + 0.65s^2 + s + K = 0$

$a_0 \quad a_1 \quad a_2 \quad a_3$

s^3	0.1	1	0
s^2	0.65	K	0
s^1	$\frac{0.65-0.1K}{0.65}$	0	
s^0	K		

For the system to be stable, $K > 0$ for s^0 & for s^1

$\frac{0.65-0.1K}{0.65} > 0 \rightarrow K < 6.5$

∴ The range of K for which system is stable is $0 < K < 6.5$

i) To determine K_{mar}, consider s^1 row.

$\frac{0.65-0.1K}{0.65} = 0 \rightarrow \underline{K = 6.5 = K_{mar}}$

Auxilary equation for s^2 row, $A(s) = 0.65s^2 + K$.

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Equati A(s) = 0.

$0.65s^2 + k = 0.$

& $K \to K_{mar}$ $\quad 0.65\, s^2 = -6.5$

$s^2 = -10.$

$s = \sqrt{-10} = \pm j\,3.162.$

$\because s = j\omega.$

$\therefore \omega = 3.162$ rad/sec.

Q. For a system with $s^4 + 22s^3 + 10s^2 + s + k = 0$. Determine marginal value of K & frequency.

$s^4 + 22s^3 + 10s^2 + s + k$

s^4	1	10	k
s^3	22	1	0
s^2	$219/22$	$\frac{22k}{22}$	0
s	$\frac{9.95 - 22k}{9.95}$	0	
s^0	K		

To determine K_{mar} $\quad \frac{9.95 - 22k}{9.95} = 0.$

$k_{mar} = \frac{9.95}{22} = \underline{\underline{0.452}}$

The auxilary eqn.

$A(s) = 9.95s^2 + k = 0$

$s^2 = \frac{-0.452}{9.95}$

$= \sqrt{-0.0452}$

$= \pm j\,0.213$ ~~rad/se~~.

$\omega = 0.213$ rad/s

Q For a system with characteristic eqn. $F(s) = s^6 + 3s^5 + 4s^4 + 6s^3 + 5s^2 + 3s + 2 = 0$. Determine the stability of the system.

s^6	1	4	5	2
s^5	3	6	3	0
s^4	2	4	2	0
s^3	0 [8]	0 [8]	0 [~~8~~]	
s^2	2	2	0	
s^1	0 [4]	0		
s^0	2			

s^3 row → special case 2.

s^1 row → $A'(s)$

$A(s) = 2s^4 + 4s^2 + 2 = 0 \quad = s^4 + 2s^2 + 1 = 0$

$\frac{dA(s)}{ds} = 8s^3 + 8s \cancel{+2} \qquad \frac{dA(s)}{ds} = 4s^3 + 4s$

$A'(s) = 2s^2 + 2$

$\frac{dA'(s)}{ds} = 4s$

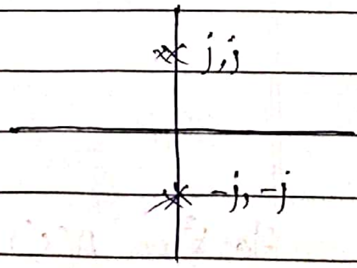
Looking at the first column of routh's array, there are no poles on RHS as there are no sign changes. But we cannot say that the system is stable as there are auxilary eqns. Once the auxillary eqn appears there will be repeated roots or non repeated roots on the imaginary axis. Looking at these roots stability of the system has to be decided.

$A(s) = s^4 + 2s^2 + 1 = 0$

$s^2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{4-4}}{2} = \frac{-2}{2} = -1, -1$

$s = \pm j, \pm j$

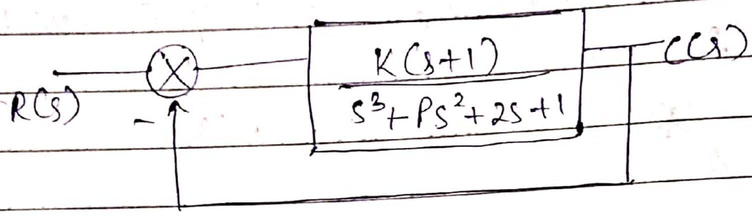
$= j, j, -j, -j$



As the roots are repeated roots on imaginary axis, the system is unstable.

Note: For non repeated roots on imaginary axis the system is marginally stable.

Q. The following system oscillates with freq of 2 rad/sec. Find the value of K_{max} & P, assume that there are no poles in RHS.



$$T.F \; \cancel{g(s)} = \frac{\frac{K(s+1)}{s^3+Ps^2+2s+1}}{1 + \frac{K(s+1)}{s^3+Ps^2+2s+1}} = \frac{K(s+1)}{s^3+Ps^2+2s+1+K(s+1)}$$

$$g(s) = s^3 + Ps^2 + s(2+K) + 1(1+K) = 0$$

s^3	1	$2+K$	0
s^2	P	$1+K$	0
s^1	$\frac{(2+K)P-(1+K)}{P}$		
s^0	$1+K$		

$\cancel{(2+K)P - 1} \cdot 2P + \frac{PK - 1 \cdot K}{P}$

$s = j\omega$

$\cancel{\omega = 2j}$

Equating it to 0 gives $K_{max} \rightarrow \frac{(2+K)P-(1+K)}{P} = 0$

$$(2+K_{max})P = 1+K_{max}$$

$$P = \frac{1+K_{max}}{2+K_{max}}$$

For s^2 row $A(s) = Ps^2 + (1+K) = 0$

$$Ps^2 = -(1+K)$$

$$s^2 = \frac{-(1+K)}{P}$$

$$s = \pm j\sqrt{\frac{1+K}{P}}$$

$$j\omega = \pm j\sqrt{\frac{1+k}{p}}$$

$$\cancel{\omega} = 2 = \cancel{\pm}\sqrt{\frac{1+K}{p}}$$

$$\frac{1+K}{p} = 4.$$

$$p = \frac{1+k}{4}.$$

$$\cancel{\frac{1+K_{max}}{4}} = \cancel{\frac{1+K_{max}}{2+K_{max}}}$$

$$2+K = 4$$

$$\underline{\underline{K_{max} = 2.}}$$

$$p = \frac{1+k}{4} = \frac{1+2}{4} = \underline{\underline{3/4}}$$

Q A negative f/b system is characterised by $G(s)H(s) = \frac{Ke^{-s}}{s(s^2+5s+9)}$. Determine the range of K for the system to be stable. [Hint: Assume that $e^{-s} \approx 1-s$]

→ $$1+G(s)H(s) = 1 + \frac{ke^{-s}}{s(s^2+5s+9)} = 0.$$

$$g(s) = s(s^2+5s+9) + ke^{-s} = 0$$

$$= s(s^2+5s+9) + k(1-s) = 0$$

$$s^3 + 5s^2 + 9s + k - sk = 0$$

$$s^3 + 5s^2 + s[9-k] + k = 0$$

s^3	1	$9-k$	0
s^2	5	k	0
s^1	$\frac{45-6k}{5}$	0	
s^0	k		

$$\frac{5(9-k)-k}{5}$$

$$\frac{45-5k-k}{5} = \frac{45-6k}{5}$$

$$\frac{45-6k}{5} = 0$$

$$45-6k=0$$

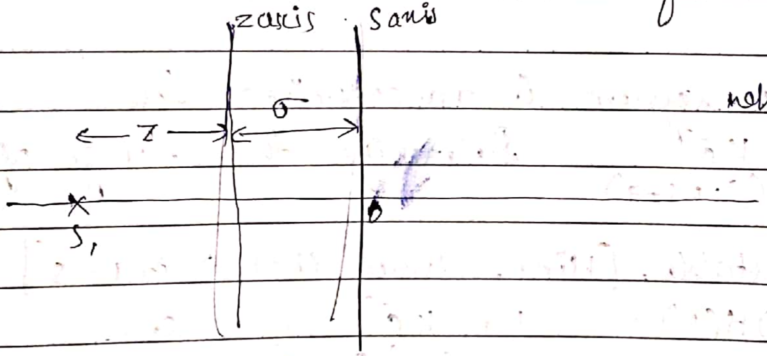
$$k=7.5$$

& $k>0$ for the system to be stable

$\therefore \quad 0<k<7.5$

Relative stability analysis:

Usually relative stability analysis is done by considering the settling time T_s which will show how fast transients will die out, But for higher order system it is better to shift the s axis to a z axis so that the roots lying on the left half of z axis are considered to be more stable than the roots on the right half of z axis. Let us consider the following fig.



where $\sigma = -s$

$$S_1 = z + \sigma$$

$$S_1 = z - s$$

Characteristic eqn. of a system is given as $s^3+7s^2+25s+39=0$ which by the routh's test can be shown to have all its roots in the left half of s plane. Let us check if all the roots of this eqn. have real parts more than -1.

Substituting $s = z-1$. [given $\sigma = 1$]

& simplyfing & rearranging the terms.

$$z^3+4z^2+14z+20=0$$

Routh's array

z^3	1	14	0
z^2	4	20	0
z^1	9		
z^0	20		

From the 1st column of routh's array, we can say that all roots of the original characteristic eqn. in S domain lie to the left half of z axis or left half of $s=-1$.
Hence the system is more stable.

Root locus

Root locus is graphical method of showing the movement of poles in the s plane when the system gain K is varied from 0 to infinity (maybe the variation of some other parameter other than the system gain K)

General steps to solve problems on root locus:

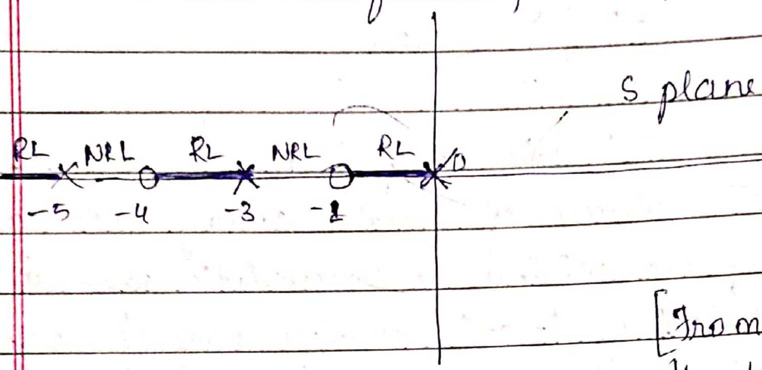
Step 1: Consider the OLTF G(s) H(s) and decide the number of poles (P=?) number of zeroes (Z=?) & Let N be the number of branches of root locus.

i) If $P>Z$; $N=P$
ii) If $P<Z$; $N=Z$
iii) If $P=Z$; $N=P=Z$

iv) $P-Z$ = number of branches approaching ∞.
v) always starting points are poles of the OLTF & terminating points are 0's of the OLTF or it will be at ∞.

