Control System representation:

Control system
- Open loop
- Closed loop [+ve feedback, -ve feedback]
  - Manually controlled C.S
  - Automatically controlled C.S

General block diagram of an open loop C.S

i/p → [controller] → [plant] → o/p

Ex1) Fan

switch (i/p) → [regulator] → [Fan (motor)] → speed (o/p)

Ex2) Toaster

Timer knob (i/p) → [Timer] → [Heater or a toaster] → degree of darkness of the bread (o/p)

Ex-3) Car [Automobile]

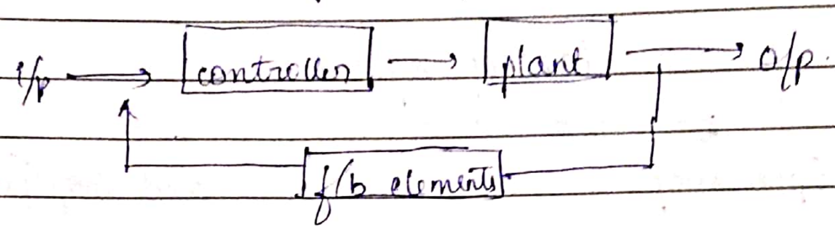
i/p (force) → [Accelerator, pedal, linkages & carburetter] → [engine vehicle] → o/p (speed)

Rate of fuel flow.

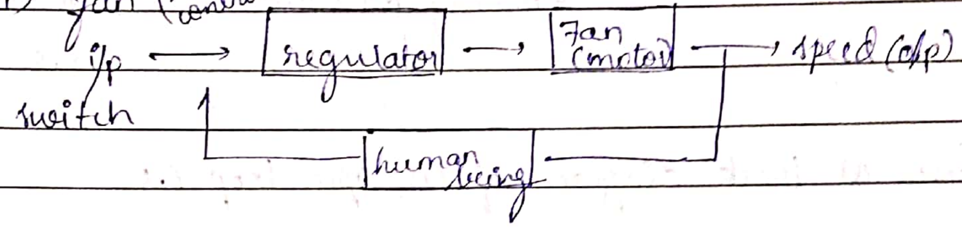
controller

plant

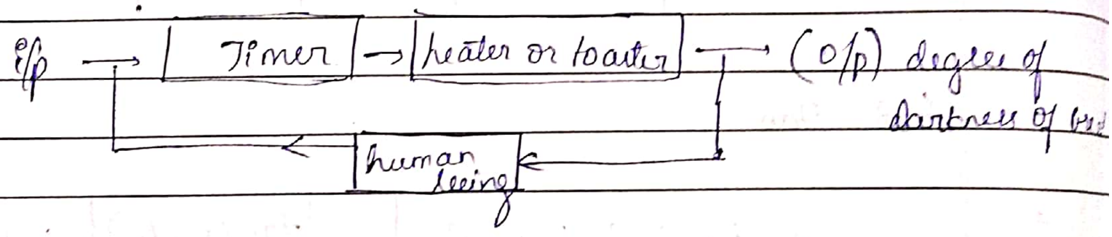
General block diagram of a closed loop C.S



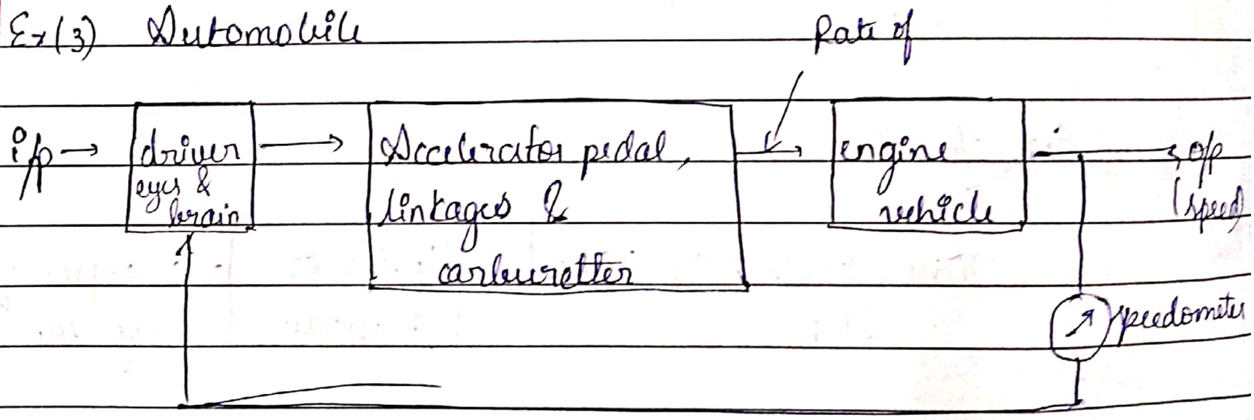
Ex 1) Fan (Manually controlled)



Ex (2) Toaster (Manually controlled)

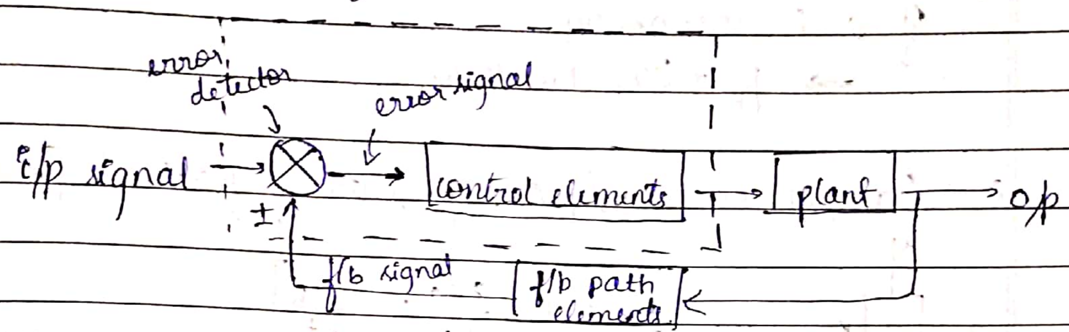


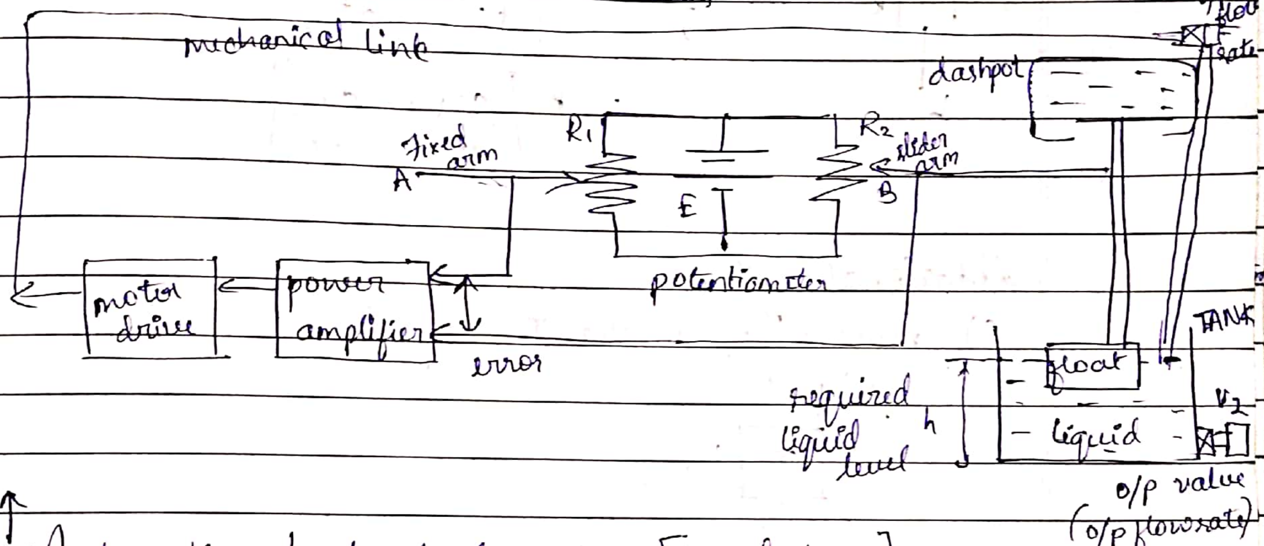
Ex (3) Automobile



When feedback signal is added to the i/p signal it is positive feedback.
When feedback signal is subtracted from i/p signal it is negative feedback.

General block diagram of Automatic controlled system.





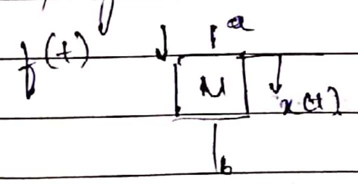
Automatic tank level system [liquid level]

In this system, float is the feedback element.

Mechanical system :— Translational system.
└ rotational system

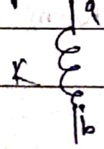
Basic elements of translational system :

i) Mass (M) in kg



$$f_M = ma = M\frac{d^2x(t)}{dt^2} \text{ Newtons}$$

ii) Spring [for the property of elasticity] K (N/m)



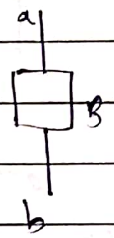
$f_K = K[x_a - x_b]$ newtons

$= K[x_b - x_a]$

If a is stationary

$\Rightarrow f_K = K x_b$ Newtons.

3. Damper or Dashpot [property of friction] [N/m/sec]

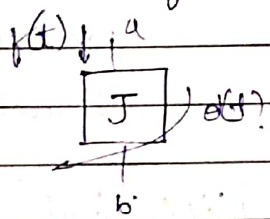


$$f_B = B\left[\frac{dx_a}{dt} - \frac{dx_b}{dt}\right]$$

$$= B\left[\frac{dx_b}{dt} - \frac{dx_a}{dt}\right]$$

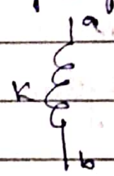
Rotational system

i) moment of inertia (J) (kg-m)



$$T_m = \frac{J d^2\theta(t)}{dt^2} \text{ N-m.}$$

ii) Spring [for the property of elasticity] [N-m/rad]



$$T_k = K[\theta_a - \theta_b] \text{ N-m}$$

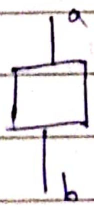
$$T_k = K[\theta_b - \theta_a]$$

If a is stationary

$T_k = K\theta_b$ Newton-m.

iii) Damper or dashpot (B) [N-m/rad/sec]

[property of friction]



$$T_B = B\left[\frac{d\theta_a}{dt} - \frac{d\theta_b}{dt}\right] \text{ N-m}$$

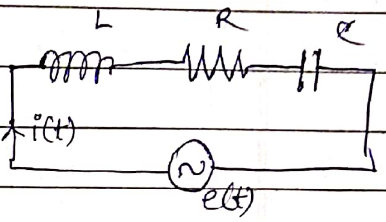
$$f_B = B\left[\frac{d\theta_b}{dt} - \frac{d\theta_a}{dt}\right]$$

In general,

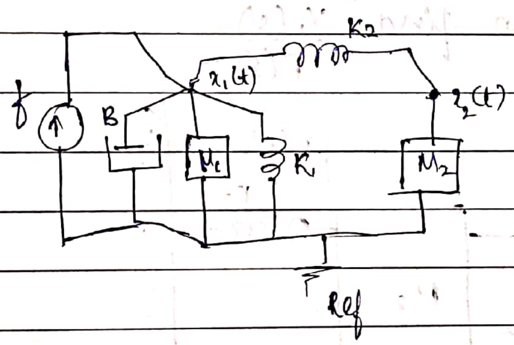
$$v(t) = \frac{dx(t)}{dt} \quad ; \quad \omega(t) = \frac{d\theta(t)}{dt}.$$

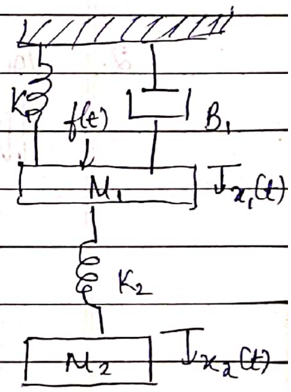
Mathematical model of the system:

$$e(t) = i(t)R + L\frac{di(t)}{dt} + \frac{1}{C}\int_0^t i(t)\,dt$$



Write the mathematical model for the following mechanical system & then determine the transfer function $\frac{X_2(s)}{F(s)}$;





The mathematical model at node $x_1(t)$ is given by

At $x_1(t)$:-

$$f(t) = M_1\frac{d^2x_1(t)}{dt^2} + B_1\frac{dx_1(t)}{dt} + K_1x_1(t) + K_2\left[x_1(t) - x_2(t)\right] \quad -(1)$$

$$F(s) = M_1s^2X_1(s) + B_1sX_1(s) + K_1X_1(s) + K_2\left[X_1(s) - X_2(s)\right] \quad -(2)$$

$$F(s) = X_1(s)\left[M_1s^2 + B_1s + K_1 + K_2\right] + X_2(s)\left[-K_2\right] \quad -(3)$$

At $x_2(t)$:-

$$0 = M_2\frac{d^2x_2(t)}{dt^2} + K_2\left[x_2(t) - x_1(t)\right] \quad -(4)$$

$$= M_2s^2X_2(s) + K_2\left[X_2(s) - X_1(s)\right] \quad -(5)$$

$$0 = X_1(S)\left[-K_2\right] + X_2(S)\left[S^2M_2 + K_2\right] \quad - \text{⑥}$$

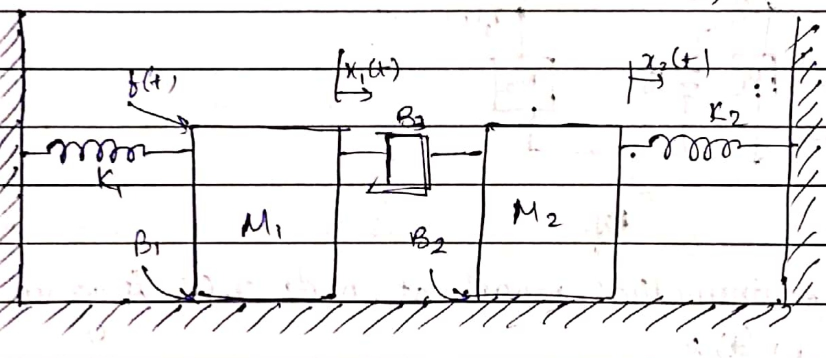
$$\begin{bmatrix} (M_1S^2 + B_1S + K_1 + K_2) & -K_2 \\ -K_2 & (M_2S^2 + K_2) \end{bmatrix} \begin{bmatrix} X_1(S) \\ X_2(S) \end{bmatrix} = \begin{bmatrix} F(S) \\ 0 \end{bmatrix}$$

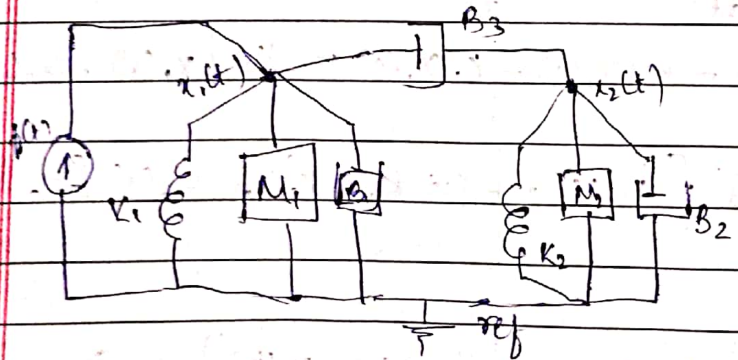
$$X_2(S) = \frac{\begin{vmatrix} M_1S^2 + B_1S + K_1 + K_2 & F(S) \\ -K_2 & 0 \end{vmatrix}}{\Delta}$$

$$X_2(S) = \frac{K_2 F(S)}{\Delta}$$

$$\frac{X_2(S)}{F(S)} = \frac{K_2}{\Delta}$$

2. For the following mechanical system. Write down the cause effect equation and then find: $\frac{X_1(S)}{F(S)}$





At $x_1(t)$,

$$f(t) = M_1\frac{d^2x_1(t)}{dt^2} + K_1x_1(t) + B_1\frac{dx_1(t)}{dt} + B_3\left[\frac{dx_1(t)}{dt} - \frac{dx_2(t)}{dt}\right]$$

$$\therefore F(s) = M_1 s^2 X_1(s) + K_1 X_1(s) + B_1 s \cdot X_1(s) + B_3 s[X_1(s) - X_2(s)]$$

$$= X_1(s)\left[M_1 s^2 + K_1 + B_1 s + B_3 s\right] + X_2(s)[\cancel{s}B_3(s)]$$

At $x_2(t)$,

$$0 = M_2 \frac{d^2 x_2(t)}{dt^2} + K_2 x_2(t) + B_2 \frac{dx_2(t)}{dt} + B_3\left[\frac{dx_2(t)}{dt} - \frac{dx_1(t)}{dt}\right]$$

$$= M_2 s^2 X_2(s) + K_2 X_2(s) + B_2 s X_2(s) + B_3 s[X_2(s) - X_1(s)]$$

$$0 = X_1(s)[-B_3 s] + X_2(s)\left[M_2 s^2 + K_2 + B_2 s + B_3(s)\right]$$

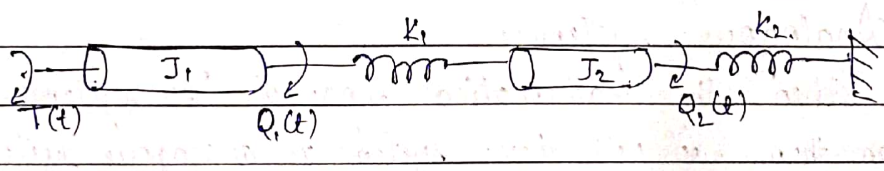
$$\begin{bmatrix} M_1 s^2 + K_1 + B_1 s + B_3 s & -B_3 s \\ -B_3 s & M_2 s^2 + K_2 + B_2 s + B_3(s) \end{bmatrix}\begin{bmatrix} X_1(s) \\ X_2(s) \end{bmatrix} = \begin{bmatrix} F(s) \\ 0 \end{bmatrix}$$

$\cancel{\dfrac{X_1(s)}{F(s)}}$

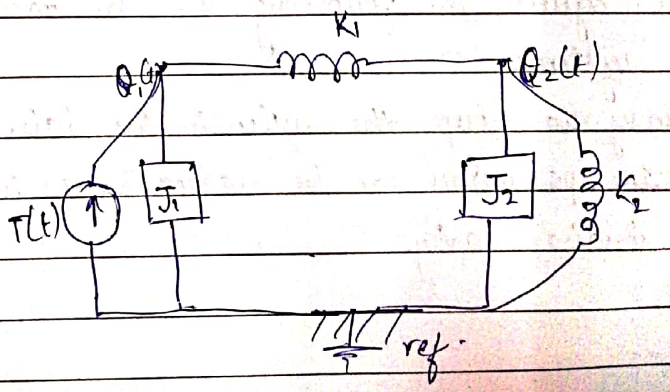
$$X_1(s) = \frac{\begin{vmatrix} F(s) & -B_3 s \\ 0 & M_2 s^2 + K_2 + B_2 s + B_3 s \end{vmatrix}}{\Delta}$$

$$\frac{X_1(s)}{F(s)} = \frac{M_2 s^2 + K_2 + B_2 s + B_3 s}{\Delta}$$

3.