Step 2: Draw the pole-zero plot and identify the section of real axis as root locus (RL) or non root locus (NRL) and also predict the minimum no. of breakaway points / breakin-in points using general prediction.

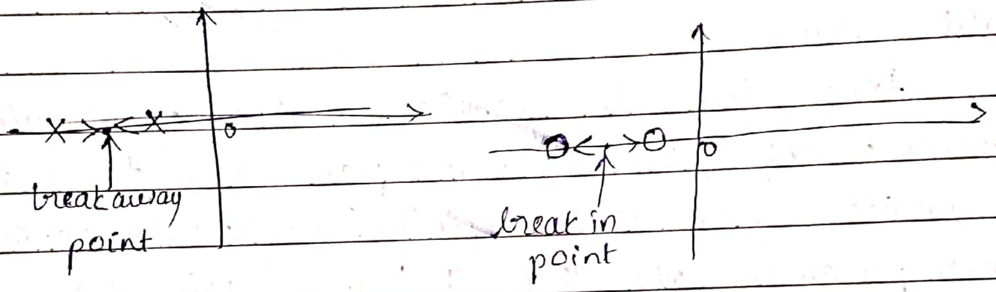
Consider the following example



odd → RL

even → NRL.

[From a given point b/w -1 & 0 consider the total no. of [P+Z]. If it is odd then the line → RL, or else NRL]



Step-3 : Determine centroid using the following equation,

$$\text{centroid} = \sigma = \frac{\Sigma \text{ real part of poles} - \Sigma \text{ real part of zeros}}{P-Z}$$

Mark this centroid on the real axis.

Step 4 : Calculate the angles of asymptotes using the equations.

$$\theta = \frac{(2q+1)180^\circ}{P-Z} \; ; \; q = 0,1,2,\ldots(P-Z-1)$$

then mark these asymptotes with their corresponding angles, so that all these asymptotes will intersect at centroid.

Note > Asymptotes are branches approaching ∞

Step 5 : Calculate the actual breakaway & break in points using the characteristic eqn.

Step 6 : Calculate the intersection points of root locus with imaginary axis. [using RH criteria]

Step - 7. - Calculate the angles of arrival or departure

Always root locus is symmetrical wrt real axis.

Step-9 : Predict the stability & performance of the given system using the root locus.

1) $G(s)H(s) = \frac{K(s+1)(s+2)}{s(s-1)}$ for $K \geq 0$. Comment on the stability

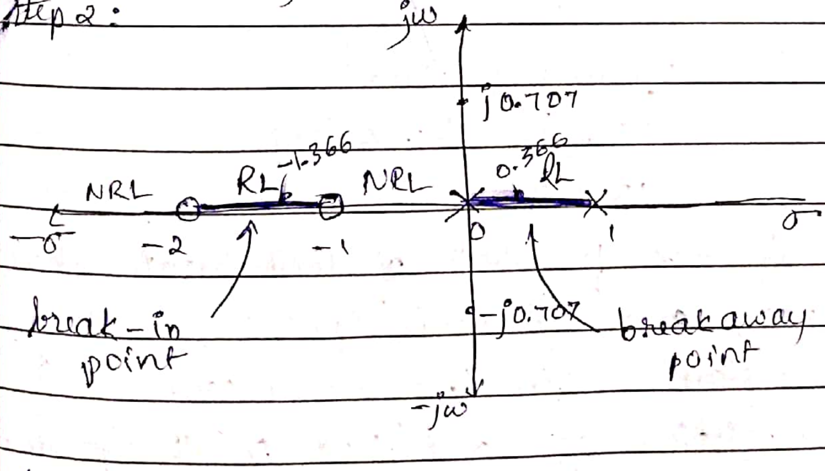
→ step 1 : $P=2$ $Z=2$ $N=P=Z=2$ branches of the root locus

$P-Z = 2-2=0$ branches approaching ∞.

Starting points : $s=0, +1$ (poles)

Terminating points : $s=-1,-2$ (zeroes)

Step 2 :



Step 3 : $\sigma = \frac{\sum R.P \text{ of poles} - \sum R.P \text{ of zeroes}}{P-Z} = \infty.$

It has centroid at infinity

Step 4 :- $P-Z=0$, ∴ there are no branches terminating at ∞ and hence no asymptotes terminating at ∞ and intersecting at centroid.

Step 5 :

$$1 + G(s)H(s) = 0$$

$$1 + \frac{K(s+1)(s+2)}{s(s-1)} = 0$$

$$K = \frac{s - s^2}{s^2 + 3s + 2}$$

$$\frac{dK}{ds} = \frac{(s^2+3s+2)(1-2s) - (s-s^2)(2s+3)}{(s^2+3s+2)^2}$$

$$0 = \frac{(s^2+3s+2)(1-2s) - (s-s^2)(2s+3)}{(s^2+3s+2)^2}$$

$$(s^2+3s+2)(1-2s) = (s-s^2)(2s+3)$$

$$\cancel{-4s^3} \quad \cancel{-2s^3 - 6s^2 - 4s} -$$

$$\Rightarrow \quad -4s^2 - 4s + 2 = 0$$

$$2s^2 + 2s - 1 = 0$$

$$s = 0.366 \ \& \ -1.366$$

Substituting the above roots in the eqn. for K :

$$K = \frac{0.366 - (0.366)^2}{0.366^2 + 3(0.366) + 2}$$

$$= 0.0718$$

$$K = \frac{-1.366 - (-1.366)^2}{(-1.366)^2 + 3(-1.366) + 2}$$

$$= 13.93$$

As K value is +ve for both of these roots, these roots are valid breakaway and break-in points.

Step 6 : Intersection with imaginary axis,

The characteristic eqn. obtained in the prev. step

$$1 + G(s)H(s) = 0$$

$$s^2(1+K) + s(3k-1) + 2k = 0$$

s^2	$1+k$	$2k$	0
s^1	$3k-1$	0	
s^0	$2k$		

to determine marginal value of K. let us consider s^1 (row) = 0

$3K - 1 = 0$

$K = 1/3 = K_{mar}$.

s^2 row → $A(s) = (1+K)s^2 + 2K = 0$

$$\left(1+\frac{1}{3}\right)s^2 + 2\left(\frac{1}{3}\right) = 0$$

$$s^2 = \frac{-\cancel{2}}{\cancel{3}} \times \frac{\cancel{3}}{\cancel{4}\,2}$$

$$s^2 = -\frac{1}{2}$$

$$s = \pm j0.707$$

Step 7: As there are no complex poles or zeroes, there ~~are~~ is no angle of departure or angle of arrival

Step 8: Complete the root locus.

It should be noted that root locus is symmetrical wrt real axis.

Step 9: comment on the stability:

~~when~~ For $0 < K < 1/3$, the root locus lies on the right half of s plane making the system unstable.

For $K = 1/3 = K_{mar}$, the root locus will lie on the imaginary axis making the system critically or marginally stable.

For ~~K~~ $1/3 < K < \infty$, root locus enters into LHS. making the system stable.

Details of 7 step:

Angle of departure for a complex conjugate pole & angle of arrival for a complex conjugate zero.

Let us consider an example to consider the angle of departure & angle of arrival.

Date
Page

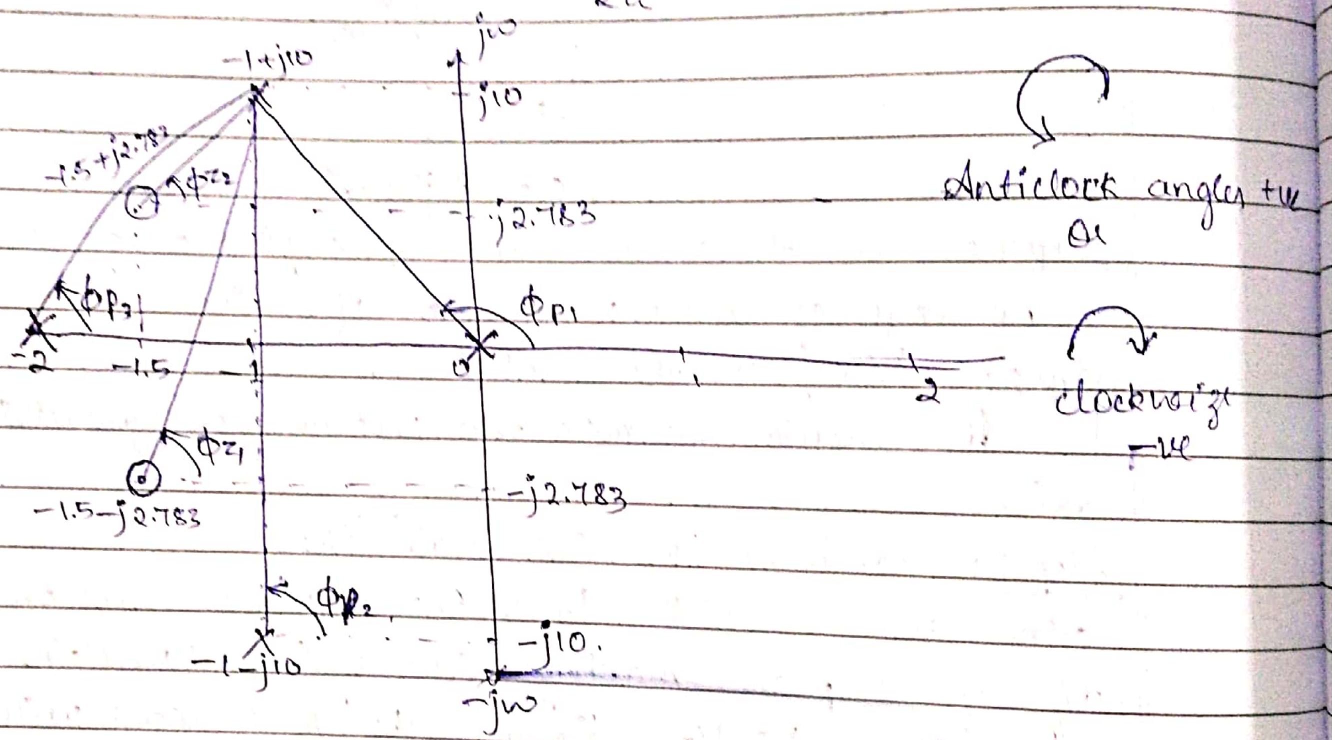
Q. Determine the angle of departure & angle of arrival for

$$G(s)H(s) = \frac{K(s^2+3s+10)}{s(s+2)(s^2+2s+101)}$$

→ There are 4 poles.