$\dfrac{\theta_1(s)}{T(s)}$:



At $\theta_1(t)$

$$T(t) = J_1 \frac{d^2\theta_1(t)}{dt^2} + K_1(\theta_1(t) - \theta_2(t)) \quad - (1)$$

$$T(s) = J_1 s^2 \theta_1(s) + K_1[\theta_1(s) - \theta_2(s)] - (2)$$

$$= \theta_1(s)[J_1 s^2 + K_1] + \theta_2(s)[-K_1]$$

At $\theta_2(t)$:

$$\cancel{T(t)}\ 0 = J_2 \frac{d^2\theta_2(t)}{dt^2} + K_2(\theta_2(t)) + K_1[\theta_2(t) - \theta_1(t)] - (3)$$

$$0 = J_2 s^2 \theta_2(s) + K_2\theta_2(s) + K_1[\theta_2(s) - \theta_1(s)] - 4$$

$$\theta_1(s)[-K_1] + \theta_2(s)[J_2 s^2 + K_2\theta_2 + K_1]$$

$$\begin{bmatrix} J_1 s^2 + K_1 & -K_1 \\ -K_1 & J_2 s^2 + K_2 + K_1 \end{bmatrix} \begin{bmatrix} \theta_1(s) \\ \theta_2(s) \end{bmatrix} = \begin{bmatrix} T(s) \\ 0 \end{bmatrix}$$

$$\theta_1(s) = \frac{\begin{bmatrix} T(s) & -K_1 \\ 0 & J_2 s^2 + K_2 + K_1 \end{bmatrix}}{\Delta}$$

$$\theta_1(s) = \frac{(J_2 s^2 + K_2 + K_1)\, T(s)}{\Delta}$$

$$\frac{\theta_1(s)}{\cancel{\theta}\, T(s)} = \frac{J_2 s^2 + K_2 + K_1}{\Delta}$$

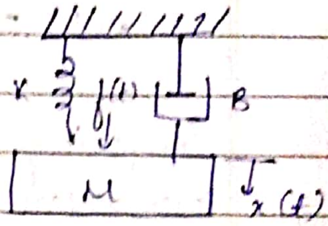
Analogous systems:

When the mathematical model of two systems are of the same form then we call these systems as analogous systems. In this topic, we are studying analogous electrical system, for the given mechanical system. The following are the advantages of analogous electrical system:

- It is easy to setup the system in the lab.
- Any of the parameters can be varied & the response of the system can be studied easily.

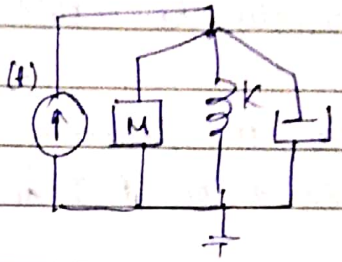
At $x(t)$:

$$f(t) = M\frac{d^2x(t)}{dt^2} + B\frac{dx(t)}{dt} + Kx(t) \quad \text{--- (1)}$$



Let $V(t) = \frac{dx(t)}{dt}$ --- (a)

$$\Rightarrow x(t) = \int_0^t v(t)\,dt \quad \text{--- (b)}$$

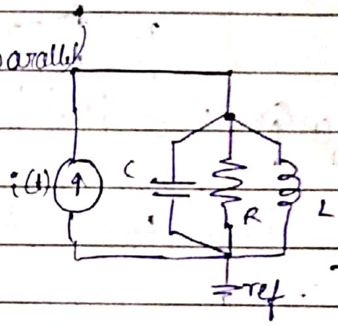


Force velocity equation will be,

$$f(t) = M\frac{dV(t)}{dt} + BV(t) + K\int_0^t v(t)\,dt \quad \text{--- (2)}$$

Let us consider a simple RLC network (parallel)

$$i(t) = C\frac{de(t)}{dt} + \frac{e(t)}{R} + \frac{1}{L}\int_0^t e(t)\,dt \quad \text{--- (3)}$$



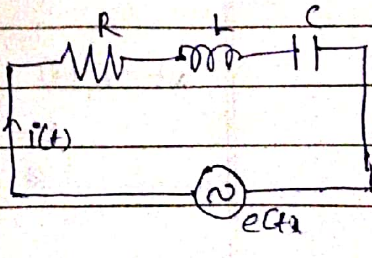
fig©.

Comparing equations (2) & (3), we can say that both the equations are of the same form. Hence the given mechanical system is analogous to electrical system considered in fig©

Comparing these eqn. the analogous set that we get is called a force current analogous set or F-I analogous set.

Rotational	Translational		Electrical (F-I)		Electrical (F-V)
$T(t)$	$f(t)$	$\Leftrightarrow$	$i(t)$	$\Rightarrow$	$e(t)$
$\omega(t)$	$v(t)$	$\Leftrightarrow$	$e(t)$	$\Rightarrow$	$i(t)$
J	M	$\Leftrightarrow$	C	$\Rightarrow$	L
B	B	$\Leftrightarrow$	$1/R$	$\Rightarrow$	R
K.	K	$\Leftrightarrow$	$1/L$.	$\Rightarrow$	$1/C$

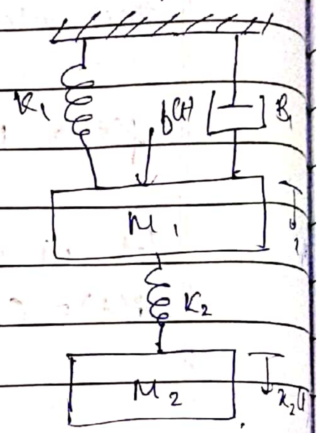
$$e(t) = i(t)R + \frac{1}{C}\int_0^t i(t)\,dt + L\frac{di(t)}{dt} \quad \text{--- (4)}$$

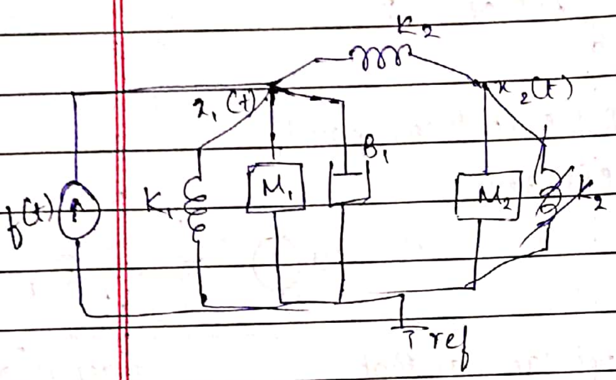


$$e(t) = L\frac{d(i(t))}{dt} + i(t)R + \frac{1}{C}\int_0^t i(t)\,dt \quad -(4)'$$

Comparing (2) & (4)', the two systems are analogous and are in force voltage analogy and corresponding analogous set can be written as electrical (F-V) [in the table]

Ex. For the following mechanical system, draw the electrical network based on F-I analogy





→ At $x_1(t)$

$$f(t) = M_1\frac{d^2x_1(t)}{dt^2} + K_1x_1(t) + B_1\frac{dx_1(t)}{dt} + K_2[x_1(t) - x_2(t)] \quad -(1)$$

~~$F(s) = M_1 s^2$~~

Let $v_1(t) = \frac{dx_1(t)}{dt}$ -(a) & $x_1(t) = \int_0^t v_1(t)\,dt$ & -(b)

$v_2(t) = \frac{dx_2(t)}{dt}$ -(c) & $x_2(t) = \int_0^t v_2(t)\,dt$ -(d)

Force velocity eqn is

$$f(t) = M_1\frac{dv_1(t)}{dt} + B_1v_1(t) + K_1\int_0^t v_1(t)\,dt + K_2\left[\int_0^t v_1(t) - v_2(t)\right]$$

F-I analysis set,

$$i(t) = C_1\frac{de_1(t)}{dt} + \frac{1}{R_1}e_1(t) + \frac{1}{L_1}\int_0^t e_1(t)\,dt + \frac{1}{L_2}\left[\int_0^t (e_1(t) - e_2(t))\,dt\right]$$

At $x_2(t)$:

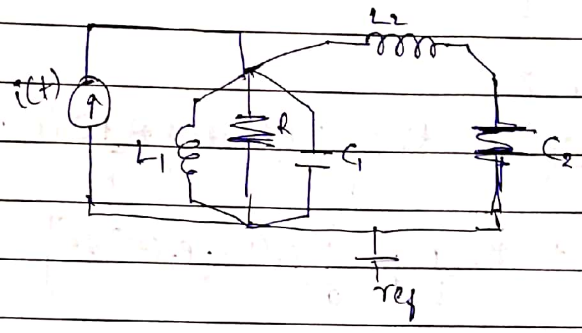
$$0 = M_2\frac{d^2x_2(t)}{dt^2} + K_2[x_2(t) - x_1(t)] \quad -(2)$$

Force velocity eqn. is

$$0 = M_2 \frac{dV_2(t)}{dt} + K_2 \left[\int_0^t (V_2(t) - V_1(t)) dt \right] \quad -⑤$$

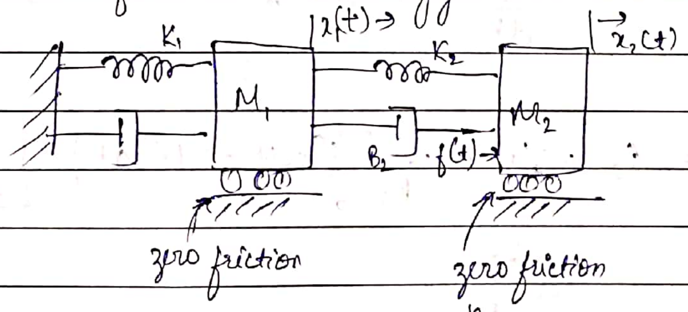
$$0 = C_2 \frac{de_2(t)}{dt} + \frac{1}{L_2} \left[\int_0^t (e_2(t) - e_1(t)) dt \right] \quad -⑥$$

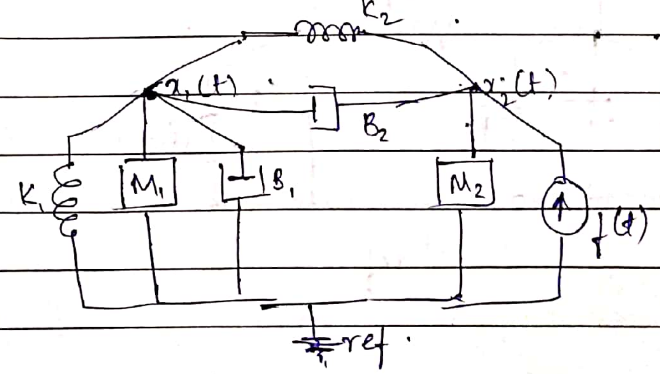
corresponding electrical network can be drawn as shown below using eqn ③ & ⑥



Analogous electrical network (F-I) analogy.

Write the differential eqn. governing the behaviour of mechanical system shown below, and then draw the analogous electrical network based on force-current analogy.





At $x_1(t)$ ①

$$0 = M_1 \frac{d^2x_1(t)}{dt^2} + K_1 \cancel{\frac{d}{dt^2}} x_1(t) + B_1 \frac{dx_1(t)}{dt} + K_2 \left[x_1(t) - x_2(t) \right] + B_2 \left[\frac{dx_1(t)}{dt} - \frac{dx_2(t)}{dt} \right]$$

$$V_1(t) = \frac{dx_1(t)}{dt} \quad -(a) \qquad V_2(t) = \frac{dx_2(t)}{dt} \quad -(b)$$

$$x_1(t) = \int_0^t V_1(t)\,dt \quad -(c) \qquad x_2(t) = \int_0^t V_2(t)\,dt \quad -(d)$$

$$0 = M_1\frac{dV_1(t)}{dt} + K_1\int_0^t V_1(t)\,dt + B_1 V_1(t) + K_2\left[\int_0^t (V_1(t) - V_2(t))\,dt\right]$$

$$+ B_2\left[V_1(t) - V_2(t)\right] \quad -(2)$$

FI analysis eqn is;

$$0 = C_1\frac{de_1(t)}{dt} + \frac{1}{L_1}\int_0^t e_1(t)\,dt + \frac{1}{R}e_1(t).$$

$$+ \frac{1}{L_2}\left[\int_0^t [e_1(t) - e_2(t)]\,dt\right] \quad -(3).$$

At $x_2(t)$:

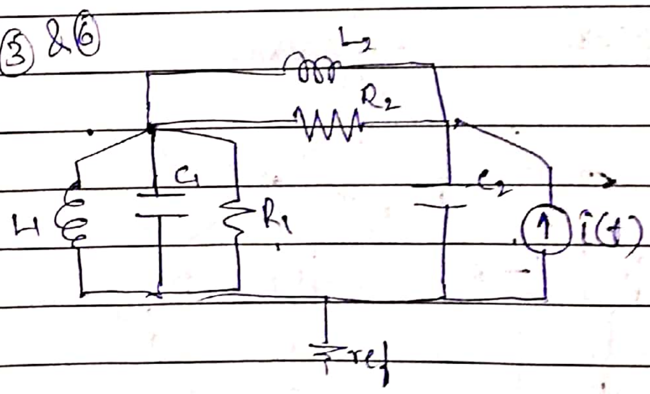
$$f(t) = M_2\frac{d^2x_2(t)}{dt^2} + B_2\left[x_2(t) - x_1(t)\right] + K_2\left[\frac{dx_2(t)}{dt} - \frac{dx_1(t)}{dt}\right]$$

~~FI analysis~~

$$f(t) = M_2\frac{dV_2(t)}{dt} + B_2\int_0^t (V_2(t) - V_1(t))\,dt + K_2\left[V_2(t) - V_1(t)\right]$$

$$i(t) = C_2\frac{de_2(t)}{dt} + \frac{1}{R_2}\left[\int_0^t (e_2(t) - e_1(t))\,dt\right] + \frac{1}{L_2}\left[e_2(t) - e_1(t)\right] \quad (6)$$

From (3) & (6)



Q. Get the corresponding network based on F-V analogy (for the same example as before).

Consider eqn (2) & (5)

$$0 = M_1 \frac{d v_1(t)}{dt} + K_1 \int_0^t v_1(t)\,dt + B_1 v_2(t) + K_2 \left[\int_0^t (v_1(t) - v_2(t))\,dt\right]$$

$$+ B_2 \left[v_1(t) - v_2(t)\right]$$

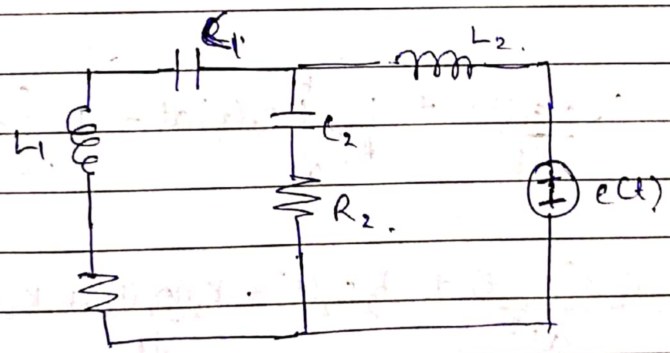
$$0 = \cancel{M_1} L_1 \frac{d i_1(t)}{dt} + \frac{1}{C_1}\int_0^t i_1(t)\,dt + R\, i_2(t) + \frac{1}{C_2}\left[\int_0^t (i_1(t) - i_2(t))\,dt\right]$$

$$+ R_2 \left[i_1(t) - i_2(t)\right] \quad - (7)$$

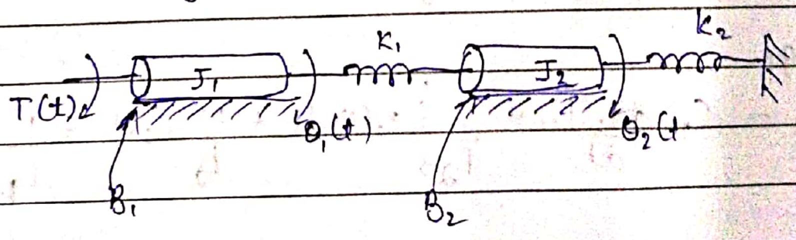
$$f(t) = M_2 \frac{d v_2(t)}{dt} + B_2 \int_0^t v_2(t) - v_1(t)\,dt + K_2\left[v_2(t) - v_1(t)\right]$$

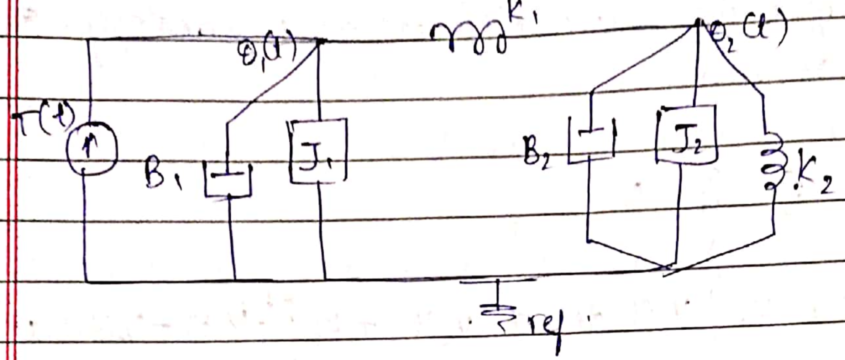
$$e(t) = \frac{L_2 d i_2(t)}{dt} + R_2 \int_0^t i_2(t) - i_1(t)\,dt + \frac{1}{C_2}\left[i_2(t) - i_1(t)\right] \quad - (8)$$

Using (7) & (8).



Q Draw the electrical network using T-I analogy as well as T-V analogy for the following rotational system.