$s = -2,\ s = 0\ \&\ s = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = -1 \pm j10$

& zeroes → $s = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = -1.5 \pm j2.783$



Angle of departure ϕ_d at a complex pole $-1+j10$

$\phi_{p_1} = 180 - \tan^{-1} 10 = 95.71°$

$\phi_{p_2} = 90°$

$\phi_{p_3} = \tan^{-1}\frac{10}{1} = 84.28°$

$\phi_{z_1} = \tan^{-1}\frac{(10+2.783)}{(1.5-1)} = 87.75°$

$\phi_{z_2} = \tan^{-1}\frac{(10-2.783)}{(1.5-1)} = 86.03°$

$\Sigma\phi_p = \phi_{p_1} + \phi_{p_2} + \phi_{p_3} = 269.39$

$\Sigma\phi_z = \phi_{z_1} + \phi_{z_2} = 173.78$

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$\phi = \Sigma\phi_p - \Sigma\phi_z = 95.61°$

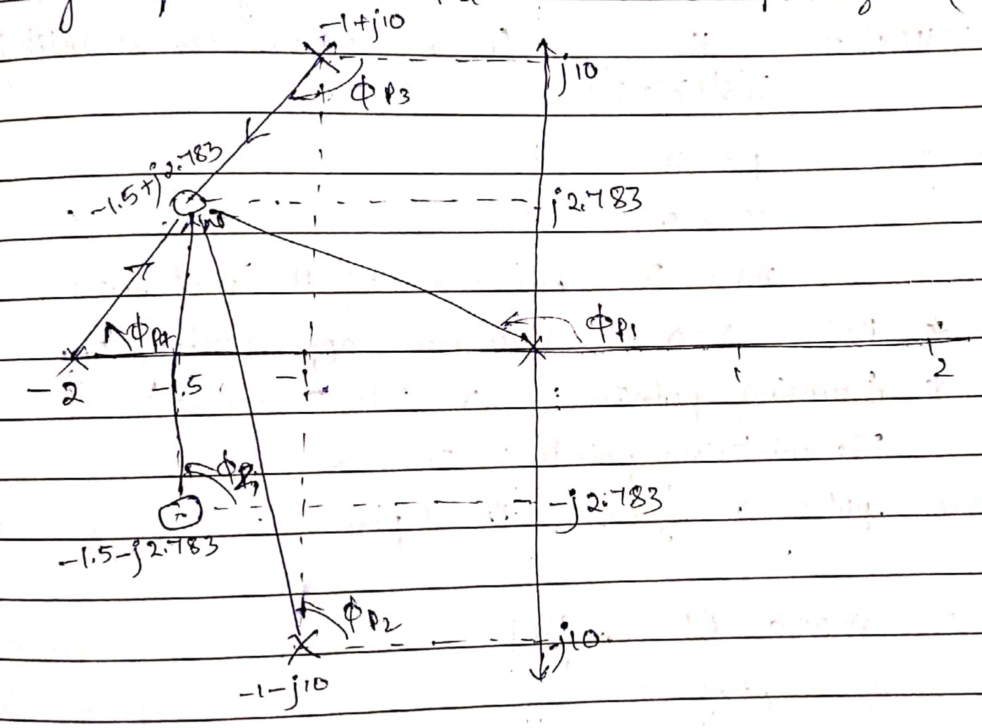
angle of departure,

$\phi_d = 180 - \phi = 84.39°$

$\phi_d = 84.39°$ (anticlockwise) at $(-1+j10)$ pole.

$|||^{ly}$ $\phi_d = -84.39°$ (clockwise) at $(-1-j10)$ pole.

Angle of arrival ϕ_a at a complex zero $(-1.5+j2.783)$



$\phi_{P_1} = 180 - \tan^{-1}\frac{2.783}{1.5} = 118.34$

$\phi_{P_2} = 180 - \tan^{-1}\frac{(10+2.783)}{(1.5-1)} = 92.24°$

$\phi_{P_3} = -90 - \tan^{-1}\left(\frac{1.5-1}{10-2.78}\right) = -93.96°$ ($\because$ clockwise)

$\phi_{P_4} = \tan^{-1}\frac{2.78}{2-1.5} = 79.8°$

$\phi_{Z_1} = 90°$

$\Sigma\phi_p = 196.42$ $\quad \Sigma\phi_z = 90°$

$\phi = \Sigma\phi_p - \Sigma\phi_z = 106.42^\circ$

$\phi_a = 180 + \phi$

$= 286.42 - 360^\circ$

$\phi_a = -73.58^\circ$ (clockwise) at $(-1.5 + j2.783)$ zero

III^{ly} $\phi_a = 73.58^\circ$ (anticlockwise) at $(-1.5 - j2.783)$ zero

Q ~~State~~ Sketch the root locus for the system having $G(s)H(s) = \dfrac{K}{s(s^2+2s+2)}$; Comment on the stability.

→ Step 1. $P = 3$

$Z = 0$

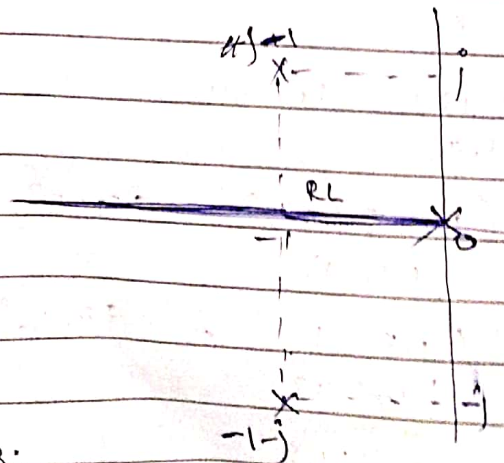
$N = 3$. branches of the root locus.

* $P - Z = 3 - 0 = 3$ branches approaching ∞.
* Starting points: $0, -1+j, -1-j$

Terminating points: ∞, ∞, ∞.

Locate poles & zeroes in s plane:

Step 2: mark RL & NRL



Step 3:

$\sigma = \dfrac{\Sigma \text{ real part poles} - \Sigma \text{ real part zeros}}{P - Z}$

$\dfrac{0-1-1}{3} = \dfrac{-2}{3} = -0.67$

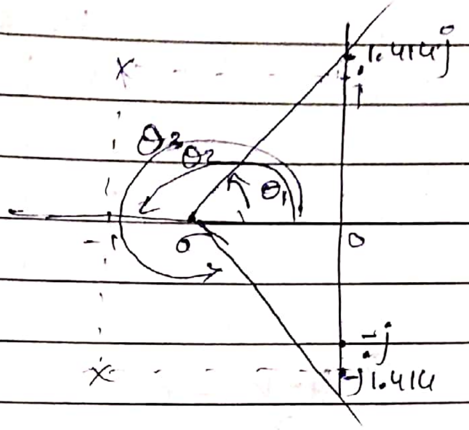
Step 4:

$$\theta = \frac{(2q+1)\,180^\circ}{P-Z}$$

As there are 3 branches approaching ~~terminating~~ at ∞, there must be 3 asymptotes terminating at ∞.

$\therefore \; q = 0, 1, 2.$

$$\theta_1 = \frac{180^\circ}{3} = 60^\circ$$



$$\theta_2 = \frac{3 \times 180^\circ}{3} = 180^\circ$$

$$\theta_3 = \frac{5 \times 180^\circ}{3} = 300^\circ$$

Step 5:
$$1 + G(s)H(s) = 0$$
$$s(s^2+2s+2) + k = 0.$$
$$s^3 + 2s^2 + 2s + k = 0.$$
$$k = -s^3 - 2s^2 - 2s$$
$$\frac{dk}{ds} = -3s^2 - 4s - 2 = 0.$$

Roots are $s_{1,2} = -0.67 \pm j\,0.4714.$

Applying angle condition, i.e., $G(s)H(s) = -1$
$\angle G(s)H(s) = \pm(2q+1)180^\circ$

When the roots are complex ones, then angle condition has to be checked to determine the validity of breakaway & break-in points.

$$G(s)H(s) = \frac{K}{s(s+1+j)(s+1-j)}$$

Put $s = -0.67 + j\,0.4714$

$$= \frac{K}{(-0.67+j0.4714)(0.33+1.47j)(0.33-j0.53)}$$

$$\angle G(s)H(s) = \frac{0^\circ}{\angle 144.87^\circ \; \angle 77.34^\circ \; \angle -58.09^\circ}$$
$$= 0 - (144.87 + 77.34 - 58.09)$$

$\left[\because \dfrac{\angle\theta_1}{\angle\theta_2 \, \angle\theta_3} = \theta_1 - (\theta_2 + \theta_3) \right.$

$$\angle G(s)H(s) = -164.11^\circ$$

As the above value is odd multiple of 180°, hence there is no breakaway & break in point for this root locus

Step 6 : Intersection with imaginary axis.