At $\theta_1(t)$

$$T(t) = J_1 \frac{d^2\theta_1(t)}{dt^2} + B_1\theta_1(t) + K_1\left[\frac{d\theta_1(t)}{dt} - \frac{d\theta_2(t)}{dt}\right] \quad -①$$

$$\frac{d\theta_1(t)}{dt} = \omega_1 \quad -ⓐ \qquad \omega_2 = \frac{d\theta_2(t)}{dt} \quad -ⓑ$$

$$\theta_1(t) = \int_0^t \omega_1 dt \quad -ⓒ \qquad \theta_2(t) = \int_0^t \omega_2(t)\,dt \quad -ⓓ$$

Torque-velocity equation,

$$T(t) = J_1 \frac{d\omega_1(t)}{dt} + B_1\int_0^t \omega(t)dt + K_1\left[\omega_1(t) - \omega_2(t)\right] \quad -②$$

T–I analogy,

$$i(t) = C_1 \frac{de_1(t)}{dt} + \frac{1}{R_1}\int_0^t e_1(t)\,dt + \frac{1}{L_1}\left[e_1(t) - e_2(t)\right] \quad -③$$

At $\theta_2(t)$

$$0 = J_2 \frac{d^2\theta_2(t)}{dt^2} + B_2\theta_2(t) + K_2\frac{d\theta_2(t)}{dt} + K_1\left[\frac{d\theta_2(t)}{dt} - \frac{d\theta_1(t)}{dt}\right]$$

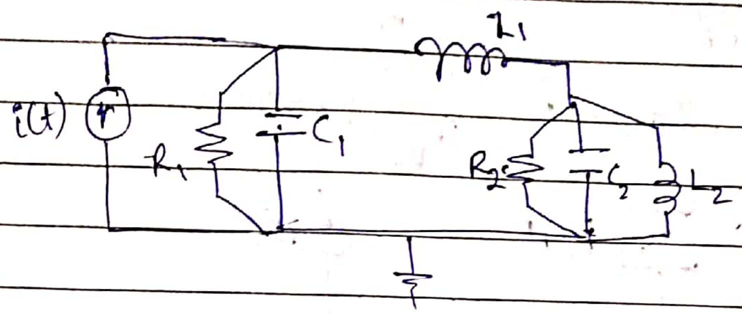
Torque velocity equation.

$$0 = J_2 \frac{d\omega_2(t)}{dt} + B_2\int_0^t \omega_2(t)\,dt + K_2\,\omega_2(t) + K_1\left[\omega_2(t) - \omega_1(t)\right] \quad ⑤$$

T–I analogy

$$0 = C_2 \frac{de_2(t)}{dt} + \frac{1}{R_2}\int_0^t e_2(t)\,dt + \frac{1}{L_2} e_2(t) + \frac{1}{L_1}\left[e_2(t) - e_1(t)\right] \quad ⑥$$

From (3) & (6)



Considering T-V analogy.

consider equation (2) & (5).

$$T(t) = J_1 \frac{d\omega_1(t)}{dt} + B_1 \int_0^t \omega_1(t)\,dt + K_1\left[\omega_1(t) - \omega_2(t)\right]$$

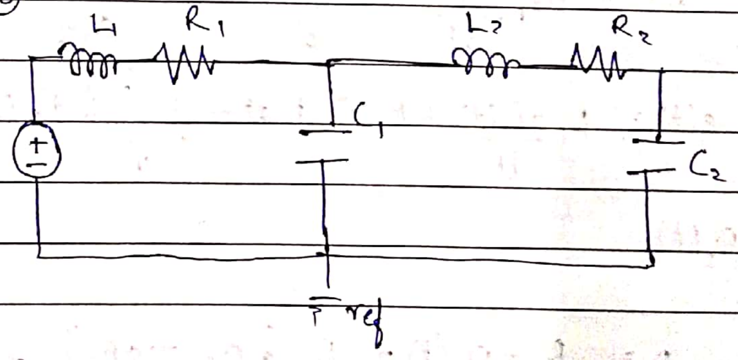
T-V analogy

$$e(t) = L_1 \frac{di_1(t)}{dt} + R_1 \int_0^t i_1(t)\,dt + \frac{1}{C_1}\left[i_1(t) - i_2(t)\right] \quad (7)$$

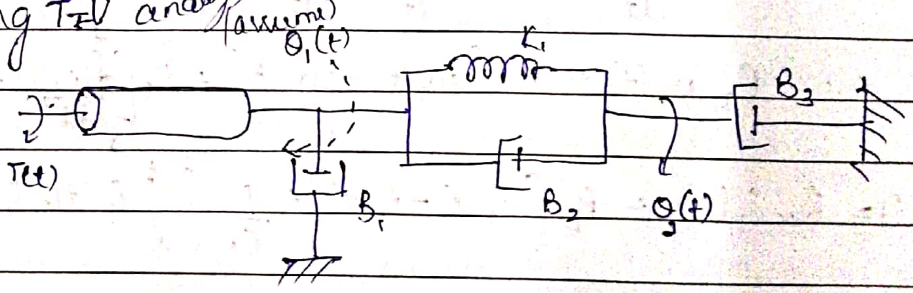
$$0 = J_2 \frac{d\omega_2(t)}{dt} + B_2 \int_0^t \omega_2(t)\,dt + K_2\,\omega_2(t) + K_1\left[\omega_2(t) - \omega_1(t)\right]$$

$$0 = L_2 \frac{di_2(t)}{dt} + R_2 \int_0^t i_2(t)\,dt + \frac{1}{C_2} i_2(t) + \frac{1}{C_1}\left[i_2(t) - i_1(t)\right] \quad (8)$$

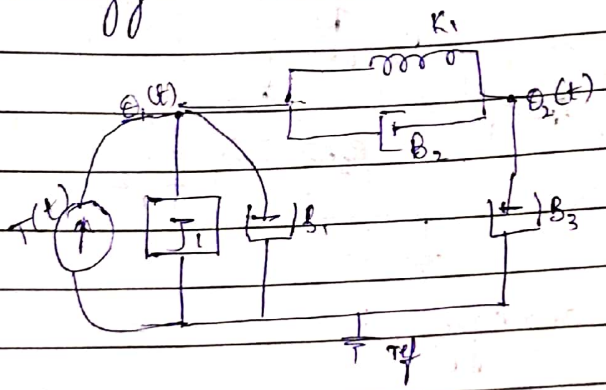
Using (7) & (8)



Q Using T-V analogy



TV analogy



At $\theta_1(t)$

$$T(t) = J_1\frac{d^2\theta_1(t)}{dt^2} + B_1\frac{d\theta_1(t)}{dt} + K_1\left[\theta_1(t) - \theta_2(t)\right] + B_2\left[\frac{d\theta_1(t)}{dt} - \frac{d\theta_2(t)}{dt}\right] \quad (1)$$

$$\frac{d\theta_1(t)}{dt} = \omega_1 \quad \text{—(a)} \qquad \theta_1(t) = \int_0^t \omega_1(t)\,dt \quad \text{—(b)}$$

$$\frac{d\theta_2(t)}{dt} = \omega_2 \quad \text{—(c)} \qquad \theta_2(t) = \int_0^t \omega_2(t)\,dt \quad \text{—(d)}$$

$$T(t) = J_1\frac{d\omega_1(t)}{dt} + B_1\omega_1(t) + K_1\int_0^t\left[\omega_1(t) - \omega_2(t)\right] + B_2\left[\omega_1(t) - \omega_2(t)\right] \quad (2)$$

TV analogy,

$$e(t) = L_1\frac{di_1(t)}{dt} + R_1 i_1(t) + \frac{1}{C_1}\int_0^t\left[i_1(t) - i_2(t)\right] + R_2\left[i_1(t) - i_2(t)\right]$$

$$e(t) = L_1\frac{di_1(t)}{dt} + R_1 i_1(t) + \frac{1}{C_1}\int_0^t (i_1(t) - i_2(t))\,dt + R_2(i_1(t) - i_2(t)) \quad (3)$$

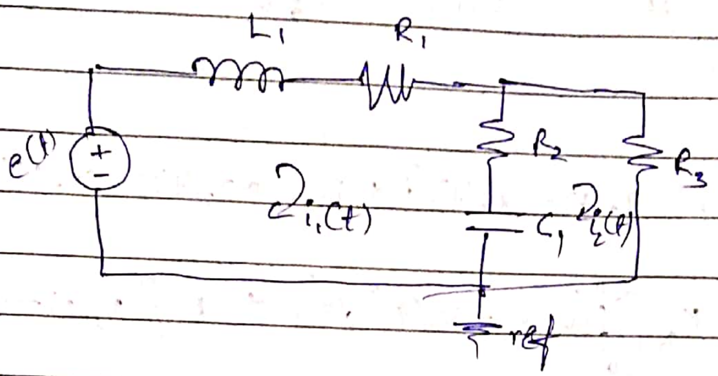
At $\theta_2(t)$

$$0 = B_3\frac{d\theta_2(t)}{dt} + B_2\left(\frac{d\theta_2}{dt} - \frac{d\theta_1}{dt}\right) + K_1(\theta_2 - \theta_1) \quad (4)$$

$$0 = B_3\,\omega_2(t) + B_2\left[\omega_2(t) - \omega_1(t)\right] + K_1\left[\int_0^t \omega_1(t) - \omega_2(t)\,dt\right] \quad (5)$$

$$0 = R_3 i_2(t) + R_2\left[i_2(t) - i_1(t)\right] + \frac{1}{C_1}\left[\int_0^t i_2(t) - i_1(t)\,dt\right] \quad (6)$$

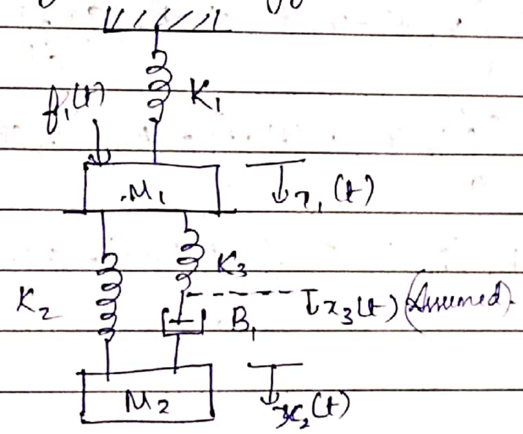
Using ③ & ⑥ equation,

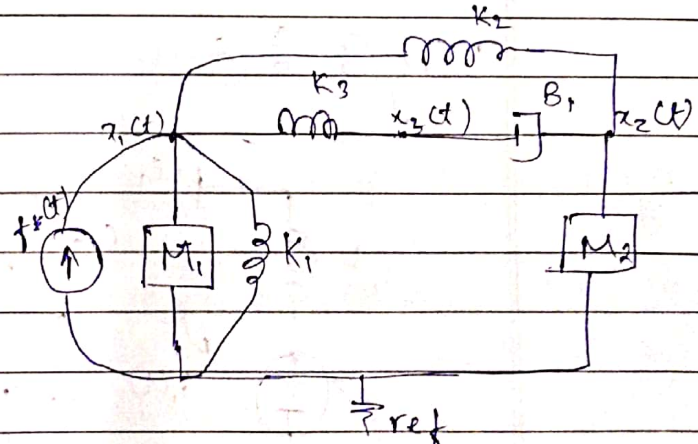


Analogous electrical network.

Q For the same circuit use T-I analogy, Design an analogous e.network.

Q. Obtain the analogous network for the following mechanical system using F-V analogy.





→ At $x_1(t)$,

$$f(t) = M_1 \frac{d^2x_1(t)}{dt^2} + K_1 x_1(t)$$
$$+ K_3 [x_1(t) - x_3(t)]$$
$$+ K_2 [x_1(t) - x_2(t)] \quad \text{— ①}$$

$$v_1(t) = \frac{dx_1(t)}{dt} \qquad v_2(t) = \frac{dx_2(t)}{dt}$$

$$x_1(t) = \int_0^t v_1(t)dt \qquad x_2(t) = \int_0^t v_2(t)\,dt.$$

$$\therefore \quad f(t) = M_1 \frac{dv_1(t)}{dt} + K_1 \int_0^t v_1(t)\,dt + K_3 \left[\int_0^t v_1(t) - v_3(t)\right] + K_2 \left[\int_0^t v_1(t) - v_2(t)\,dt\right]$$

②

$$e(t) = L_1 \frac{d i_1(t)}{dt} + \frac{1}{C_1}\int_0^t i_1(t)dt + \frac{1}{C_3}\int_0^t i_1(t) - i_3(t)\,dt + \frac{1}{C_2}\int_0^t i_1(t) - i_2(t)\,dt \quad - (3)$$

At $x_3(t)$,

$$0 = K_3[x_3(t) - x_1(t)] + B_1\left[\frac{dx_3(t)}{dt} - \frac{dx_2(t)}{dt}\right] \quad - (4)$$

$$0 = K_3\left[\int_0^t (V_3(t) - V_1(t))dt\right] + B_1\left[V_3(t) - V_2(t)\right] \quad - (5)$$

$$0 = \frac{1}{C_3}\left[\int_0^t i_3(t) - i_1(t)\,dt\right] + R_1\left[i_3(t) - i_2(t)\right] \quad - (6)$$

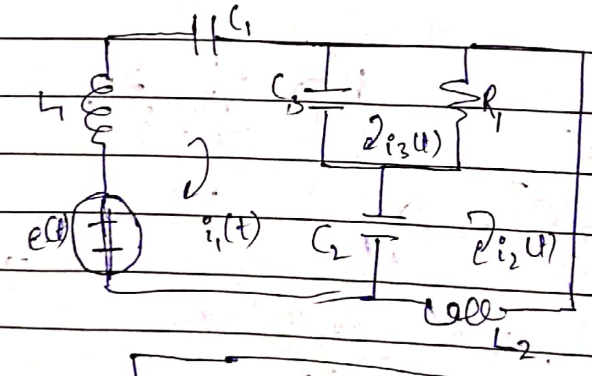
At $x_2(t)$

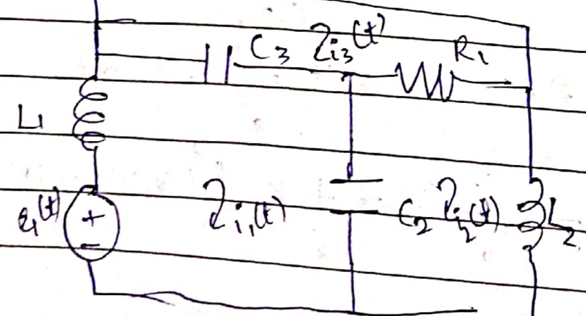
$$0 = M_2 \frac{d^2 x_2(t)}{dt^2} + K_2\left[x_2(t) - x_1(t)\right] + B_1\left[\frac{dx_2(t)}{dt} - \frac{dx_3(t)}{dt}\right] \quad - (7)$$

$$= M_2 \frac{dV_2(t)}{dt} + K_2\left[\int_0^t V_2(t) - V_1(t)dt\right] + B_1\left[V_2(t) - V_3(t)\right] \quad - (8)$$

$$= L_2 \frac{di_2(t)}{dt} + R_1(i_2 - i_3) + \frac{1}{C_2}\int_0^t i_2(t) - i_1(t)\,dt \quad - (9)$$

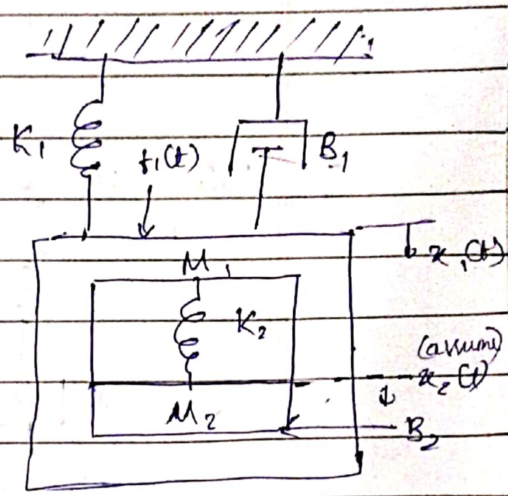
Using (3), (6), (9)



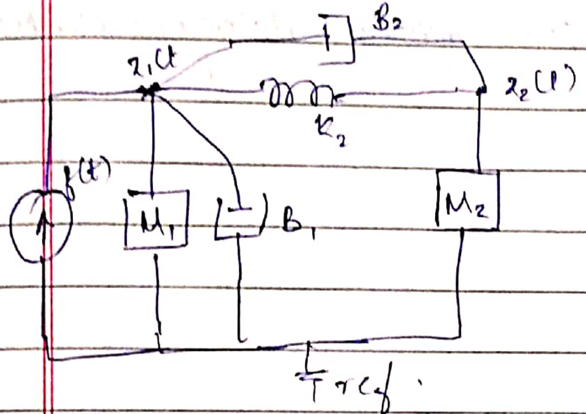


Q. Using FI analogy, solve the above examples.

Q. Draw the mechanical network, mathematical model & analogous ele. network based on F-V analogy for the following mechanical system.



→ Mechanical network,



At $x_1(t)$

$$f(t) = M_1 \frac{d^2x_1(t)}{dt^2} + B_1\left[\frac{dx_1(t)}{dt}\right] + B_2\left[\frac{dx_1(t)}{dt} - \frac{dx_2(t)}{dt}\right] + K_2\left[x_1(t)-x_2(t)\right] \quad (1)$$

$$v_1 = \frac{dx_1(t)}{dt} \qquad v_2 = \frac{dx_2(t)}{dt}$$

$$x_1(t) = \int_0^t v_1(t)\,dt \qquad x_2(t) = \int_0^t v_2(t)\,dt.$$

$$f(t) = M_1 \frac{dv_1(t)}{dt} + B_1 v_1(t) + B_2\left[v_1(t) - v_2(t)\right] + K_2\int_0^t (v_1(t)-v_2(t))\,dt \quad (2)$$

$$e(t) = L_1 \frac{di_1(t)}{dt} + R_1 i_1(t) + R_2\left[i_1(t)-i_2(t)\right] + \frac{1}{C_2}\int_0^t i_1(t)-i_2(t)\,dt \quad (3)$$

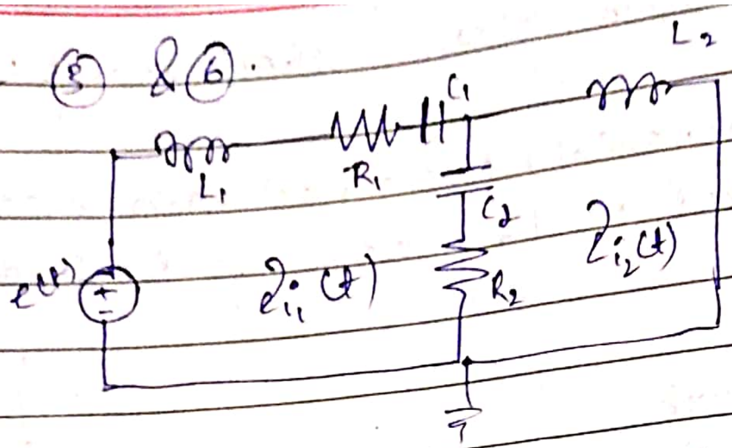
At $x_2(t)$

$$0 = M_2 \frac{d^2x_2(t)}{dt^2} + K_2\left[x_2(t)-x_1(t)\right] + B_2\left[\frac{dx_2(t)}{dt} - \frac{dx_1(t)}{dt}\right] \quad (4)$$

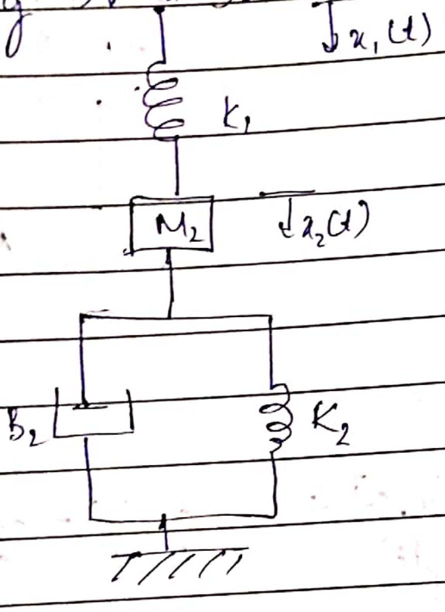
$$= M_2 \frac{dv_2(t)}{dt} + K_2\left[\int_0^t v_2(t)-v_1(t)\,dt\right] + B_1\left[v_2(t)-v_1(t)\right] \quad (5)$$

$$= \cancel{M_2}\; L_2 \frac{di_2(t)}{dt} + C_2\int_0^t i_2(t)-i_1(t)\,dt + R_2\left[i_2(t)-i_1(t)\right] \quad (6)$$

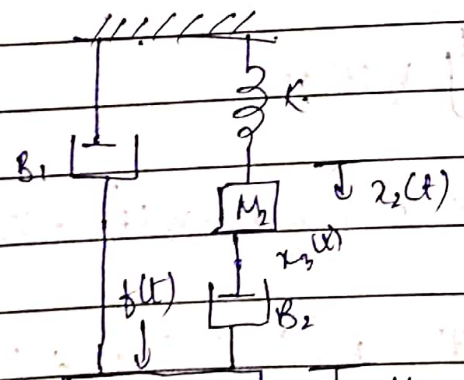
Using (5) & (6).



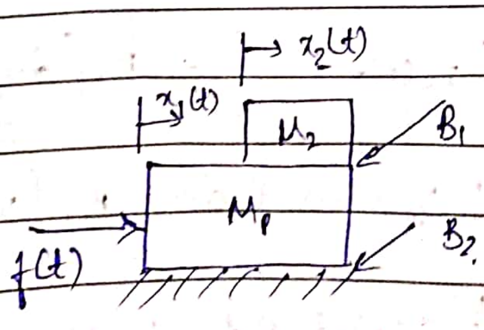
Q. Using FV & FI analogy, draw equivalent analogous ele. netw

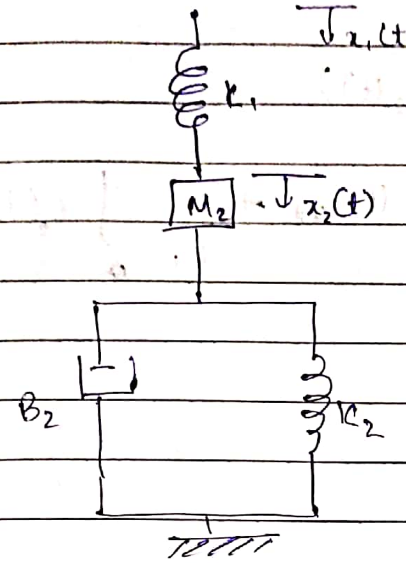


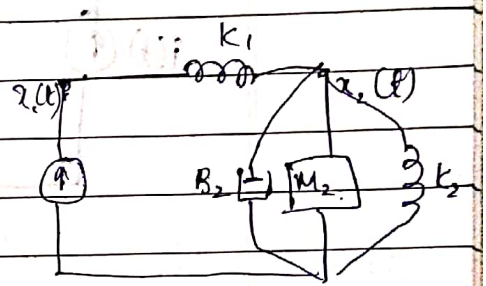
Q.



Q.







At $x_1(t)$;

$$f(t) = K_1(x_1(t) - x_2(t)) \quad - (1)$$

$$e(t) = \frac{1}{C_1}\left[\int_0^t i_1(t) - i_2(t)\, dt\right] \quad - (2)$$

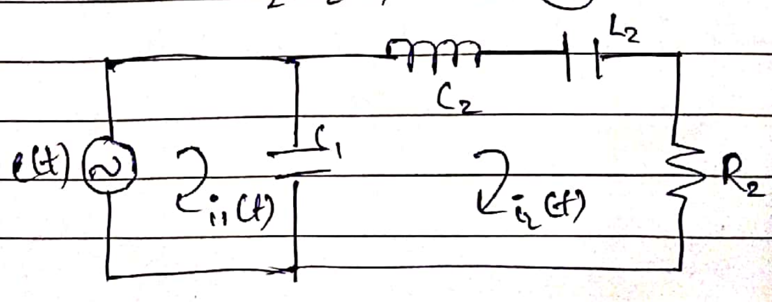
$$i(t) = \frac{1}{L_1}\left[\int_0^t e_1(t) - e_2(t)\, dt\right] \quad - (3)$$

at $x_2(t)$

$$0 = M_2 \frac{d^2 x_2(t)}{dt^2} + K_2 x_2(t) + B_2 \, dx_2(t) + K_1\left[x_2(t) - x_1(t)\right]$$

$$0 = M_2 \frac{dv_2(t)}{dt} + K_2 \int_0^t v_2(t)\, dt + K_1 \int v_2(t) - v_1(t)$$
$$+ B_2 v_2(t) \quad - (5)$$

$$0 = L_2 \frac{di_2(t)}{dt} + \frac{1}{C_2}\int_0^t i_2(t)\, dt + \frac{1}{C_1}\int i_2(t) - i_1(t)\, dt$$
$$+ R_2 i_2(t) \quad - (6)$$



classmate

Date

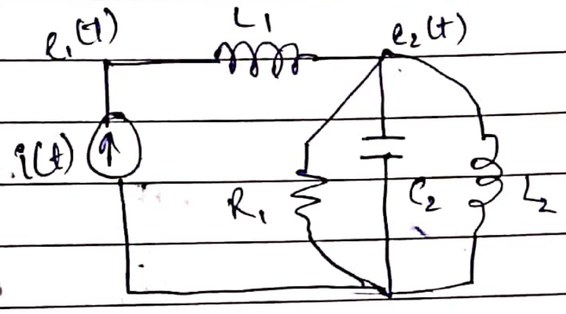
Page

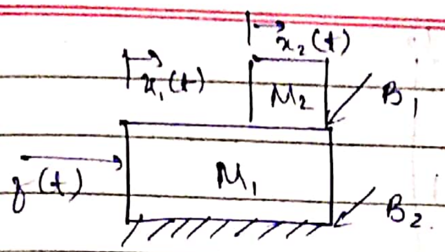
FI analogy

$$i(t) = \frac{1}{L_1}\int_0^t (e_1(t) - e_2(t))$$

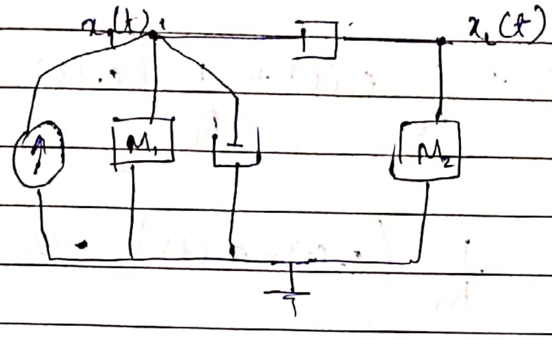
$$0 = C_2 \frac{de_2(t)}{dt} + \frac{1}{L_2}\int_0^t e_2(t)\,dt + \frac{1}{L_1}\int e_2(t) - e_1(t)\,dt$$

$$+ \frac{1}{R_2} e_2(t)$$





Mechanical network



At node $x_1(t)$

$$f(t) = M_1 \frac{d^2 x_1(t)}{dt^2} + B_2 \frac{dx_1(t)}{dt} + B_1\left[\frac{dx_1(t)}{dt} - \frac{dx_2(t)}{dt}\right] \quad -(1)$$

$$f(t) = M_1 \frac{dv_1(t)}{dt} + B_2 v_1(t) + B_1\left(v_1(t) - v_2(t)\right) \quad -(2)$$

F-V analysis set

$$e(t) = L_1 \frac{di_1(t)}{dt} + R_2 i_1(t) + R_1\left[i_1(t) - i_2(t)\right] \quad -(3)$$

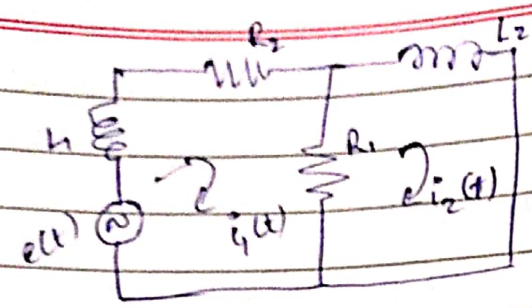
At node $x_2(t)$

$$0 = M_2 \frac{d^2 x_2(t)}{dt^2} + B_1\left[\frac{dx_2(t)}{dt} - \frac{dx_1(t)}{dt}\right] \quad -(4)$$

$$0 = M_2 \frac{dv_2(t)}{dt} + B_1\left[v_2(t) - v_1(t)\right] - (5)$$

FV analysis set

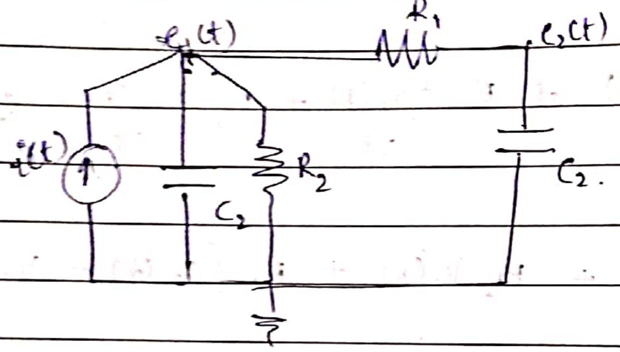
$$0 = L_2 \frac{di_2(t)}{dt} + R_1\left(i_2(t) - i_1(t)\right) - (6)$$



FI .

$$i(t) = C_1 \frac{de_1(t)}{dt} + \frac{1}{R_2} e_1(t) + \frac{1}{R_1}\left[e_1(t) - e_2(t)\right] \quad (7)$$

$$0 = C_2 \frac{de_2(t)}{dt} + \frac{1}{R_1}\left(e_2(t) - e_1(t)\right) \quad — (8)$$

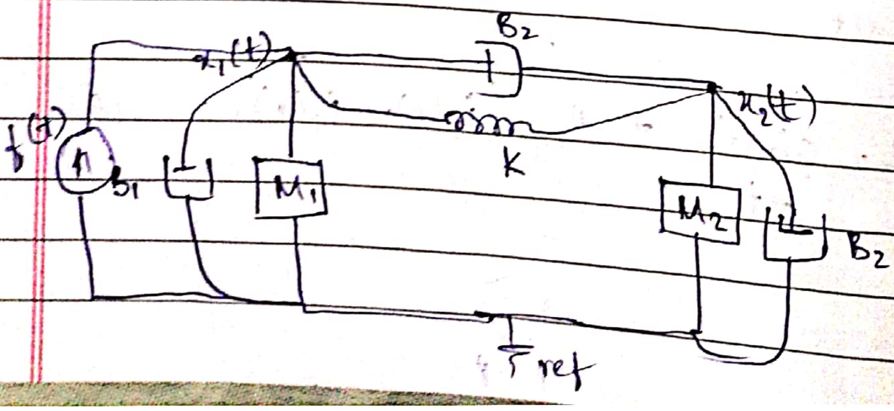


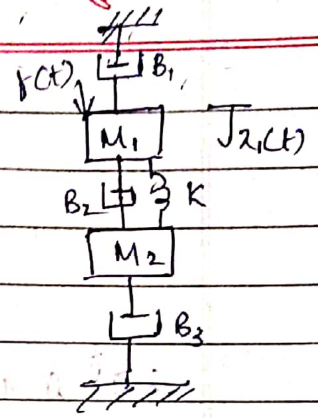
Not in syllabus

Q. Draw the mechanical network, mechanical system & for the mathematical model given below.

$$f(t) = M_1 \frac{d^2x_1(t)}{dt^2} + B_1 \frac{dx_1(t)}{dt} + B_2\left(\frac{dx_1(t)}{dt} - \frac{dx_2(t)}{dt}\right) + K\left(x_1(t) - x_2(t)\right) \quad — (1)$$

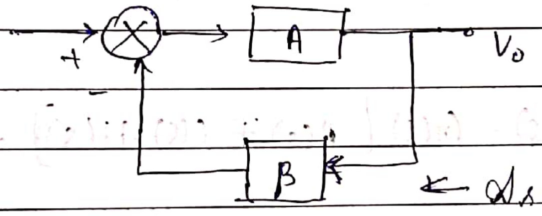
$$0 = M_2 \frac{d^2x_2(t)}{dt^2} + B_3 \frac{dx_2(t)}{dt} + B_2\left(\frac{dx_2(t)}{dt} - \frac{dx_1(t)}{dt}\right) + K\left(x_2(t) - x_1(t)\right) \quad — (2)$$





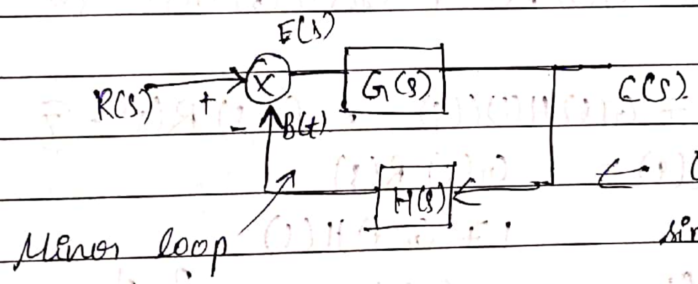
Chapter 2
Block diagram & signal flow graphs:

Block diagram is a pictorial representation of the given system in a simple way, without showing the details of the complicated circuit / system at each point.



$$A_f = \frac{A}{1+A\beta}$$

← A simple feedback amplifier.



← General block diagram of a simple closed loop system.

Referring to the block diagram, given in fig(2), we can say that the following are the basic elements to represent a closed loop system in terms of blocks.

i) Blocks
ii) Transfer function inside the block.
iii) summary points
iv) Take off points
v) Arrows.

Referring to fig (2)

$R(s) \longrightarrow r(t)$ (R(s) is the laplace transform of r(t))
$E(s) \longrightarrow e(t)$ (error)
$B(s) \longrightarrow b(t)$ (feedback)
$C(s) \longrightarrow c(t)$ (output)
$G(s) \longrightarrow g(t)$

G(s) is the transfer function of forward path & H(s) is of feedback path.

$$E(s) = R(s) \mp B(s) \quad -(1)$$
$$C(s) = G(s) \cdot E(s) \quad -(2)$$
$$B(s) = C(s) \cdot H(s) \quad -(3)$$

sub (1) in (2)

$$C(s) = G(s)\left[R(s) \mp B(s)\right] \quad -(4)$$

sub (3) in (4).

$$C(s) = G(s)\left[R(s) \mp C(s) \cdot H(s)\right] \quad -(5)$$

$$C(s) = G(s)R(s) \mp G(s)C(s)H(s) \quad -(6)$$

$$C(s) \mp G(s)H(s)C(s) = G(s)R(s) \quad -(7)$$

$$C(s) = \frac{G(s)R(s)}{1 \mp G(s)H(s)}$$

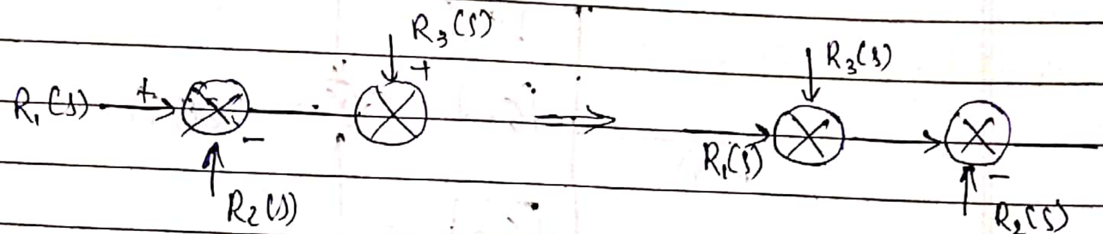
$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 \mp G(s)H(s)}$$ } Transfer function (standard).

Note : Here for -ve feedback it is $1 + G(s)H(s)$ and for +ve feedback is $1 - G(s)H(s)$.

Block diagram reduction rules:

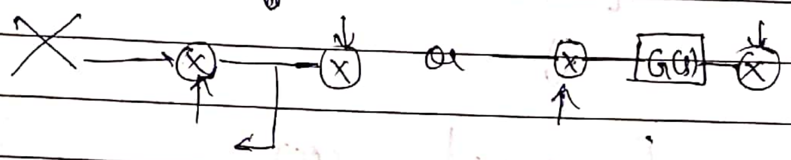
• To reduce the given complicated block into its ... and to determine the overall transfer function, there are 11 rules. Among these 11 rules, 9 rules are usually ... and the other two are critical rules and the application of these two rules has to be avoided as far as possible.

i) Associative law:

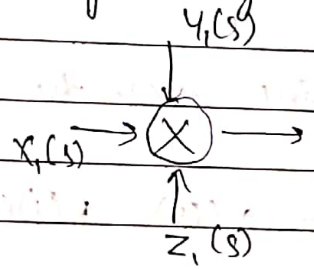


$C(s) = R_1(s) - R_2(s) + R_3(s)$ $\qquad$ $C(s) = R_1(s) + R_3(s) - R_2(s)$,

⇓

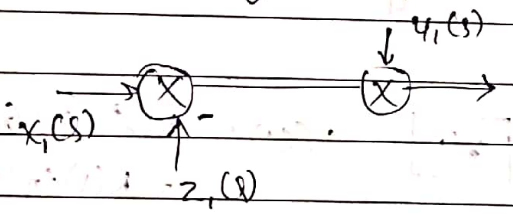


ii) Splitting the given summing point.



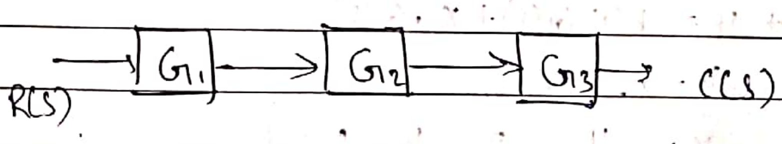
$C(s) = X_1(s) + Y_1(s) + Z_1(s)$

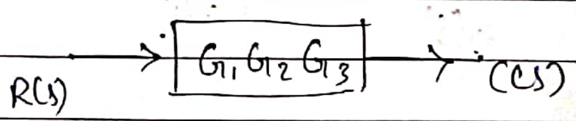
⇓

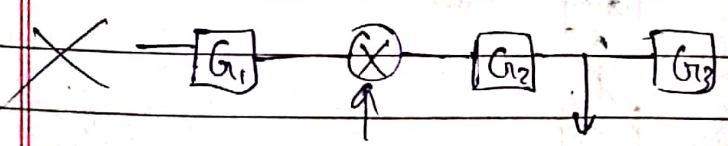


$C(s) = X_1(s) - Z_1(s) + Y_1(s)$.