The characteristic eqn. obtained is

$$s^3 + 2s^2 + 2s + k = 0$$

s^3	1	2	0
s^2	2	k	0
s	$\frac{4-k}{2}$	0	
k	k		

$$\frac{4-k}{2} = 0$$

$$\therefore k = 4 = k_{max}$$

$$A(s) = 2s^2 + \cancel{2}k = 0$$

$$\therefore 2s^2 = -4.$$

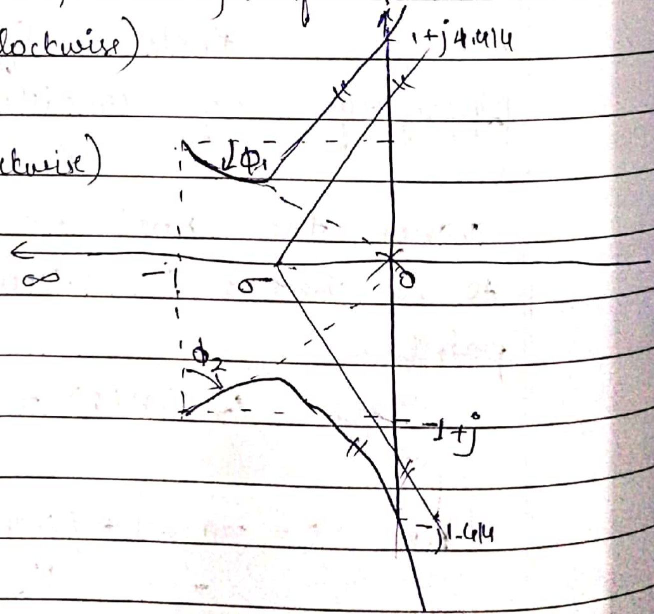
$$s^2 = -2.$$

$$s = \pm j1.414$$

Step 7 : There are complex roots, ~~the~~ angle of departure is absent

$\phi_d = -45^\circ$ at $(-1+j)$ (clockwise)

$\phi_d = +45^\circ$ at $(-1-j)$ (anticlockwise)



Step 8 : ~~Comment on~~ ~~the~~ complete the root locus.

Step 9 : Comment on stability

$0 \leq k < 4$. for this range the root locus is on left half of the s plane making the system stable.

When $k = 4 = k_{max}$, the root locus exists on the imaginary axis making the system marginally or critically stable.

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As k increases i.e., $4 \leq k < \infty$ the root locus enters into RHS making the system unstable.

Q Draw the root locus for the closed loop system having OLTF $G(s)H(s) = \frac{K}{s(s+5)(s+10)}$. Comment on the stability.

→ Step 1 :- $P = 3$

$Z = 0$

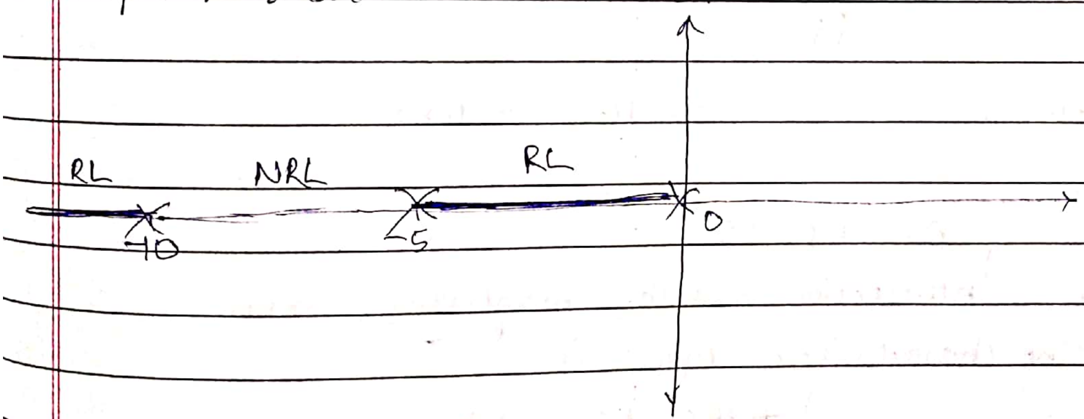
$N = 3$ branches of root locus.

* $P - Z = 3 - 0 = 3$ branches approaching ∞.
* Starting points : $0, -5, -10$.

Terminating points : ∞, ∞, ∞

Locating poles and zeroes in s plane

Step 2 : Mark RL & NRL

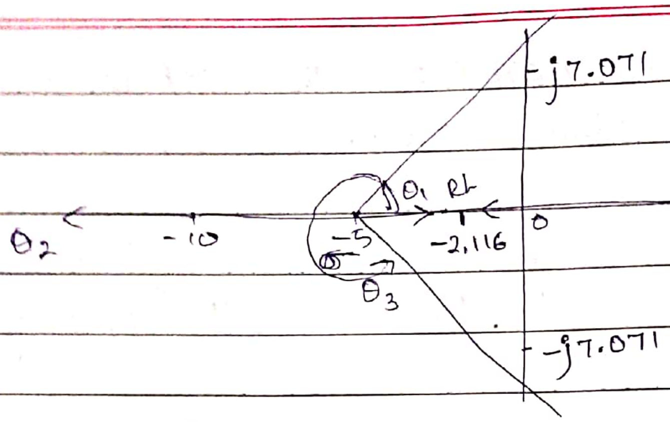


Step 3 :- $\sigma = \frac{\Sigma \text{R.P of poles} - \Sigma \text{real part of zeroes}}{P - Z}$

$= \frac{0 - 5 - 10}{3} = \frac{-15}{3} = -5$.

Step 4 :- $\theta = \frac{(2q+1)}{P-Z} 180°$

$\theta_1 = \frac{180}{3} = 60$ $\theta_2 = \frac{180 \times 3}{3} = 180°$ $\theta_3 = \frac{180 \times 5}{3} = 300°$



As per the general prediction, there exists one breakaway point b/w poles (at 0 & −5)

Step 5: Actual value of breakaway point

$$1 + G(s)H(s) = 0$$

$$s(s+5)(s+10) + K = 0.$$

$$s^3 + 10s^2 + 5s^2 + 50s + k = 0.$$

$$s^3 + 15s^2 + 50s + k = 0$$

$$K = -s^3 - 15s^2 - 50s.$$

$$\frac{dK}{ds} = -3s^2 - 30s - 50. = 0.$$

Roots are, $s = -2.116, -7.88$

$$K = 48.112$$

Step 6: Intersection with imaginary axis.

The characteristic eqn. is

$$s^3 + 15s^2 + 50s + k = 0$$

s^3	1	50	0
s^2	15	k	0
s^1	$\frac{750-k}{15}$	0	
s^0	k		

for s^1 row $\frac{750-k}{15} = 0.$

$$K_{max} = 750$$

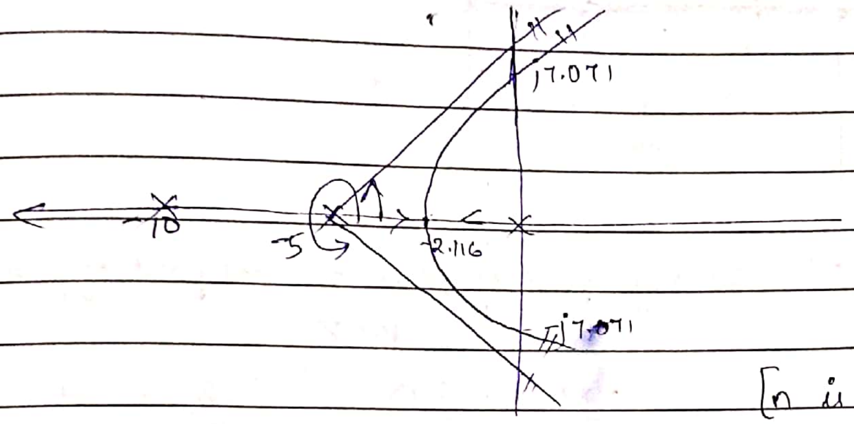
For s^2 row, $A(s) = 15s^2 + k = 0$

$$\cancel{15}s^2 = -\frac{750}{50}$$

$s = \pm j7.07$

Step 7: As there are no complex poles or zeroes, there is no angle of departure or angle of arrival.

Step 8: Complete the root locus:



$\frac{180^\circ}{n} - \frac{180^\circ}{2} = 90^\circ$

[n is the no. of branch from asymptote]

Step 9: Comment on stability.

$0 \le k \le 750$, in this range the root locus is on the left of the s plane making the system absolutely stable. When $k = 750 = k_{max}$, the root locus will be on imaginary axis making the system critically or marginally stable.

For $750 < k < \infty$ the root locus lies on the right half of s plane making the system unstable.

HW

Q Sketch the root locus for $G(s)H(s) = \frac{k(s+4)}{s(s^2+2s+2)}$. Comment on the stability.

Q Draw the root locus of a system where characteristic eqn. is given by $s^3 + 9s^2 + ks + k = 0 \Rightarrow 1 + G(s)H(s) = 0$

~~1+G(s)+1~~ $G(s)H(s) = \frac{k(s+1)}{s^2(s+9)}$

Step i) $P = 3$ $Z = 1$ $N = 3$ branches of the root locus

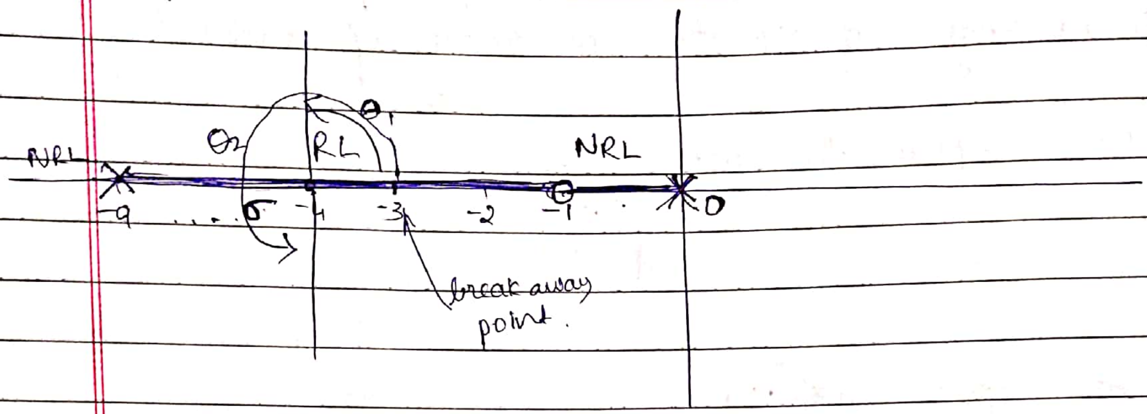
$P - Z = 2$ branches approaching ∞

Starting points - $0, 0, -9$

terminating points - $-1, \infty, \infty$

Locating poles & zeroes in s plane.

Step 2 : Mark RL & NRL



Step 3: $\sigma = \frac{\Sigma \text{ real part of poles} - \Sigma \text{ real part of zeroes}}{P-Z}$

$$= \frac{0+0-9-(-1)}{2} = \frac{-8}{2} = -4.$$

Step 4: $\theta = \frac{(2q+1)180°}{P-Z}$

As there are 2 branches approaching ∞, $q = 0, 1$

$$\therefore \theta_1 = \frac{180}{2} = 90°$$

$$\theta_2 = \frac{3 \times 180}{2} = 270°$$

As per the prediction there are no breakaway points

Step 5 : Actually value of breakaway point.

$$1 + G(s)H(s) = 0$$

$$s^3 + 9s^2 + ks + 1 = 0$$

$$k = \frac{-s^3 - 9s^2}{1+s}$$

$$\frac{dk}{ds} = \frac{(1+s)(-3s^2 - 18s) - (-s^3 - 9s^2)}{(1+s)^2}$$

$$= \frac{-3s^2 - 18s - 3s^3 - 18s^2 + s^3 + 9s^2}{(1+s)^2}$$

$$0 = \frac{-2s^3 - 12s^2 - 18s}{(1+s)^2}$$

$$-2s^3 - 12s^2 - 18s = 0$$

$$= \cancel{-s(-2s^2 - 12s}$$

$$-2s[s^2 + 6s + 6] = 0$$

$s = \cancel{0}, -3, -3.$

Sub $s = -3$ in $k = \dfrac{-s^3 - 9s^2}{(1+s)}$

$\therefore k = 27.$

Step 6: intersection with imaginary axis

$$s^3 + 9s^2 + ks + k = 0$$

s^3	1	k	0
s^2	9	k	0
s^1	$\frac{9k-k}{9}$		
s^0	k		

$$\frac{9k-k}{9} = \frac{8k}{9} = 0$$

$$k = 0. = k_{max}$$

$$A(s) = 9s^2 + k = 0$$

$$9s^2 + 0 = 0$$

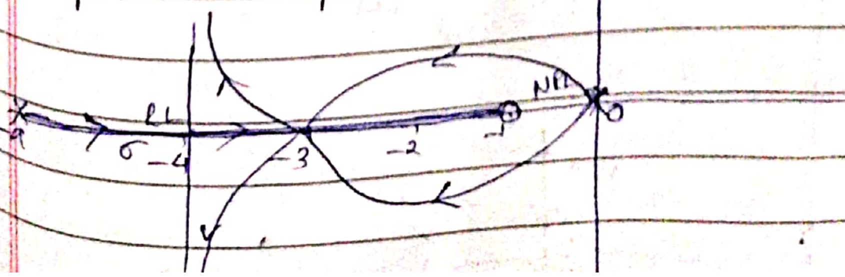
$$9s^2 = 0$$

$$s = 0$$

$\therefore$ The root locus does not intersect with the imaginary axis. [As row of zeroes is obtained when k=0 & system doesn't work with k=0] ~~Hen~~

Step 7: Angle of arrival & departure are absent as there are no complex poles & zeroes.

Step 8: Complete the root locus.



Step-9: Since the root locus is completely on the left half of s plane, the system is absolutely stable

Chapter 5

Graphical method

I) Bode plot

Important frequency domain specifications which are required for bode plot are:

i) gain crossover freq (ωgc)
ii) phase crossover freq (ωpc)
iii) gain margin (GM)
iv) phase margin (PM)

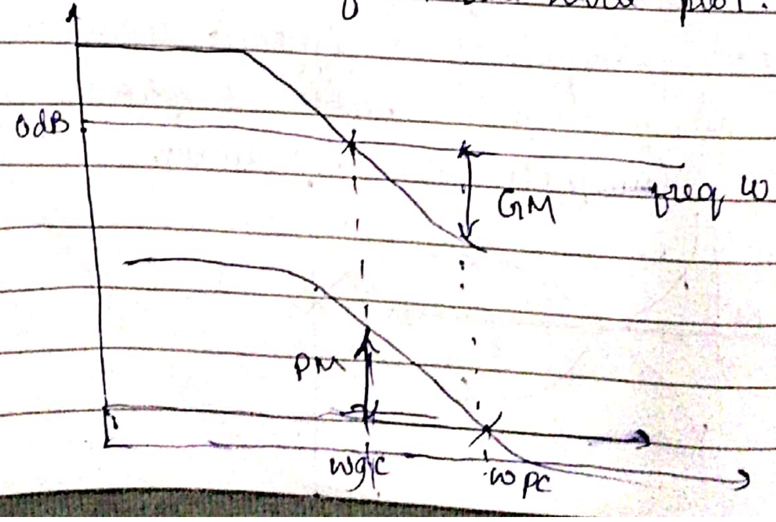
- Always the open loop transfer f/n G(s) H(s) must be in the time constant form in order to draw the magnitude and phase plot of a bode plot.
- If the given OLTF is not in time constant form then terms have to be rearranged & write this transfer function in time constant form.

General eqn is given by:

$$G(S)\,H(S) = \frac{K\,(1+T_aS)(1+T_bS)\dots}{S(1+T_1S)(1+T_2S)\dots}$$

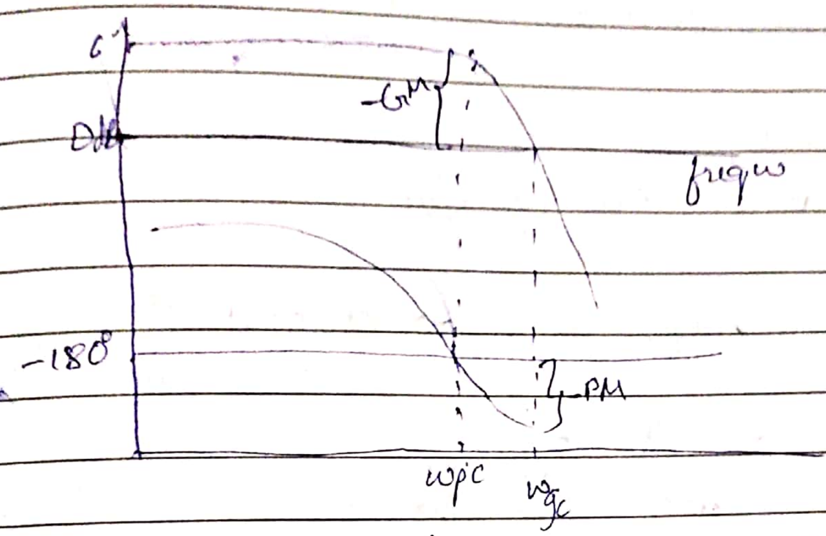
Stability ~~of~~ condition from the bode plot.

i)



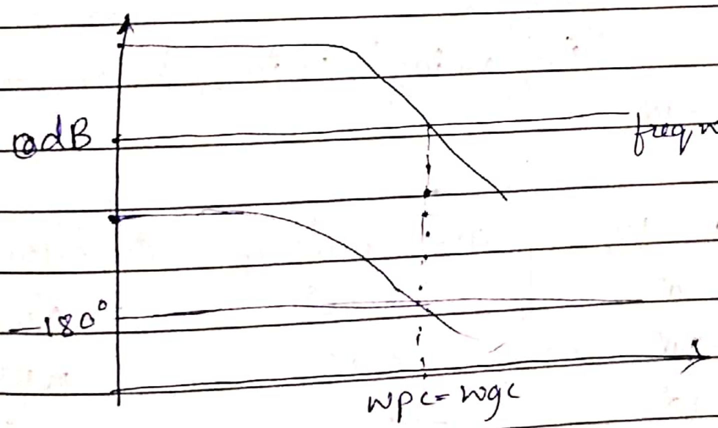
when $\omega_{pc} > \omega_{gc}$ & GM, PM are +ve
System is stable

ii)



when $\omega_{gc} > \omega_{pc}$ & GM, PM are -ve,
System is unstable.

iii)



when $\omega_{pc} = \omega_{gc}$, & GM = PM = 0,
System is marginally stable.

Q1 A unity f/b system has system eqn as $G(s)H(s) = \frac{80}{s(s+2)(s+20)}$.
Draw the bode plot. Determine the values of GM, PM, ω_{gc} & ω_{pc}. Comment on the stability

→ Given transfer function is not in time constant form.
Rearranging the eqn, $\frac{2}{s(1+s/2)(1+s/20)}$

Step 1: Identify the factors.
when $k = 2$, $20\log 2 = 6dB$.

* $1/s$ term → is a pole at the origin → $-20\,dB/dec$

* $\frac{1}{(1+\frac{s}{2})}$ term ⇒ $-20\,dB/dec$ ⇒ $\frac{1}{1+T_1 s}$

$T_1 = 1/2$

$\omega c_1 = \cancel{20}\ 2\ rad/sec$

* $\frac{1}{(1+s/20)}$ is a simple pole → $-20\,dB/dec$ ⇒ $\frac{1}{1+T_2 s}$

$T_2 = 1/20$

$\omega c_2 = 20\ rad/sec.$

Step 2: phase plot. substitute $s = j\omega$

$$G(j\omega)H(j\omega) = \frac{(2+j\omega)^{\to 0}}{j\omega(1+j\omega/2)(1+j\omega/20)}$$

$1/j\omega \Rightarrow -90^\circ$

ω	$\phi_1 = 1/j\omega$	$\phi_2 = -\tan^{-1}\omega/2$	$\phi_3 = -\tan^{-1}\omega/20$	$\phi_4 = \phi_1+\phi_2+\phi_3$
0.1	-90°	-2.86°	-0.569°	
0.2	-90°	-5.7°	-0.57°	
2	-90°	-45°	-5.7°	
8	-90°	-75.96°	-21.8°	
10	-90°	-78.69°	-26.56°	
20	-90°	-84.28°	-45°	
40	-90°	-87.13°	-63.43°	
∞	-90°	-90°	-10°	

$\left.\begin{array}{l}\omega_{gc} = 2.1\ rad/sec \\ \omega_{pc} = 6.36\ rad/sec.\end{array}\right\}$ from graph.

$GM = 21\,dB$

$PM = 38^\circ$

$\omega_{pc} > \omega_{gc}$, GM & PM +ve ⇒ Stable system

2. A unity feedback sys. has $G(s) = \frac{K}{s(s+2)(s+10)}$. Determine the marginal value of K using bode plot.

→ Rearranging the terms.

$$\frac{K}{s \cdot 2(1+s/2) \cdot 10(1+s/10)} = \frac{K/20}{s(1+s/2)(1+s/10)}$$

$$= \frac{K'}{s(1+s/2)(1+s/10)}$$

Step 1: As the value of K is unknown, let us draw the magnitude plot without K

* Factor $1/s$ ⇒ is a pole at the origin → $-20dB/dec$.