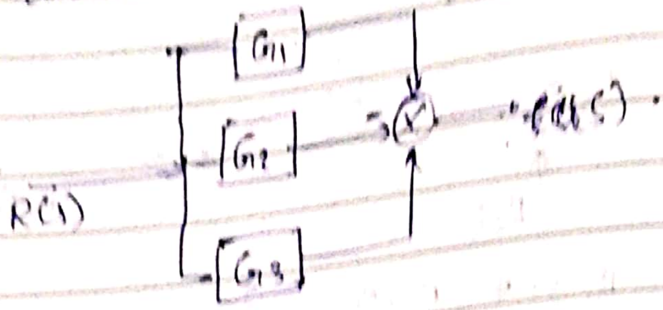
ii) When blocks are connected in series



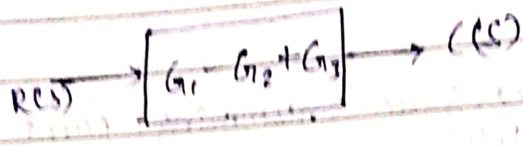




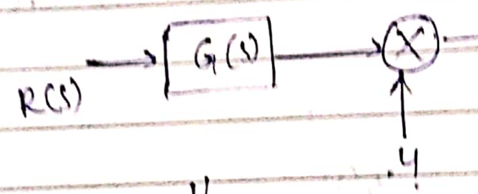
iii) When blocks are connected in parallel



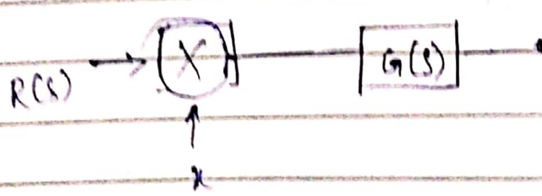
$$C(s) = R(s)\left[G_1 - G_2 + G_3\right]$$



iv) Shifting the summing point before the block.



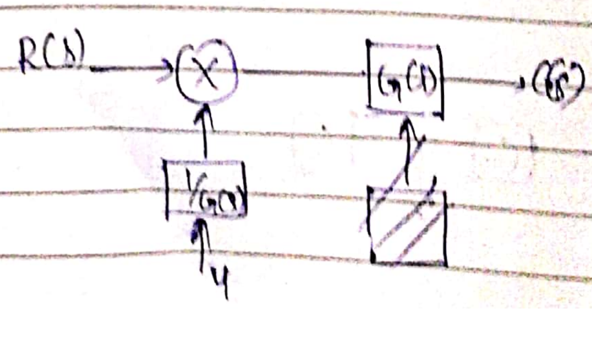
$C(s) = R(s)G(s) + y$



$C(s) = (R(s) + x)\,G(s)$
$= R(s)G(s) + x \cdot G(s)$

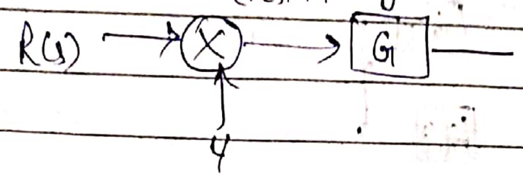
$$R(s)G(s) + y = R(s)G(s) + x \cdot G(s)$$
$$y = x \cdot G(s)$$
$$x = \frac{y}{G(s)} = y \cdot \left(\frac{1}{G(s)}\right)$$

$\therefore$

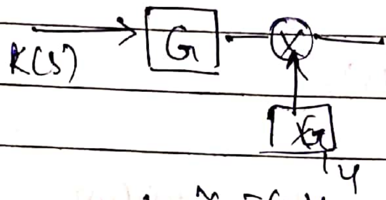


v) Shifting the summing point after the block.

(R(s)+Y)



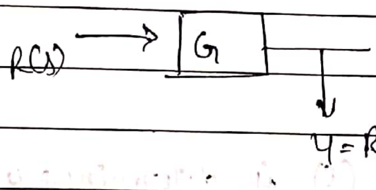
$C(s) = (R(s)+Y)\,G = GY + R(s)G.$



$C(s) = R(s)G + X.$

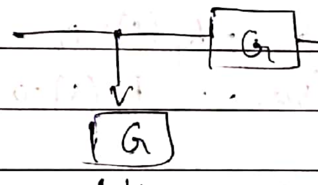
$\therefore X = GY.$

vi) Shifting the take-off point before the block.



$C(s) = R(s)\cdot G$

⇓

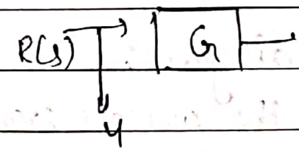


$C(s) = R(s)G.$

$Y = R(s)G.$

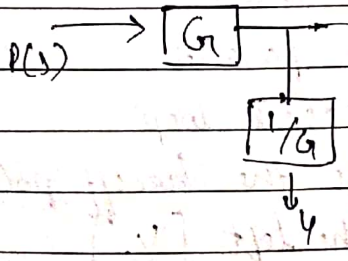
(It is same as shifting summing point after the block)

vii) Shifting the take-off point after the block.



$C(s) = R(s)\cdot G$

⇓

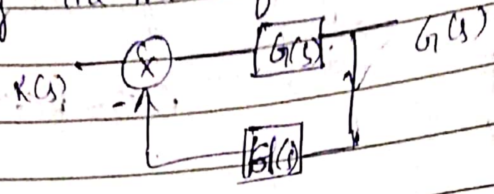


$C(s) = R(s)\cdot G$

$= R(s)\,G \times \frac{1}{G} = R(s)$

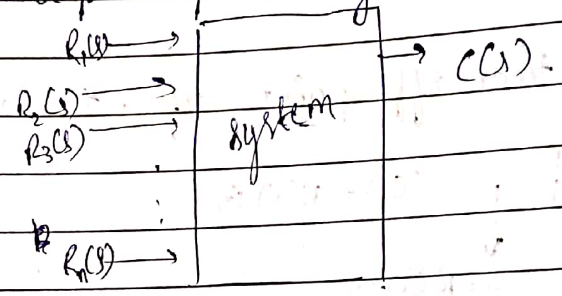
(It is same as shifting summing point before the block)

viii) Removing the minor feedback loop



$$C(s) = R(s)\left[\frac{G(s)}{1+G(s)H(s)}\right]$$

ix) For multiple input single output system



For the above system output C(s) is determined using superposition principle.

Let $C_1(s) \rightarrow R_1(s)$; $R_2(s) = R_3(s) = \dots R_n(s) = 0$

$C_2(s) \rightarrow R_2(s)$; $R_1(s) = R_3(s) = \dots R_n(s) = 0$

⋮

$C_n(s) \rightarrow R_n(s)$; $R_1(s) = R_2(s) = R_3(s) \dots = 0$

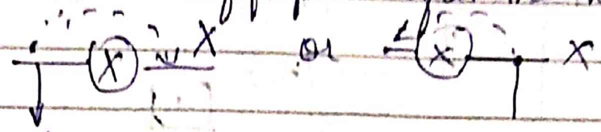
$\therefore C(s) = C_1(s) + C_2(s) + C_3(s) + \dots + C_n(s)$

Procedure to reduce a block diagram:

Solve programs to determine the overall transfer function.

i) Reduce the blocks connected in series
ii) Reduce the blocks connected in parallel
iii) Reduce the minor internal feedback loop
iv) As far as possible try to shift the take off points towards right & summing points to the left. $\left[\frac{1}{G}\right]$

v) Never shift the take off point after the summing point & vice-versa.

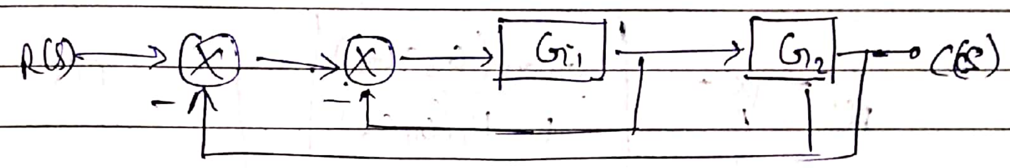


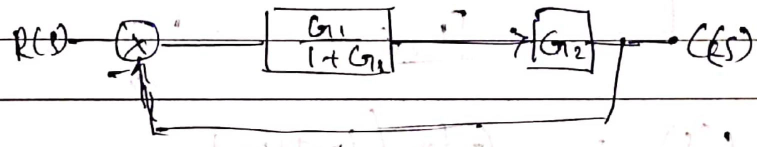
vi) Repeat the steps from 1 to 4 till a simple loop is obtained.

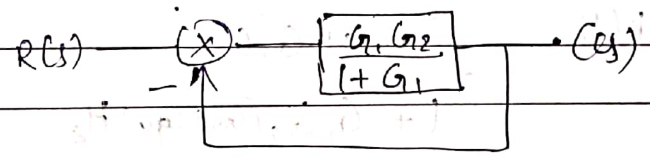
vii) Using the standard transfer function, determine overall transfer function of the given block diagram.

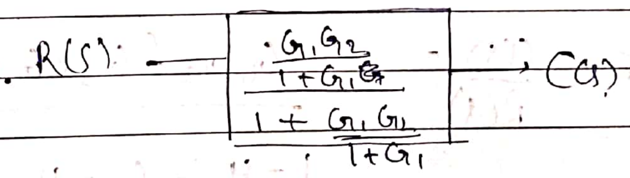
1. Determine the overall T.F for the following block diagram using block diagram reduction techniques.

1)



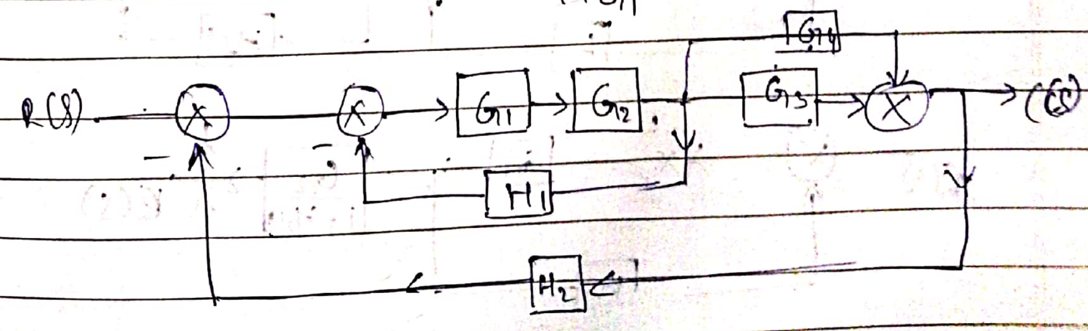


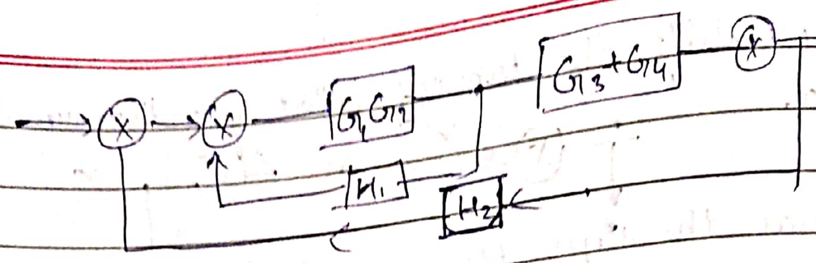


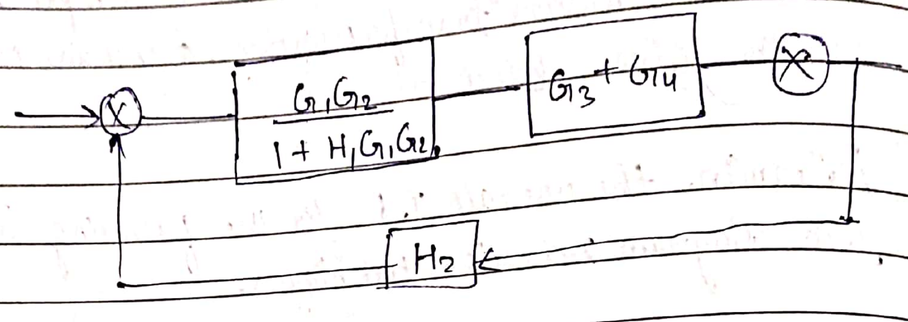


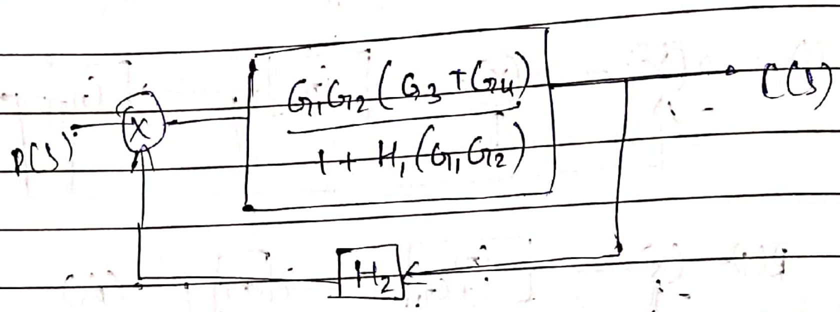
$$\frac{C(s)}{R(s)} = \frac{\dfrac{G_1 G_2}{1+G_1}}{\dfrac{1+G_1G_2+G_1}{1+G_1}} = \frac{G_1 G_2}{1+G_1+G_1G_2}$$

2)





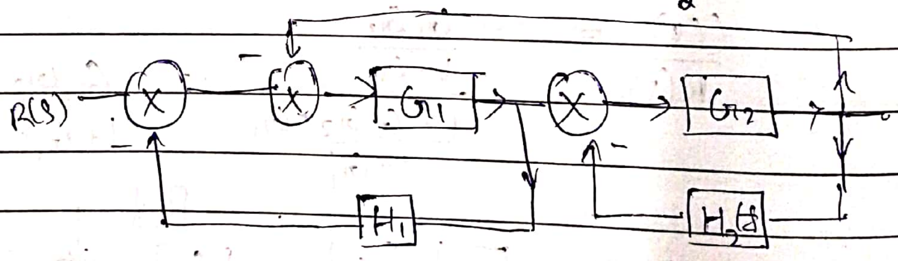


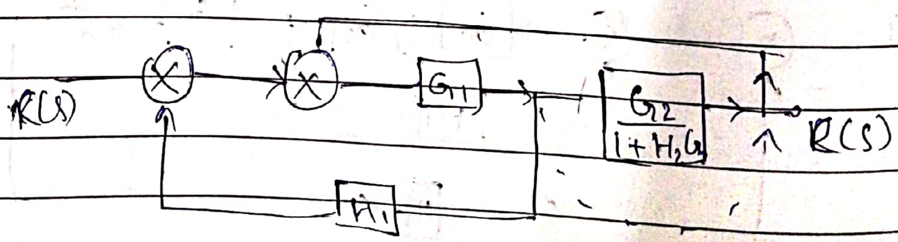


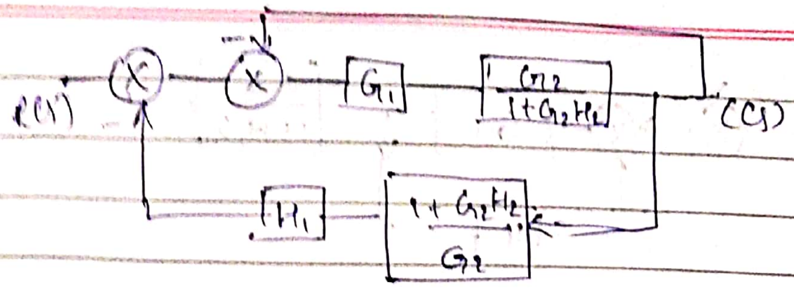
$$\frac{C(s)}{R(s)} = \frac{\dfrac{G_1G_2(G_3+G_4)}{1+H_1G_1G_2}}{1+\dfrac{G_1G_2(G_3+G_4)H_2}{1+H_1G_1G_2}}$$

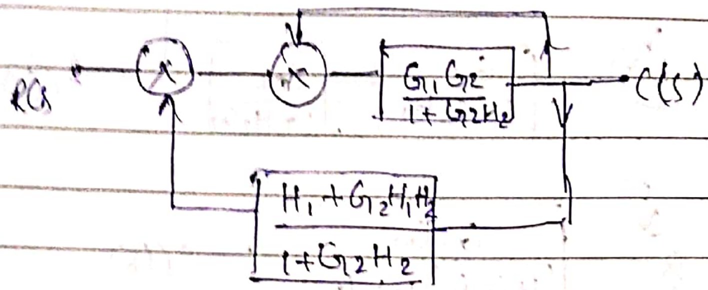
$$TF = \frac{C(s)}{R(s)} = \frac{G_1G_2(G_3+G_4)}{1+H_1G_1G_2+H_2G_1G_2(G_3+G_4)}$$

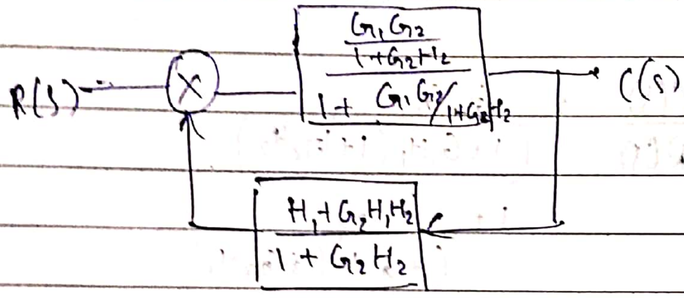
3)











$$\frac{C(s)}{R(s)} = \frac{\frac{G_1G_2}{1+G_2H_2}}{1+\frac{G_1G_2}{1+G_2H_2}} = \frac{G_1G_2}{1+G_2H_2+G_1G_2}$$

$$\cancel{1+H_1}\; 1+\frac{H_1+G_2H_1H_2}{(1+G_2H_2)} \times \frac{G_1G_2}{(1+G_2H_2+G_1G_2)}$$

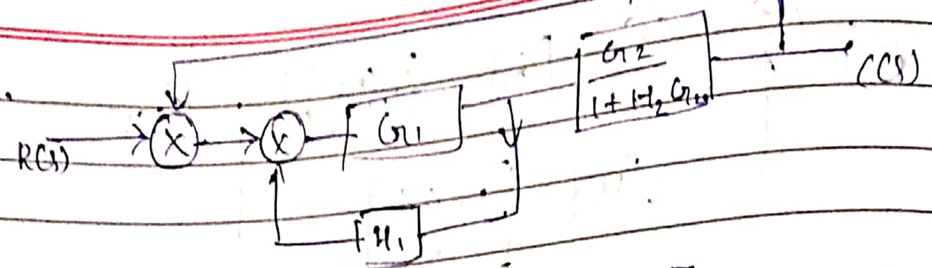
$$\cancel{TF} = \frac{\frac{G_1G_2}{\cancel{1+G_2H_2+G_1G_2}}}{\frac{(1+G_2H_2)(1+G_2H_2+G_1G_2)+(H_1+G_2H_1H_2)(G_1G_2)}{(1+G_2H_2)\cancel{(1+G_2H_2+G_1G_2)}}}$$

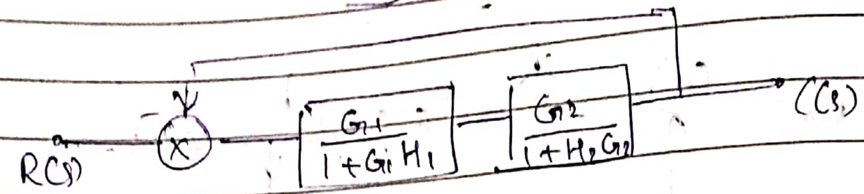
$$= \frac{G_1G_2(1+\cancel{G_2H_2})}{H_1(1+\cancel{G_2H_2})(G_1G_2)+(1+\cancel{G_2H_2})(1+G_2H_2+G_1G_2)}$$

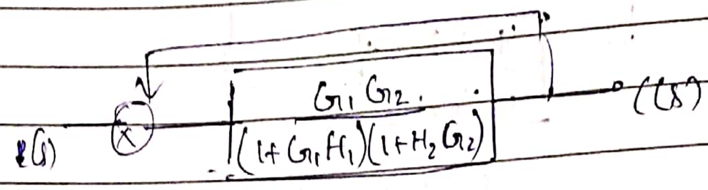
$$= \frac{G_1G_2}{H_1G_1G_2+1+G_1G_2+G_2H_2}$$

$$= \frac{G_1G_2}{1+G_1G_2+H_2G_2+H_1G_1G_2}$$

OR.