* $\frac{1}{(1+s/2)}$ → simple pole → $-20dB/dec$

⇒ resultant = $-20+-20 = -40dB/dec$.

$$= \frac{1}{1+T_1 s}$$ $T_1 = 1/2$.

$\omega c_1 = 2$ rad/sec.

* Factor $\frac{1}{(1+s/10)}$ ⇒ A simple pole → $-20dB/dec$

resultant $-40+-20 = -60dB/dec$

$$\frac{1}{1+s/10} = \frac{1}{1+T_2 s}$$ $T_2 = 1/10$.

→ $\omega C_2 = 10$ rad/sec.

Step 2: phase plot: substitute $s = j\omega$.

$$G(j\omega)H(j\omega) = \frac{K'}{j\omega(1+j\omega/2)(1+j\omega/10)}$$

ω	$\phi_1 = 1/j\omega$	$\phi_2 = -\tan^{-1}\omega/2$	$\phi_3 = -\tan^{-1}\omega/10$	$\phi_4 = \phi_1+\phi_2+\phi_3$
0.1	$-90°$	-2.86	$-0.57°$	$-93.43°$
1.0 ~~2~~	$-90°$	$-26.56°$	$-5.7°$	$-122.26°$
2	$-90°$	$-45°$	$-11.323°$	$-146.3°$
5	$-90°$	$-68.19°$	$-26.56°$	$-184.75°$
10	$-90°$	$-78.69°$	$-45°$	$-213.69°$
20	$-90°$	$-84.28°$	$-63.43°$	$-237.71°$
~~40~~	~~$-90°$~~	~~$-87.13°$~~		
~~∞~~	~~$-90°$~~	~~$-90°$~~		

~~For~~ From the phase plot $\omega_{pc} = 40.7\,rad/sec$.

$\omega_{gc} = \omega_{pc} = 4.7\,rad/sec$ GM = 0dB & PM = 0°.

A→B shift = upward = 22dB = $20 \log K$.

$$K' = \text{Antilog}\left(\frac{22}{20}\right) =$$

$$K = 251.76 = K_{max}$$

Verification of marginal value ~~of~~ using RH criteria.

$$s^3 + 12s^2 + 20s + k = 0.$$

s^3	1	20	0
s^2	12	k	0
s	$\frac{240-k}{12}$	0	
s^0	k		

For s' row,

making → $\frac{240-k}{12} = 0.$

$$\boxed{k = 240 = k_{max}}$$

3) Using the bode plot determine the value of gain so that
a) GM is 6dB b) PM is 25°

→

We can draw a magnitude plot without K.

Step 1 - Factors $\frac{1}{s}$ poles of the origin ⇒ slope $-20dB/dec$

Factor - $\frac{1}{(1+0.5s)}$ → a simple pole ⇒ $-20dB/dec$ - slope;

Resultant slope = $-40dB/dec$.

$\frac{1}{1+T_1 s}$ ⇒ $T_1 = 0.5\,sec$ ∴ $\omega c_1 = \frac{1}{T_1} = 2\,rad/sec$

Factor $\frac{1}{1+0.2S}$ → A simple pole → Slope $-20dB/dec$;
Resultant slope - $-60dB/dec$.

$\frac{1}{1+T_2S}$ → $T_2 = 0.2$ $\omega C_2 = \frac{1}{T_2} = 5$ rad/sec.

Step 2 - Phase plot ; substitute $s = j\omega$.

$$G(s)H(s) = \frac{K}{j\omega(1+0.5j\omega)(1+0.2j\omega)}$$

ω	ϕ_1	$\phi_2 = -\tan^{-1} 0.5\omega$	$\phi_3 = -\tan^{-1} 0.2\omega$	$\phi_4 = \phi_1 + \phi_2 + \phi_3$
0.1	$-90°$	$-2.862°$	-1.145	$-98°$
0.2	$-90°$			$-106.$
~~2~~.	$-90°$			$-156.8°$
5	$-90°$			$-203.2°$
10	$-90°$			$-232.12°$

From the graph $\omega_{pc} = 3.5$ rad/sec.

a) $K = ?$ for $GM = 6dB$.
shift $B \to A$ ⟹ $9dB = 20\log K$ $\boxed{\overset{\text{shift}}{15 - 6 = 9dB}}$
⇒ $K = 2.81$.

b) $K = ?$ for $PM = 25°$
$180 - 25 = 155°$
Shift from $C \to C'$ ⇒ $6dB = 20\log K$
⇒ $K = 1.99.. \approx \underline{\underline{2}}$

4) Draw the Bode plot for the following T.F. Determine the value of

$$G(S)H(S) = \frac{KS^2}{(1+0.2S)(1+0.02S)}$$

K for the gain crossover frequency of 5 rad/sec.

5 Given, $G(s)H(s) = \frac{K}{s(1+0.1s)(1+s)}$. Determine the value of K so that gain margin is 30dB. ~~PM~~ What is the corresponding PM = ?

ii) Given PM = 30°. Find the value of K & corresponding GM.

→ Step (i). Factor $\frac{1}{s}$ ⇒ A pole at the origin ⇒ slope = −20dB

Factor $\frac{1}{1+s}$ ⇒ A simple pole with slope −20dB/dec

→ Resultant = −40dB/dec

$\frac{1}{1+T_1 s}$ ⇒ $T_1 = 1$ sec $\omega_{C_1} = 1$ rad/sec.

Factor $\frac{1}{(1+0.1s)}$ ⇒ A simple pole with slope −20dB/dec.

→ resultant = −60dB/dec.

$T_2 = 0.1s$ $\omega_{C_2} = 10$ rad/sec.

Step 2: $G(s)H(s) = \frac{K}{s(1+0.1s)(1+s)}$

ω	$\phi_1 = 1/j\omega$	$\phi_2 = \tan^{-1}\omega$	$\phi_3 = \tan^{-1}0.1\omega$	$\phi_4 = \phi_1+\phi_2+\phi_3$
0.1	−90°	−5.7°	−0.57°	−96.2°
1	−90°	−45°	−5.7°	−140.7°
3	−90°	−71.55°	−16.6°	−178.25°
5	−90°	−78.6°	−26.5°	−194.16°
10	−90°	−84.2°	−45°	−219.2°

From graph, $\omega_{pc} = 3.3$ rad/sec.

~~AF~~ A→B shift is downward −10dB = 20 log K

$K = \text{AntiLog}\left(\frac{-10}{20}\right)$

$= 0.316$ − K_{max} → PM = 68°

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b) PM = 30°

Shift C → D (upward) = 6dB = 20 log k

$\Rightarrow k = 2.$

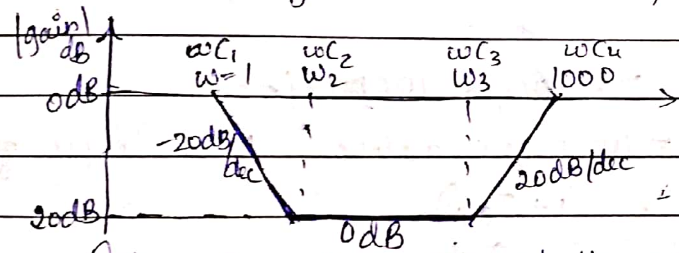
$\omega_{gc} = 1.4$ rad/sec.

corresponding GM = 14dB.

H.W.

6 $G(s)H(s) = \dfrac{ks^2}{(1+0.2s)(1+0.02s)}$

Determining the OLTF (Transfer function) from the given Bodeplot [magnitude plot or asymptotic plot]

1) 

Determine the TF of the system whose asymptotic plot is as shown in the above figure.

→ Step 1 → As the starting slope is zero. There are no poles or zeroes at the origin.

2 → At $\omega = 1$ rad/s. ; shift = 0dB = 20 log K ; $\boxed{k=1}$

~~3 → At~~

3 → At $\omega = 1$ rad/s $= \omega c_1$; A simple pole is added. $\dfrac{1}{1+T_1 s}$

where $T_1 = \dfrac{1}{\omega c_1} = 1$ sec.

$\dfrac{1}{(1+T_1 s)} = \boxed{\dfrac{1}{1+s}}$

4 → $y = mx + c$

mag, slope, $\log\omega$, constant.

Let $\omega c_1 = 1$ rad/sec.

$0 = 20\text{dB/dec} \times \log\omega c_1 + c$; $c = 0$dB

* $y = mx + c$ at the corner frequency ωc_2 = ?

slope = -20dB/dec

$-20\text{dB} = -20\text{dB/dec} \times \log\omega c_2 + 0$

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$\omega C_2 = 10$ rad/sec.

5 → At $\omega C_2 = 10$ rad/sec; a simple zero is added, Factor $(1+T_2 S)$

$\Rightarrow \omega C_2 = \frac{1}{T_2}$; $T_2 = \frac{1}{\omega C_2} = \frac{1}{10} = 0.1$ sec.

the factor is $(1+T_2 S) = (1+0.1S)$

6 → As $\omega_3 = \omega C_3$ is ~~En~~ not known, let us consider the next nearer frequency $\omega C_3 = 1000$ rad/sec and let us apply $y = mx + c$ att this frequency $\omega C_4 = 1000$ rad/sec

$0 = 20\text{dB/dec} \cdot \log 1000 + C.$

$\boxed{C_1 = -60\text{dB}}$

7 → Then at ωC_3 (not known) apply $y = mx + c$

$-20\text{dB} = 20\text{dB/dec} \times \log \omega C_3 - 60$

$\rightarrow \omega C_3 = 100$ rad/sec.