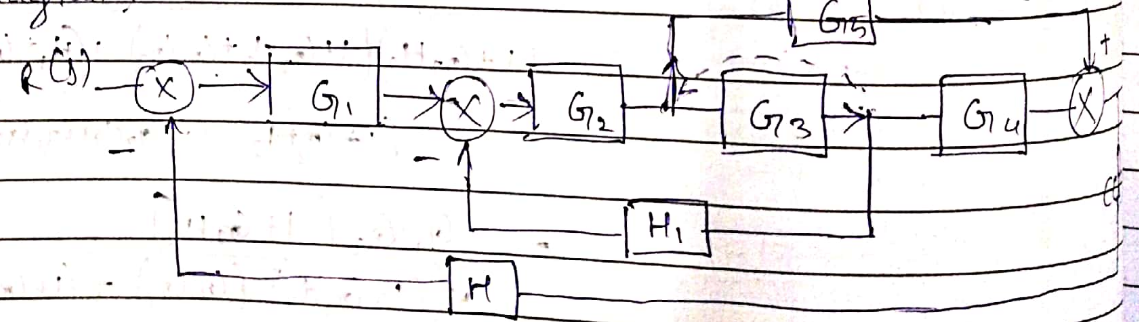


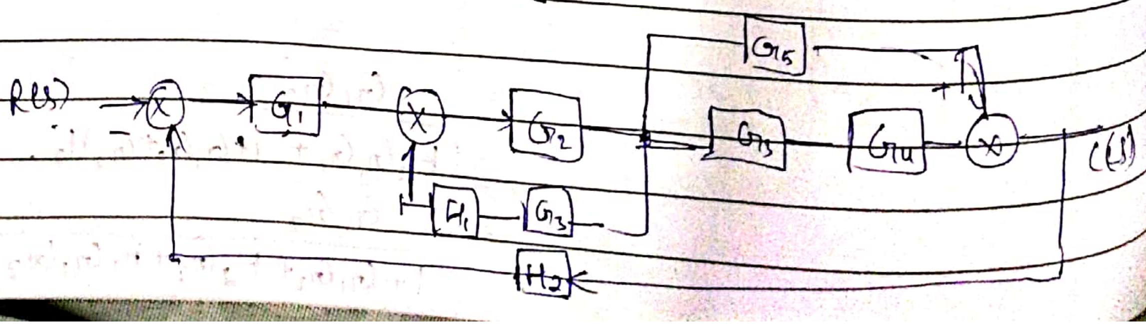
$$\frac{C(s)}{R(s)} = \frac{\dfrac{G_1G_2}{(1+G_1H_1)(1+G_2H_2)}}{1+\dfrac{G_1G_2}{(1+G_1H_1)(1+G_2H_2)}}$$

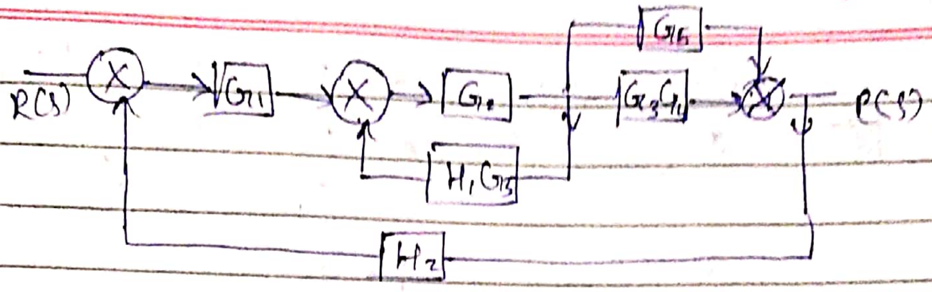
$$= \frac{G_1G_2}{(1+G_1H_1)(1+G_2H_2)+G_1G_2}$$

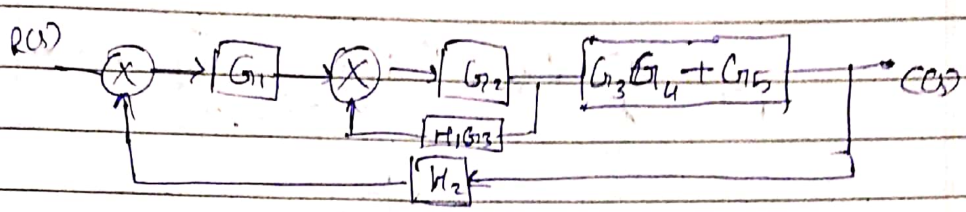
$$\frac{C(s)}{R(s)} = \frac{G_1G_2}{1+G_1H_1+G_2H_2+G_1G_2+G_1G_2H_1H_2}$$

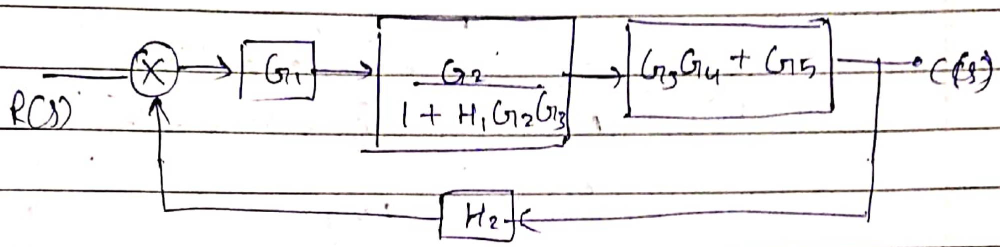
1) Determine the transfer function $\frac{C(s)}{R(s)}$ for the following block diagram.

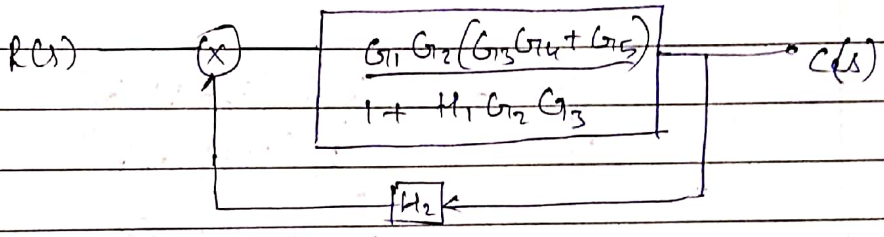








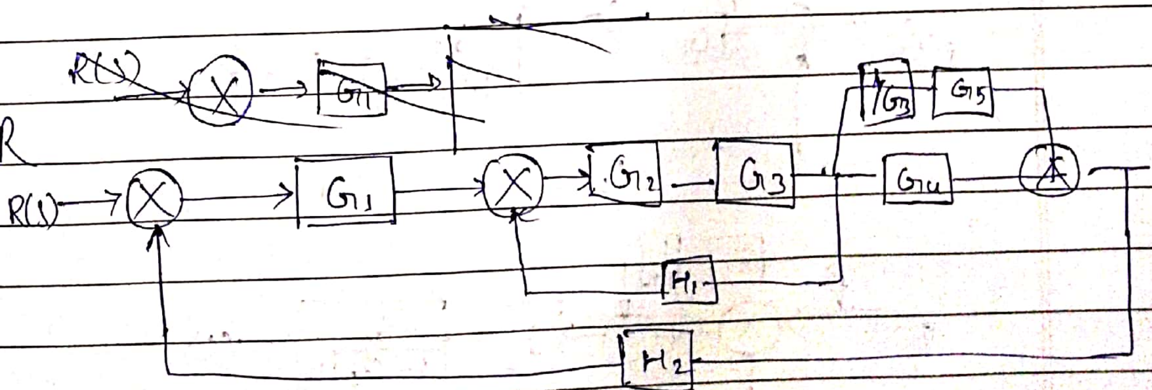


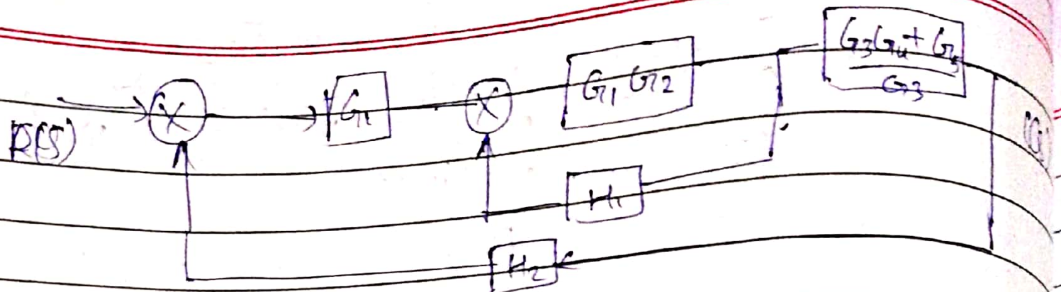


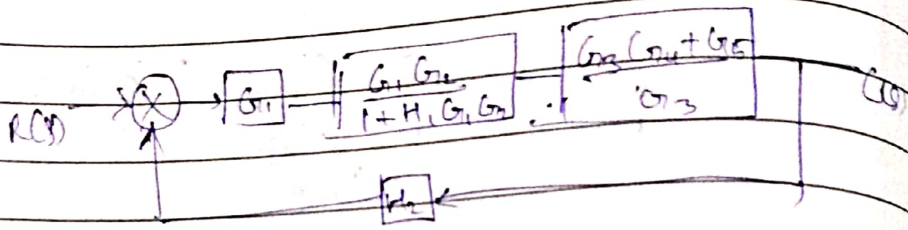
$$\frac{C(s)}{R(s)} = \frac{\dfrac{G_1 G_2 (G_3 G_4 + G_5)}{1 + H_1 G_2 G_3}}{1 + \dfrac{G_1 G_2 (G_3 G_4 + G_5) H_2}{1 + H_1 G_2 G_3}}$$

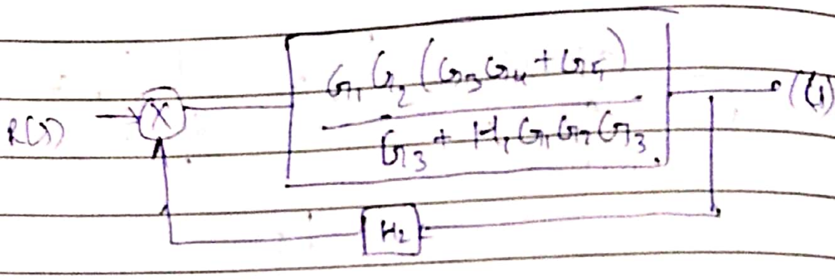
$$\frac{C(s)}{R(s)} = \frac{G_1 G_2 (G_3 G_4 + G_5)}{1 + H_1 G_2 G_3 + H_2 G_1 G_2 (G_3 G_4 + G_5)}$$

OR



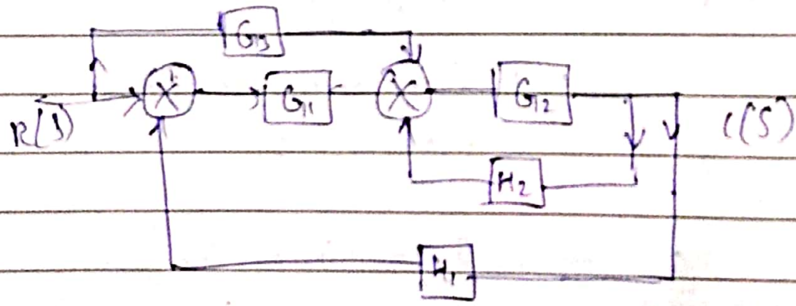


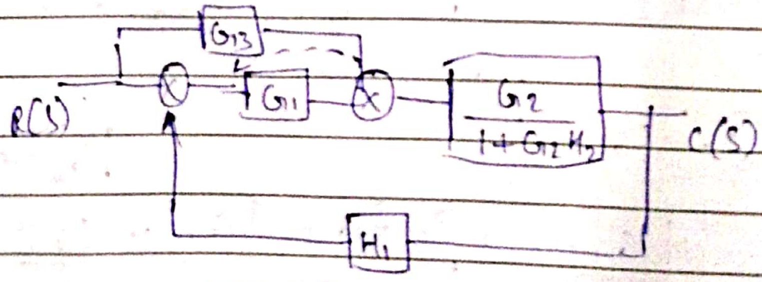


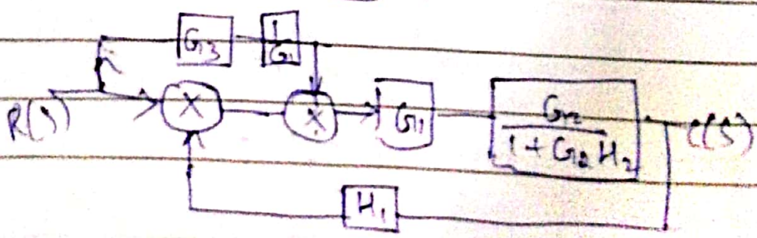


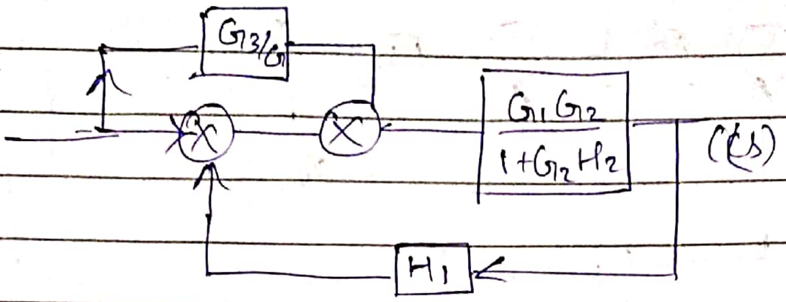
$$\frac{C(s)}{R(s)} = \frac{\dfrac{G_1G_2(G_3G_4+G_5)}{G_3(1+H_1G_1G_2)}}{1+\dfrac{H_2G_1G_2(G_3G_4+G_5)}{G_3(H_1G_1G_2+1)}}$$

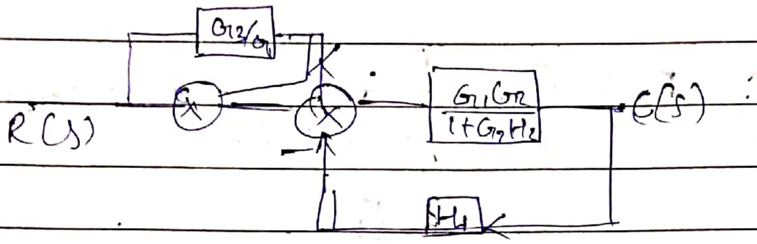
5)

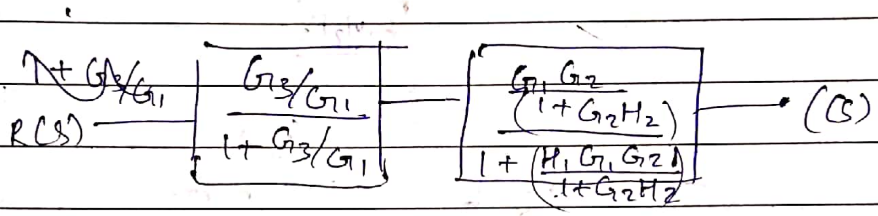


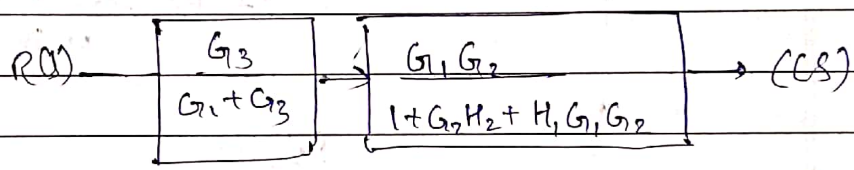






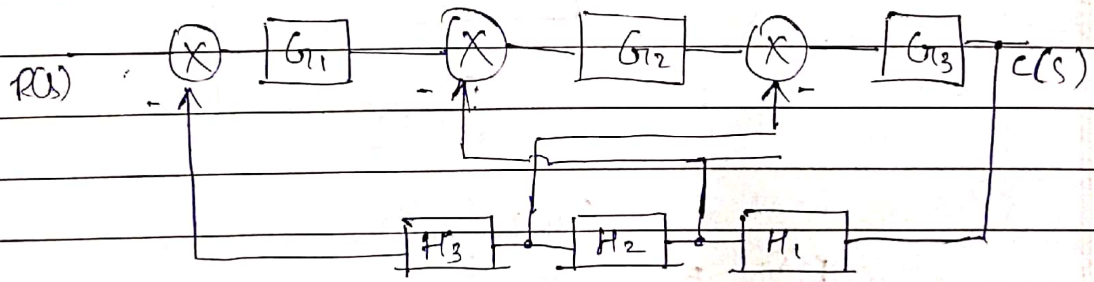


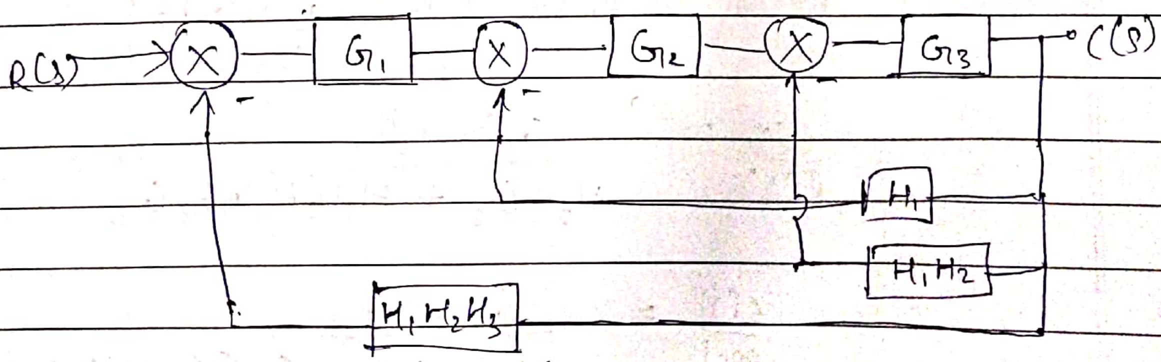


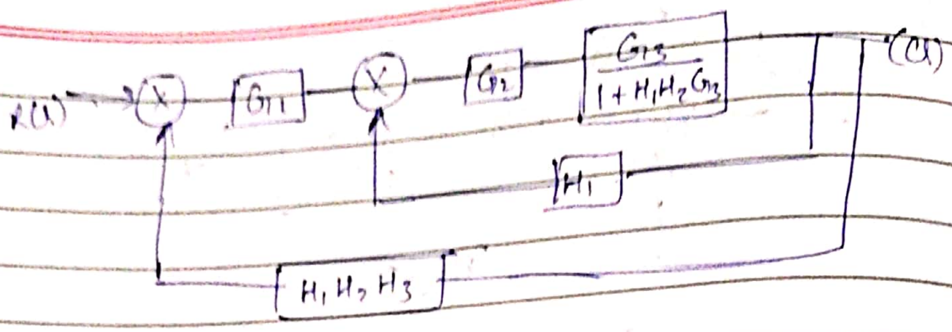


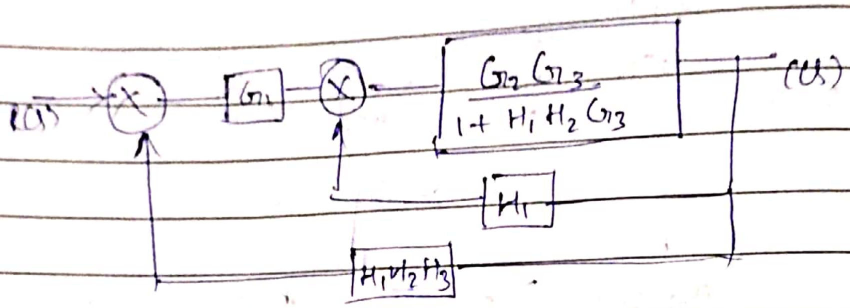
$$\frac{C(s)}{R(s)} = \frac{G_3(G_1+G_2)}{(1+G_2H_2+H_1G_1G_2)(G_1+G_3)}$$

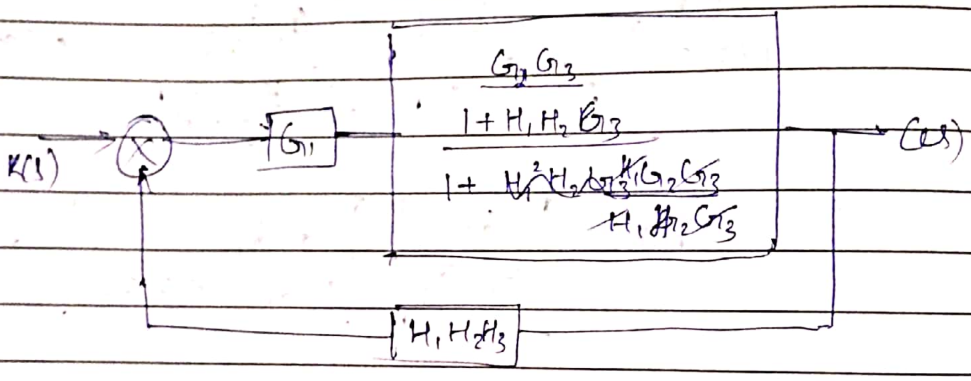
6.

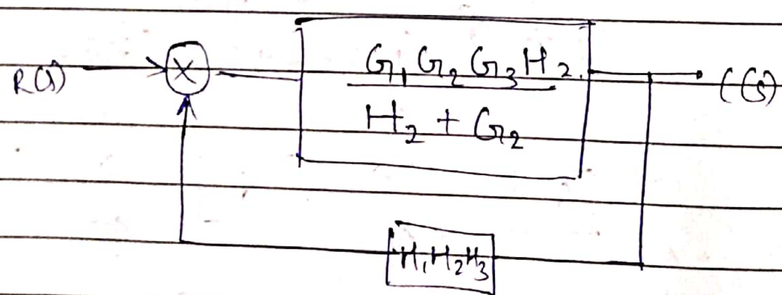










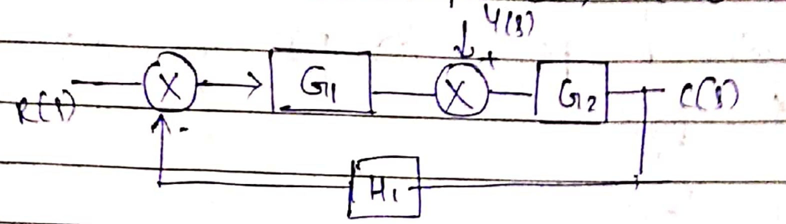


$$\frac{C(s)}{R(s)} = \frac{\dfrac{G_1 G_2 G_3 H_2}{H_2 + G_2}}{1 + H_1 H_2 H_3 \left(\dfrac{G_1 G_2 G_3 H_2}{H_2 + G_2}\right)}$$

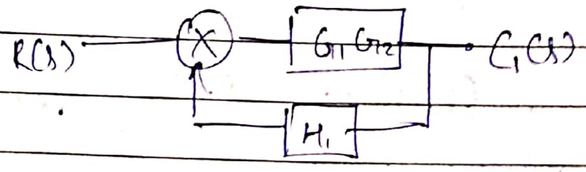
$$\frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3 H_2}{H_2 + G_2 + H_1 H_2^2 H_3 G_1 G_2 G_3}$$

$$\frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3}{1 + G_3 H_1 H_2 + G_2 G_3 H_1 + G_1 G_2 G_3 H_1 H_2 H_3}$$

7) Determine the resultant output C(s) due to R(s) & Y(s)

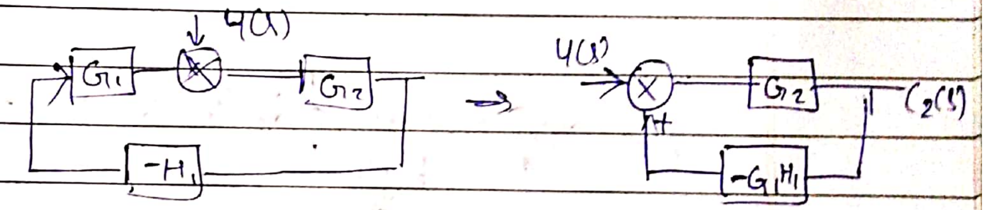


Applying superposition principle,
Let $Y(s) = 0$



$$C_1(s) = \frac{G_1 G_2 \, R(s)}{1 + H_1 G_1 G_2}$$

Let $R(s) = 0$.

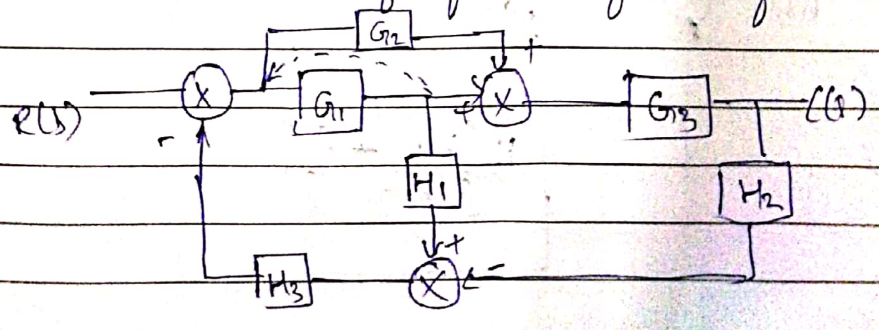


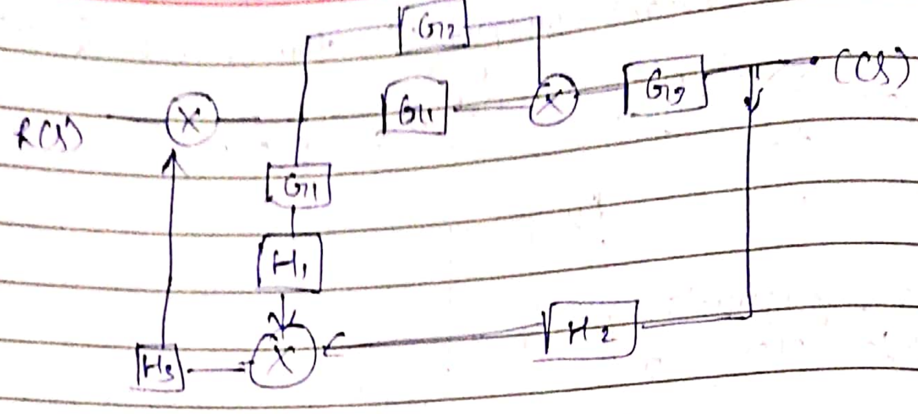
$$C_2(s) = \frac{G_2 \, Y(s)}{1 + G_1 H_1 G_2}$$

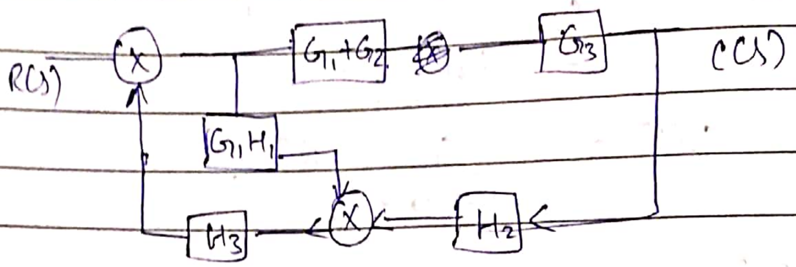
$$C(s) = C_1(s) + C_2(s) = \frac{G_1 G_2 \, R(s)}{1 + G_1 G_2 H_1} + \frac{G_2 \, Y(s)}{1 + G_1 G_2 H_1}$$

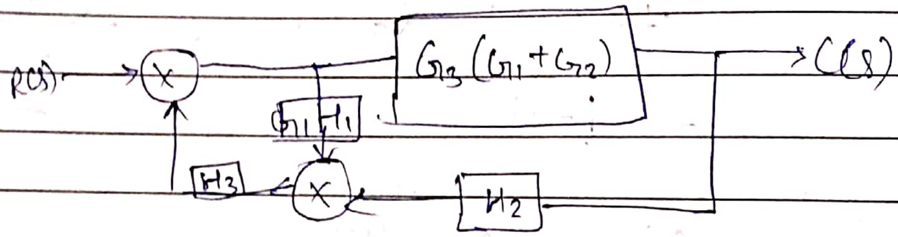
$$C(s) = \frac{G_1 G_2 R(s) + G_2 Y(s)}{1 + G_1 G_2 H_1}$$

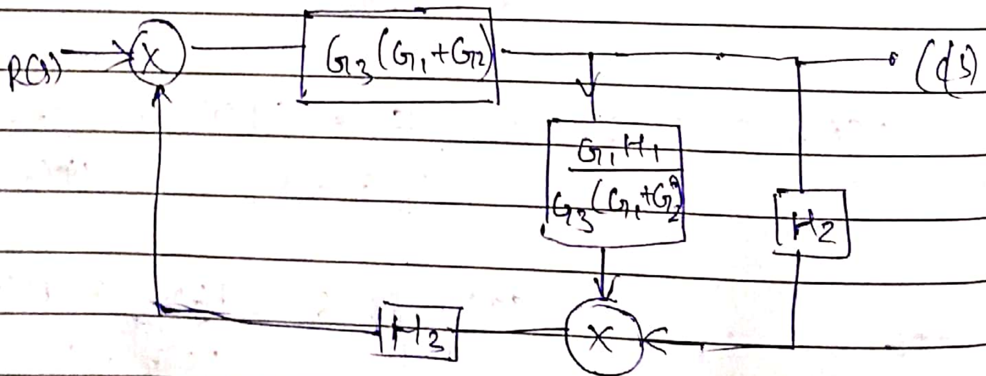
8) Find the overall transfer function for the following diagram,

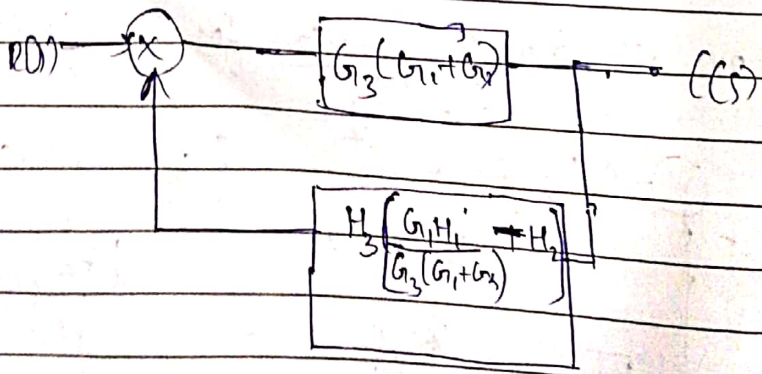


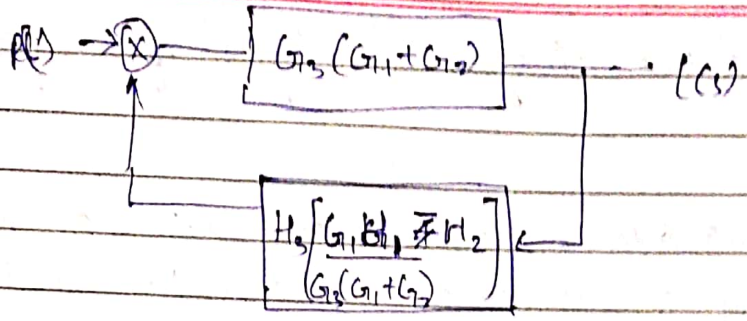

R(s)
G2
G1
G2
G3
C(s)
H1
H2
H3


R(s)
G1+G2
G3
C(s)
G1H1
H3
H2


R(s)
G3(G1+G2)
C(s)
G1H1
H3
H2


R(s)
G3(G1+G2)
C(s)
G1H1 / G3(G1+G2)
H2
H3


R(s)
G3(G1+G2)
C(s)
H3[G1H1 / G3(G1+G2) + H2]



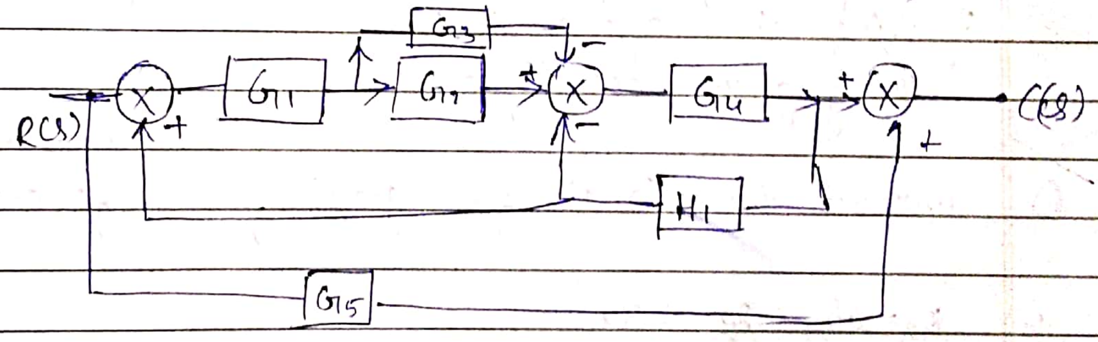
$$\frac{C(s)}{R(s)} = \frac{G_3(G_1+G_2)}{1+G_3(G_1+G_2)\left[H_3\left(\frac{G_1H_1-H_2}{G_3(G_1+G_2)}\right)\right]}$$

$$= \frac{G_3(G_1+G_2)}{1+\cancel{G_3(G_1+G_2)}\left[\frac{H_3H_1G_1-H_2H_3(G_3(G_1+G_2))}{\cancel{G_3(G_1+G_2)}}\right]}$$

$$\frac{C(s)}{R(s)} = \frac{G_3(G_1+G_2)}{1+H_3H_1G_1-H_2H_3G_3(G_1+G_2)}$$

HW

9)



Block diagram & signal flow graphs:

It is required to draw the block after each & every step after applying a rule or a technique. This is time consuming & also if any one rule is applied wrongly. Then the overall transfer function will go wrong. Hence the alternate method of determining overall transfer function is using a single formula called as mason's gain formula.

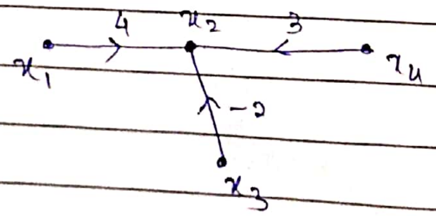
Signal flow graph is graphical representation of a system consisting of different nodes which are connected by branches with an arrow mark to show the signal flow & the gain of the branch (transmittance) will be denoted on the branch.

Signal flow graph can be drawn using

i) Using given algebraic equations
ii) using given block diagram.

Q Construct the signal flow graph for the following equations

1) $x_2 = 4x_1 - 2x_3 + 3x_4$



2) ~~Where~~ V_i = input node & V_0 is output node & V_1 to V_3 are system variables.

$V_1 = 2V_i + 3V_2$

$V_2 = 4V_1 + 5V_3 + 2V_2$

$V_3 = 5V_2 + V_0$

$V_0 = 6V_3$

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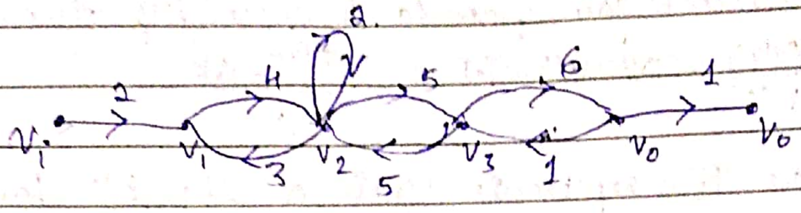


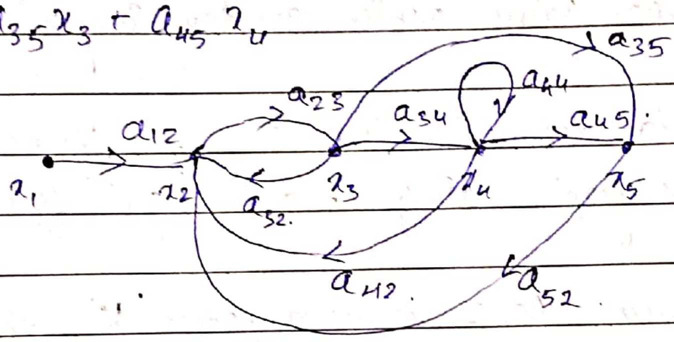
Q. Draw the signal flow graph for the following algebraic equations where x_1 is the input variable & x_5 is the output variable.

$$x_2 = a_{12}x_1 + a_{32}x_3 + a_{42}x_4 + a_{52}x_5$$

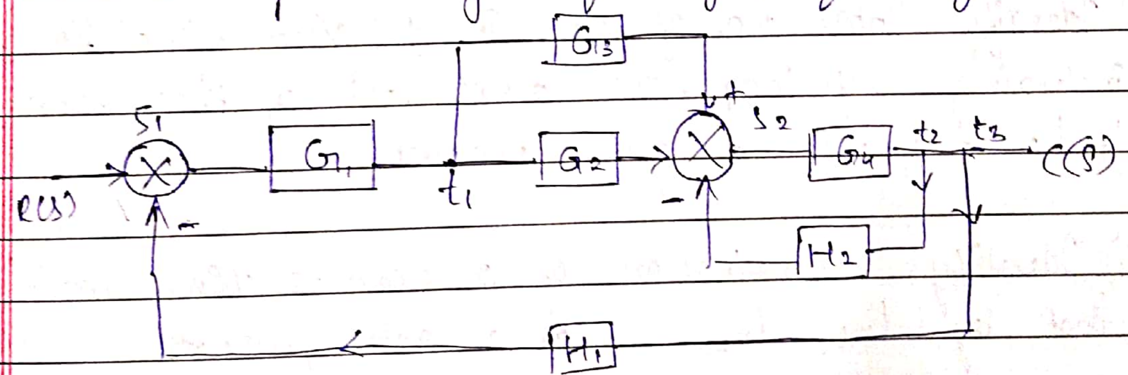
$$x_3 = a_{23}x_2$$

$$x_4 = a_{34}x_3 + a_{44}x_4$$

$$x_5 = a_{35}x_3 + a_{45}x_4$$



Q. Constructing the signal flow graph for the given block diagram.



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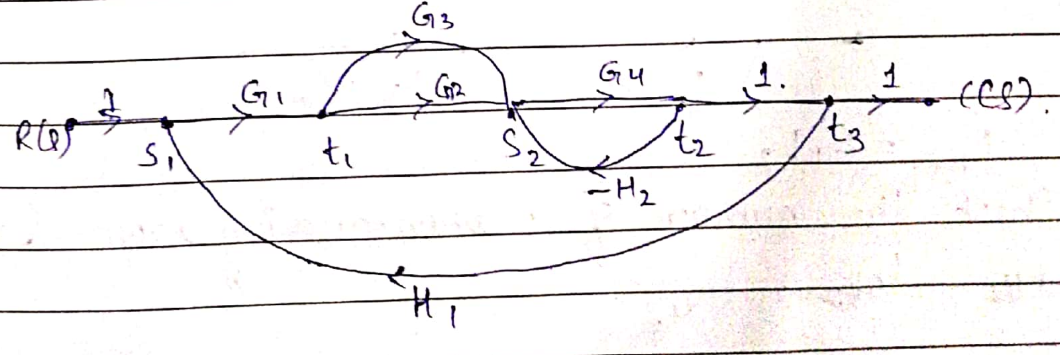


The Mason's gain formula is given by the following equation

$$\frac{C(s)}{R(s)} = \text{overall T.F} = \frac{\sum T_k \Delta k}{\Delta}$$

where T_k represents gain of the k^{th} forward path & k represents no. of forward paths & Δ represents the system determinant which is equal to [1 − (sum of all individual feedback loop gain including the self loop) + (sum of gain products of all possible combinations of two non touching loops)] − (sum of gain products of combination of 3 non touching loops) + . . .]