8 → At $\omega C_3 = \omega_3 = 100$ rad/sec ; A simple zero is added

→ Factor is $(1+T_3 S)$

→ $\boxed{(1+0.01S)}$

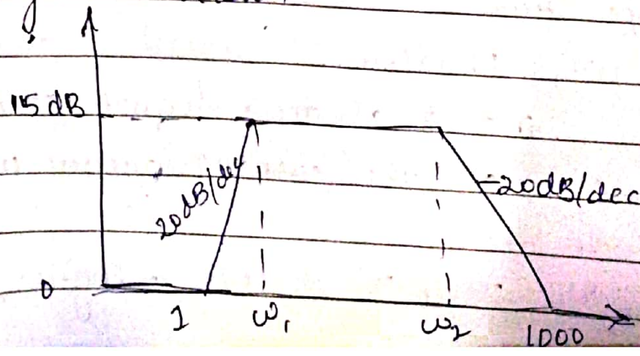
9 → At $\omega C_4 = 1000$ rad/sec ; A simple pole is added

The factor is $\frac{1}{1+T_4 S} = \boxed{\frac{1}{1+0.001S}}$

10 → The OLTF is,

$$TF = \frac{1(1+0.01S)(1+0.01S)}{(1+S)(1+0.001S)}$$

2. Determine the transfer fun. of the system whose Bode plot is given below.



Step 1 → As the starting slope is zero. There are no poles & zeroes at the origin.

2 → $\omega = 1$; shift = 0dB = +20 log K ; K = 1.

3 → At $\omega = 1$ rad/s $= \omega_{c_1}$; A simple zero is added factor is $(1+T_1 S) = \boxed{(1+S)}$

4 → $y = mx + c$

$\omega c_1 = 1$ rad/sec.

$0 = -20dB/dec \times \log \omega c_1 + C.$

$\boxed{C = 0}$

~~5~~ → * $y = mx + c$ at $\omega c_2 = ?$

slope = 20dB/dec

$15dB \cancel{20dB} = 20dB/dec \times \log \omega c_2 + 0$

$\boxed{\omega c_2 = 5.623}$

$\cancel{\omega c_2 = 10 rad/sec.}$

5 → At $\omega c_2 = 5.623$ rad/sec. ; a simple pole is added

Factor $\frac{1}{(1+T_2 S)}$.

→ $\omega c_2 = \frac{1}{T_2}$; $T_2 = \frac{1}{\omega c_2} = \frac{1}{5.623} = 0.1778$ sec

The factor is $\frac{1}{(1+T_2 S)} = \boxed{\frac{1}{(1+0.1775)}}$

6 → As $\omega_3 = \omega c_3$ is not known, let us consider the next nearer frequency $\omega c_3 = 1000$ rad/sec. and let us apply $y = mx + c$ at this frequency $\omega c_4 = 1000$ rad/sec.

$0 = -20dB/dec \cdot \log 1000 + c$

$\boxed{C = 60dB}$

7 → At ωc_3 , $y = mx + c$

$15 = -20dB/dec \times \log \omega c_3 + 60$

$\boxed{\omega c_3 = 177.82 \text{ rad/sec}}$

8 → At $\omega c_3 = \omega_3 = 177.82$ rad/sec ; A simple pole is added.

→ Factor is $\frac{1}{(1+T_3S)}$

→ $\boxed{\frac{1}{1+0.00562}}$ $\quad T_3 = \frac{1}{177.92}$

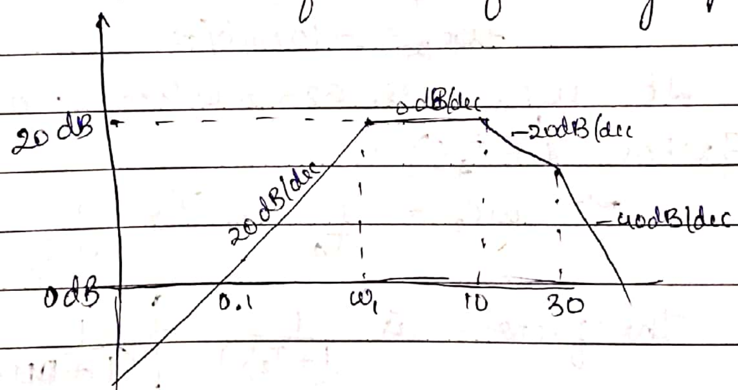
9 → At $\omega_{C_4} = 1000$ rad/sec ; A simple zero is added

Factor is $(1+T_4S) = \boxed{(1+0.001S)}$

10 → The OLTF is

$$TF = \frac{(1+S)(1+0.001S)}{(1+0.0005625)(1+0.177S)}$$

3. Determine the T.F for the following plot.



→ Step 1 → As the starting slope is +20dB/dec, there is one zero at the origin.

$\boxed{S}$

Step 2 → At $\omega_1 = \omega_{c_1} = ?$

At $\omega = 0.1$ rad/sec

$y = mx + c$

$0 = 20\text{dB/dec} \log 0.1 + C$

$\boxed{C = 20}$

* At $\omega_{C_1} = \omega_1 = ?$

$y = mx + C$

$20\text{dB} = 20 \log \omega_{C_1} + 20$

$20 \log \omega_{C_1} = 0$

$\omega_{C_1} = 1.$

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At this point a simple pole exists $\frac{1}{(1+T_1 s)}$

$$T_1 = \frac{1}{\omega_{c_1}} = \frac{1}{1} = 1.$$

$$\therefore \boxed{\left(\frac{1}{1+s}\right)}$$

Step 3 → $\omega_{c_2} = 10$ rad/s ; slope $= -20$ dB/dec

A simple pole is added ; Factor is $\left(\frac{1}{1+T_2 s}\right) = \boxed{\left(\frac{1}{1+0.1s}\right)}$

Step 4 → 20 log K = 20.

$$20 \log K = 20$$

$$\boxed{K = 10.}$$

Step 5 → $\omega_{c_4} = 30$ rad/sec ; slope $= -40$ dB/de.

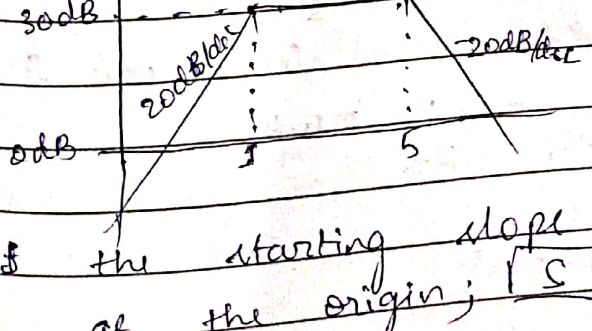
A simple pole is added.

$$\text{Factor} = \frac{1}{1+T_1 s} = \boxed{\frac{1}{1+0.033s}} \qquad T_1 = \frac{1}{30} = 0.033$$

Step 6 → The OLTF is given by

$$TF = \frac{10s}{(1+s)(1+0.1s)(1+0.033s)}$$

4. Determine the T.F for the following Bode plot.



→ Step 1 → As the starting slope is 20dB/dec, there is a zero at the origin; $\boxed{s}$

Step 2 → At $\omega_1 = \omega_{c_1} = 1$ rad/sec

$$\text{Factor } \frac{1}{1+T_1 s} = \boxed{\frac{1}{1+s}}$$

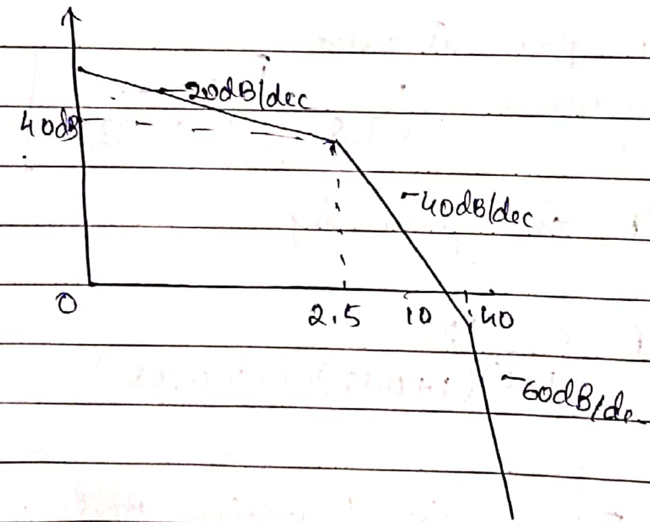
At $\omega_2 = 5$ rad/sec. $= \omega_{c2}$.
A pole is added.
Factor is $\boxed{\frac{1}{(1+0.2S)}}$ $T_2 = \frac{1}{5} = 0.2$

$30 = 20\log K$

$\boxed{K = 31.62}$

5) $T.F = \frac{31.62\,S}{(1+S)(1+0.2S)}$

5. Determine the Tranfer function for the following Bode plot.



→ 1) As the starting slope is -20dB/dec. There is one pole at the origin; The factor is $1/s$.

2) At $\omega c_1 = 2.5$ rad/sec. ; A simple pole is added & factor is $\frac{1}{1+T_1 S}$;- $T_1 = \frac{1}{\omega c_1} = \frac{1}{2.5} = 0.4$.

$\therefore \frac{1}{1+T_1 S} = \frac{1}{1+0.4S}$.

3) At $\omega c_2 = 40$ rad/sec , a simple pole is added.
Factor is $\frac{1}{1+T_2 S}$ & $T_2 = \frac{1}{\omega c_2} = \frac{1}{40} = 0.025$

Factor is $\frac{1}{1+T_2 s} = \frac{1}{1+0.025s}$

4) To determine the value of k, at $\omega = 1$ rad/sec.

At $\omega = 2.5$ rad/sec.

$$y = mx + c.$$

$$40 = -20\log(2.5) + c$$

$$\therefore c = 47.96\,dB.$$

At $\omega = 1$ rad/sec.

$$y = mx + c$$

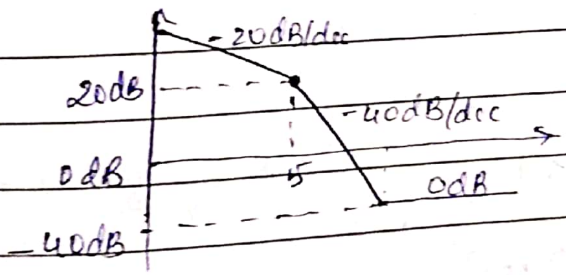
$$y = -20(\log 1) + 47.95$$

$$y = \underline{\underline{147.95\,dB}}$$

$$y = 47.95\,dB = 20\log K$$

$$K = 249.74$$

6. Determine the transfer function of the system whose bode plot is as shown below.



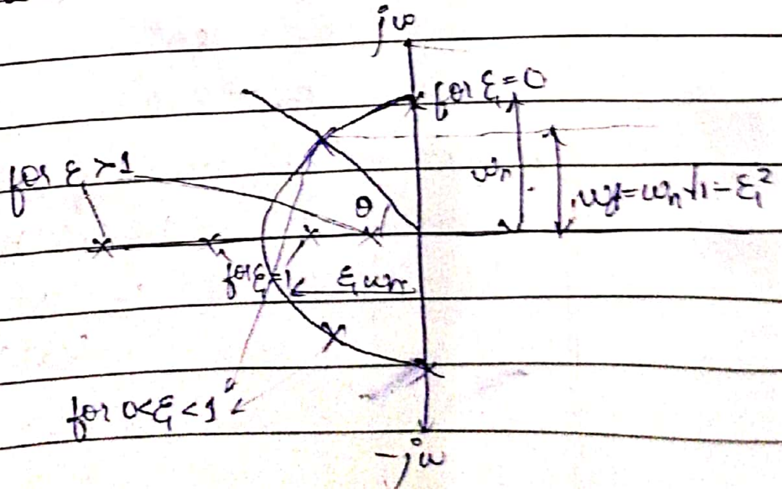
$$\omega_{c_2} = 158.113 \text{ rad/sec.}$$

$$TF = \frac{50(1+0.00632s)^2}{s(1+0.2s)}$$

$$\cos\theta = \frac{adj}{hyp} = \frac{\xi\omega_n}{\omega_n} = \xi$$

$$\cos\theta = \xi$$

$$\theta = \cos^{-1}\xi$$



Q) Draw the root locus for OLTF $G(s)H(s) = \frac{K}{s(s+2)(s+5)}$ and determine the a) breakaway point
b) line for $\xi = 0.5$ & the corresponding value of K.
c) The frequency at which root locus crosses the imaginary axis and the corresponding value of K.

$$G(s)H(s) = \frac{K}{s(s+2)(s+3)}$$

step 1: $P = 3$
$Z = 0$
$N = P - Z = 3$ branches of the root locus.
$\therefore$ $P - Z = 3 \Rightarrow 3$ asymptotes approaching ∞.
* Starting points $\Rightarrow 0, -2, -5$.
* Terminating " $\Rightarrow \infty, \infty, \infty$.

Locating poles & zeros in s plane.

Step 2: Mark ~~At~~ RL & NRL. Predict the breakaway point.

Step 3: $\sigma = \frac{\Sigma \text{RP of poles} - \Sigma \text{R.P of zeros}}{P-Z} = \frac{0-2-5}{3} = -2.33$

Step 4: $\theta = \frac{(2q+1)180°}{P-Z}$

As there are 3 branches approaching ∞, $q = 0, 1, 2$

$\theta_1 = 60°$
$\theta_2 = 180°$
$\theta_3 = 300°$

Step 5: The value of actual breakaway point,

$$1 + G(s)H(s) = 0$$

$$1 + \frac{K}{s(s+2)(s+5)} = 0$$

$$s(s+2)(s+5) + K = 0$$

$$k = -s^3 - 7s^2 - 10s$$

$$\frac{dk}{ds} = -3s^2 - 14s - 10 = 0$$

$$s = -3.78 , -0.88$$

$\therefore s = -0.88$ [$\because$ 3.88 in NRL.

$$\therefore K = 4.032.$$

Step 6: Intersection with imaginary axis.

$$s^3 + 7s^2 + 10s + k = 0$$

Routh's array

s^3	1	10	0
s^2	7	K	
s	$\frac{70-k}{7}$	0	
s^0	K		

For s' row, $\frac{70-k}{7} = 0$

$$K_{mar} = 70.$$

For s^2 row, $A(s) = 7s^2 + k = 0.$

$$7s^2 = -k$$

$$7s^2 = -70$$

$$s = \pm\sqrt{10}j$$

$$s = \pm 3.16j$$

here $s = j\omega =$ ~~3.16 rad/sec.~~ $3.16j$

$\therefore \omega_{max} = 3.16$ rad/sec.

D.Imp Step

Step 7: For $\xi = 0.5$; $K = ?$ $\Rightarrow \cos\theta = \xi$

$$\theta = \cos^{-1}\xi$$

$$\theta = 60^\circ$$

Intersection on 60° line with root locus $= -0.7 + j1.25 = s.$

There are 2 ways to determine K value.

i) using magnitude condition : $|G(s)H(s)| = |-1|$

$$\frac{K}{s(s+2)(s+5)} = 1$$

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$\therefore K = |s(s+2)(s+5)|$ [where $s = -0.7 + j1.25$]

$= |(-0.7 + j1.25)(1.3 + j1.25)(4.3 + j1.25)|$.

$= \sqrt{0.7^2 + 1.25^2} \cdot \sqrt{1.3^2 + 1.25^2} \cdot \sqrt{4.3^2 + 1.25^2}$.

$= 11.53$

) $K = \dfrac{\text{product of phasor length of poles}}{\text{" " " " of zeros}}$ $= 4.5 \times 1.4 \times 1.8$

$= \dfrac{11.53}{1}$

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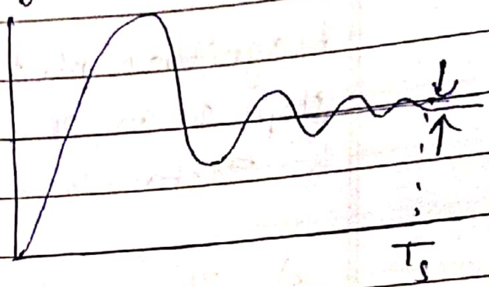
(c) Determine the time required for the output to reach 98% of steady state output.

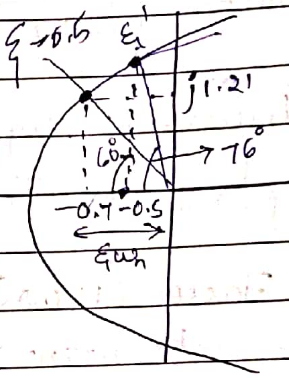
→ This time is nothing but settling time.

$$T_S = \frac{4}{\xi \omega_n}$$

$$= \frac{4}{0.7}$$

$$T_S = 5.7 \text{ sec.}$$





d) For $T_s = 8$ sec. $\xi_1 = ?$, $\theta = ?$

$$\xi \omega_n = \frac{4}{T_s}$$

$$\xi_1 \omega_n = 0.5$$

$\xi_1' = \cos\theta'$ $\theta' = 76°$ [From graph]

$\underline{\xi_1' = 0.242.}$

To determine the value of K.

$$|G(s)H(s)| = |-1|$$

Q. On a bode plot, for a system with $G(s)H(s) = \frac{242(s+5)}{s(s+1)(s^2+5s+121)}$

Draw the bode plot determine ω_{gc}, ω_{pc}, g_m & P_m and comment on the stability.

→ To make the given equation in time constant form

$$\frac{242 \cdot 5\left(1+s/5\right)}{s(1+s) \cdot 121\left(\frac{s^2}{121} + \frac{5}{121}s + 1\right)}$$

$$= \frac{10\left(1+s/5\right)}{s(1+s)\left(\frac{s^2}{121} + \frac{5}{121}s + 1\right)}$$

2) For the quadratic factor $s^2+5s+121=0$

$s^2+2\xi_1\omega_n s+\omega_n^2=0$

$2\xi_1\omega_n=5$ $\quad$ $\omega_n^2=121$; $\underline{\omega_n=11 \text{ rad/sec}}$

$\xi_1=\frac{5}{2\omega_n}$ $\quad$ $\underline{\xi_1=0.2}$. $\underline{=0.227}$

Magnitude plot.

Step 1 → $20\log K = 20\log 10 = 20dB$.

b) Factor $\frac{1}{s}$ → pole at the origin → slope $= -20dB/dec$

c) Factor $\left(\frac{1}{1+s}\right)$ → A simple pole → slope = $-20dB/dec$

Resultant slope $= -20 + -20\,\cancel{dB} = -40dB/dec$

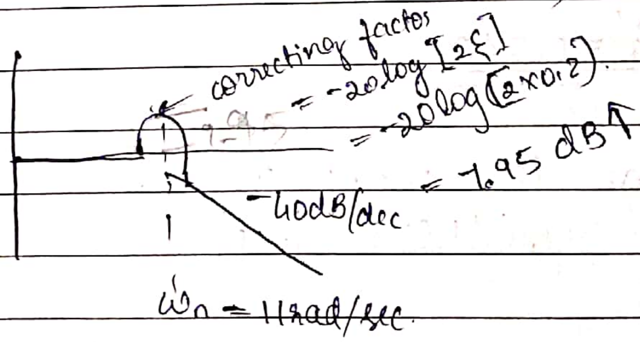
$\frac{1}{1+T_1 s}$ → $T_1=1$ $\quad$ $\omega c_1 = 1 \text{ rad/sec}$

d) Factor $(1+0.2s)$ ⇒ A simple zero → slope $= 20dB/dec$

Resultand $= -40+20 = -20dB/dec$.

$(1+T_2 s)$; $T_2=0.2$; $\omega c_2 = 5 \text{ rad/sec}$.

e) quadratic pole $\left(\frac{s^2}{121}+\frac{5s}{121}+1\right)$



Phase plot:

$\phi_4 = \tan^{-1}\left(\frac{\omega\, 0.041}{1-\frac{\omega^2}{121}}\right)$

ω	$\phi_1 = \frac{1}{j\omega}$	$\phi_2 = -\tan^{-1}\omega$	$\phi_3 = \tan^{-1} 0.2$	~~$\phi_4 = \phi_1+\phi_2+\phi_3$~~
0.1	$-90°$	$-\cancel{1.14}$ -5.7	1.14	$-0.23°$
1	$-90°$	$\cancel{+11.3}$ $-45°$	11.3	$-2.36°$
5	$-90°$	$\cancel{+45°}$ $-78.5°$	45°	$-14.4°$
8	$-90°$	$-82.8°$		$-32.8°$
10	$-90°$	$-84.2°$	63.4	$-67.0°$
20	$-90°$	$-87.13°$	75.9	$\phi - 180° = 19.5 - 180° = -160.4°$

$\phi_5 = \phi_1 + \phi_2 + \phi_3 + \phi_4$

-94.7°

-126.0°

-138°

-149.8°

-177.8°

-261.53°

From graph, As $\omega_{pc} > \omega_{gc}$.
System is stable.