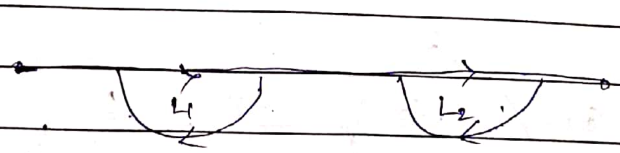
Δk represents the cofactor of k^{th} forward path by eliminating all loop gains which are touching this k^{th} forward path.

Procedure to determine the transfer function of a given signal flow graph, using mason's gain formula.

i) First determine, no. of forward paths (K) then consider forward paths T_1 to T_k as a product of corresponding forward paths

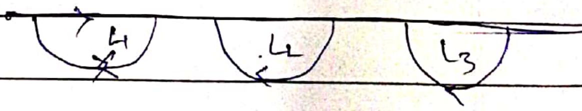
ii) Identify Total no. of loops in the given signal flow graph including self loop, then denote each of those loops as $L_1, L_2, L_3 \ldots$ as a gain product.

iii) Identify combination of 2 non touching loops & denote that by $L_{nt_1}, L_{nt_2} \ldots$ as a gain product.



$L_{nt_1} = L_1 L_2$

* Check the number of 3 non touching loops & denote it by $L_{nt_1}, L_{nt_2}, L_{nt_3} \ldots$



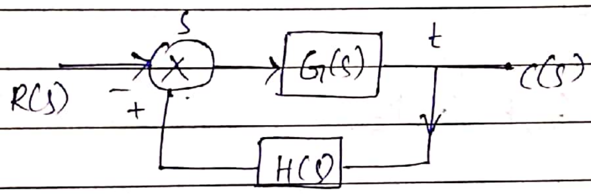
Ex − $L_{nt_1} = L_1 L_2 L_3$

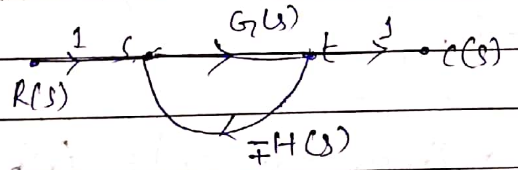
iv) Determine the system determinant Δ as
$$\Delta = 1 - [L_1 + L_2 + L_3 \ldots] + [L_{nt_1} + L_{nt_2} + \ldots] - [L_{nt_{t_1}} + L_{nt_{t_2}} + \ldots]$$

v) Determine the cofactor Δk by removing the loop gains which are touching the particular forward path. & denote them as $\Delta_1, \Delta_2, \Delta_3 \ldots$

vi) The overall transfer function $\frac{C(s)}{R(s)} = \frac{T_1\Delta_1 + T_2\Delta_2 + \ldots}{\Delta}$

Q Determine the overall transfer function of the following block diagram using Mason's gain formula.

1) 

→ 

1) $k = 1$ $\quad T(1) = G(s)$

2) No. of loops = 1
$L_1 = \mp G(s)H(s)$

3) $L_{n_t} = 0$
$* \; L_{n_{t_t}} = 0$

4) $\Delta = 1 - [\mp G(s)H(s)] + 0 - 0 \ldots$
$\therefore \Delta = 1 \pm G(s)H(s)$

5) Cofactor $\Delta_1 = 1$

6) T.F $\frac{C(s)}{R(s)} = \frac{T_1\Delta_1}{\Delta} = \frac{G(s)}{1 \pm G(s)H(s)}$

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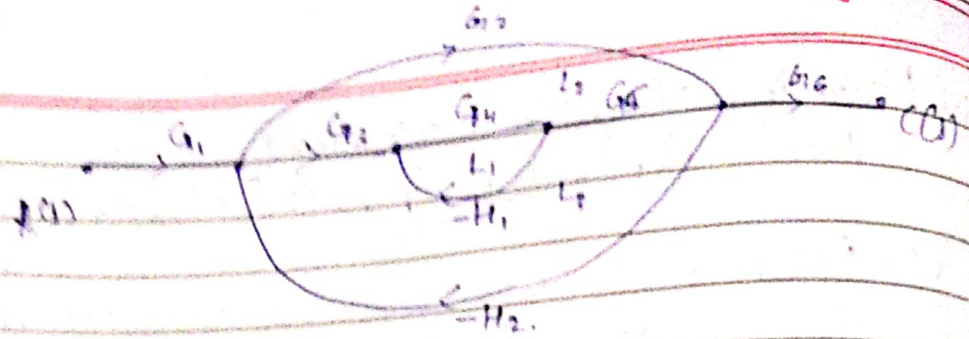
2)



1) $K = 2$ [no. of forward paths]

$T_1 = G_1 G_3 G_4 G_5 G_6$

$T_2 = G_1 G_7 G_6$

2) No. of loops $= 3$

$L_1 = -G_4 H_1$

$L_2 = -G_3 G_4 G_5 H_2$

$L_3 = -G_7 H_2$

3) $L_{nt_1} = L_1 L_3 = G_4 H_1 G_7 H_2$.

$L_{nt_2} = 0$.

4) $\Delta = 1 - [L_1 + L_2 + L_3] + [L_{nt_1}]$

$= 1 - [-G_4 H_1 - G_3 G_4 G_5 H_2 - G_7 H_2] + G_4 G_7 H_1 H_2$

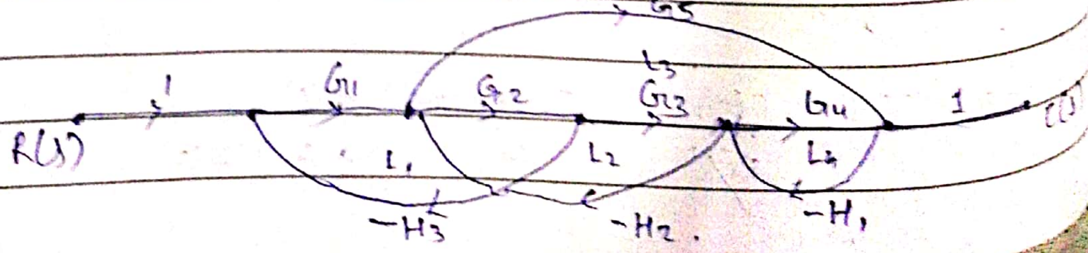
$= 1 + G_4 H_1 + G_3 G_4 G_5 H_2 + G_7 H_2 + G_4 G_7 H_1 H_2$

5) $\Delta_1 = 1$

$\Delta_2 = 1 - L_1 = 1 + G_4 H_1$

6) $$\frac{C(s)}{R(s)} = \frac{G_1 G_3 G_4 G_5 G_6 + (1 + G_4 H_1) G_1 G_7 G_6}{1 + G_4 H_1 + G_3 G_4 G_5 H_2 + G_7 H_2 + G_4 G_7 H_1 H_2}$$

3)



i) K = 2

$T_1 = G_1 G_5$

$T_2 = G_1 G_2 G_3 G_4$

ii) no. of loops 4.

$L_1 = -G_1 G_2 H_3$

$L_2 = -G_2 G_3 H_2$

$L_3 = +G_5 H_1 H_2$.

$L_4 = -G_4 H_1$

iii) $L_{nt_1} = L_1 L_4 = +G_1 G_2 H_3 G_4 H_1 = G_1 G_2 G_4 H_1 H_3$

$L_{nt_{11}} = 0$

iv) $\Delta = 1 - [-G_1 G_2 H_3 - G_2 G_3 H_2 + G_5 H_1 H_2 - G_4 H_1] + [G_1 G_2 G_4 H_1 H_3]$

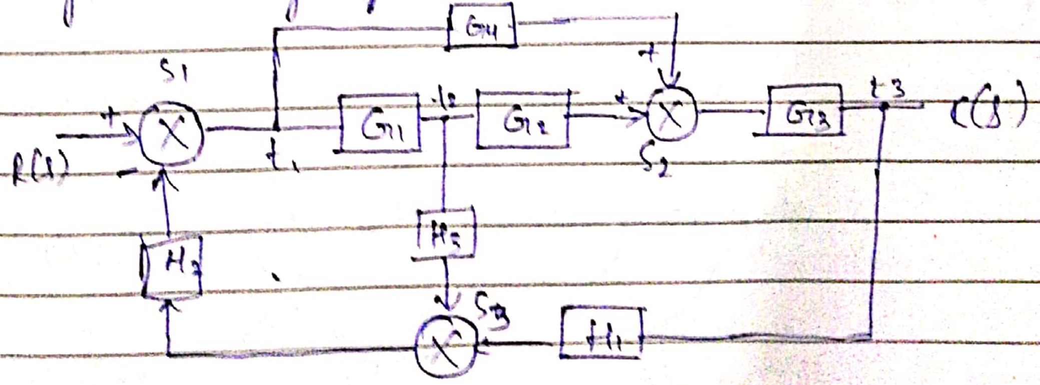
$= 1 + G_1 G_2 H_3 + G_2 G_3 H_2 - G_5 H_1 H_2 + G_4 H_1 + G_1 G_2 G_4 H_1 H_3$

5) $\Delta_2 = 1$.

$\Delta_1 = 1 - L_1$

6) $$\frac{C(s)}{R(s)} = \frac{G_1 G_5 + G_1 G_2 G_3 G_4}{1 + G_1 G_2 H_3 + G_2 G_3 H_2 - G_5 H_1 H_2 + G_4 H_1 + G_1 G_2 G_4 H_1 H_3}.$$

4) Determine the transfer function for the following block diagram using Mason's gain formula.



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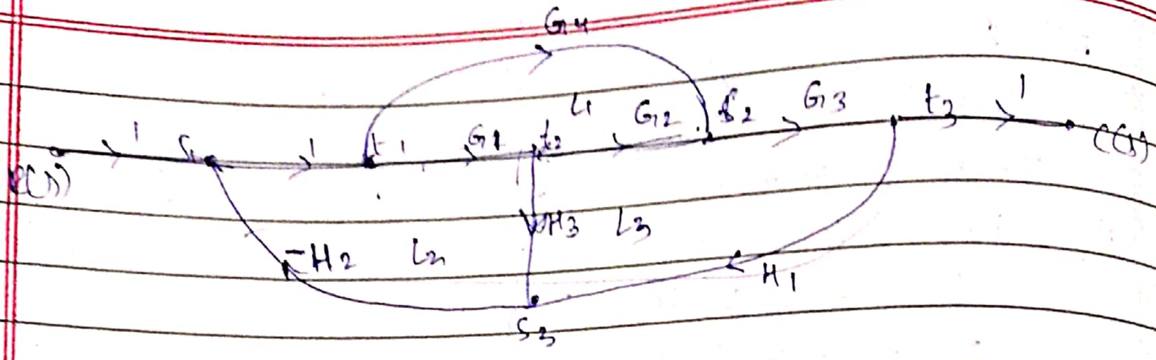


i) $K = 2$

ii) $T_1 = G_1 G_2 G_3$

$T_2 = G_4 G_3$

ii) no of loops - 3

$L_1 = -G_4 G_3 H_1 H_2$

$L_2 = -G_1 H_3 H_2$

$L_3 = -G_1 G_2 G_3 H_1 H_2$

iii) $L_{nt} = 0$

$L_{n_{t_1}} = 0$.

iv) $\Delta = 1 - [-G_4 G_3 H_1 H_2 - G_1 H_3 H_2 - G_1 G_2 G_3 H_1 H_2]$

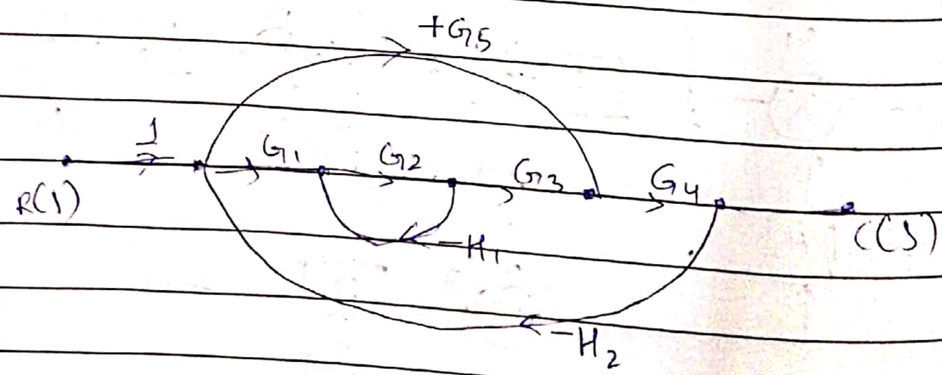
$= 1 + G_3 G_4 H_1 H_2 + G_1 H_2 H_3 + G_1 G_2 G_3 H_1 H_2$

v) $\Delta_1 = 1$

$\Delta_2 = 1$

vi) $\dfrac{C(s)}{R(s)} = \dfrac{G_1 G_2 G_3 + G_3 G_4}{1 + G_3 G_4 H_1 H_2 + G_1 H_2 H_3 + G_1 G_2 G_3 H_1 H_2}$

5)



i) $K = 2$

$T_1 = G_5 G_4$

$T_2 = G_1 G_2 G_3 G_4$

ii) No of loops = 3

$L_1 = -G_4 G_5 H_2$

$L_2 = -G_2 H_1$

$L_3 = -G_1 G_2 G_3 G_4 H_2$.

iii) $L_{nt_1} = L_1 L_2$

$= + G_2 G_4 G_5 H_1 H_2$.

iv) $\Delta = 1 - [-G_4 G_5 H_2 - G_2 H_1 - G_1 G_2 G_3 G_4 H_2] + [G_2 G_4 G_5 H_1 H_2]$

$= 1 + G_4 G_5 H_2 + G_2 H_1 + G_1 G_2 G_3 G_4 H_2 + G_2 G_4 G_5 H_1 H_2$.

v) $\Delta_1 = 1 - l_2$

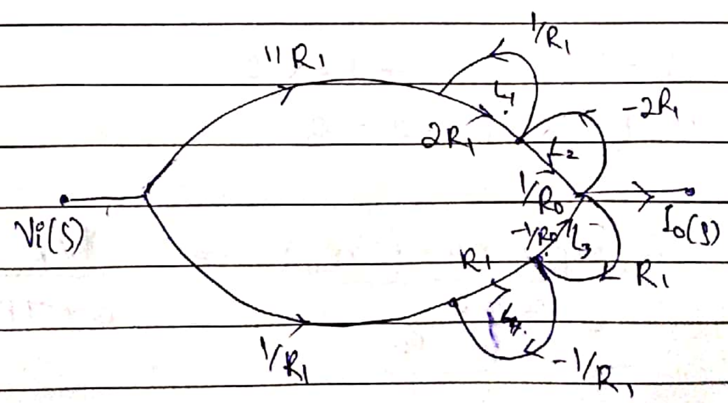
$= 1 + G_2 H_1$

$\Delta_2 = 1$.

vi) $$\frac{C(S)}{R(S)} = \frac{(1 + G_2 H_1) G_5 G_4 + G_1 G_2 G_3 G_4}{1 + G_4 G_5 H_2 + G_2 H_1 + G_1 G_2 G_3 G_4 H_2 + G_2 G_4 G_5 H_1 H_2}$$

6) Determine transfer function $\frac{I_o(S)}{V_i(S)}$ for the following SFG which corresponds to an electrical network.

→



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i) K = 2.

ii) $T_1 = \frac{1}{\cancel{R_1}} \times 2\cancel{R_1} \times \frac{1}{R_0} = 2/R_0$

$T_2 = \frac{1}{\cancel{R_1}} \times \cancel{R_1} \times -1/R_0 = -1/R_0.$

ii) No of loops. = 4

$L_1 = \frac{-1}{\cancel{R_1}} \times 2\cancel{R_1} = -2.$

$L_2 = -2R_1/R_0$

$L_3 = -R_1/R_0$

$L_4 = \cancel{R_1} \times \frac{-1}{\cancel{R_1}} = -1$

iii) $L_{nT_1} = L_1 L_3 = +2R_1/R_0$

$L_{nT_2} = L_2 L_4 = 2R_1/R_0$

$L_{nT_3} = L_1 L_4 = 2.$

~~iv)~~ $L_{nt_{k_1}} = 0$

iv) $\Delta = 1 - [L_1 + L_2 + L_3 + L_4] + [L_{nT_1} + L_{nT_2} + L_{nT_3}]$

$= 1 - [-2 - 2R_1/R_0 - R_1/R_0 - 1] + [2R_1/R_0 + 2R_1/R_0 + 2]$

$= 1 + 2 + 2R_1/R_0 + R_1/R_0 + 1 + 2R_1/R_0 + 2R_1/R_0 + 2.$

$= \underline{\underline{6 + 7R_1/R_0}}$

v) $\Delta_1 = 1 - L_4 = 1 + 1 = 2.$

$\Delta_2 = 1 - L_1 = 1 + 2 = 3$

vi) $\frac{C(S)}{R(S)} = \frac{2(2/R_0) + 3 \times (-1/R_0)}{6 + 7R_1/R_0} = \frac{1/R_0}{6 + 7R_1/R_0}$

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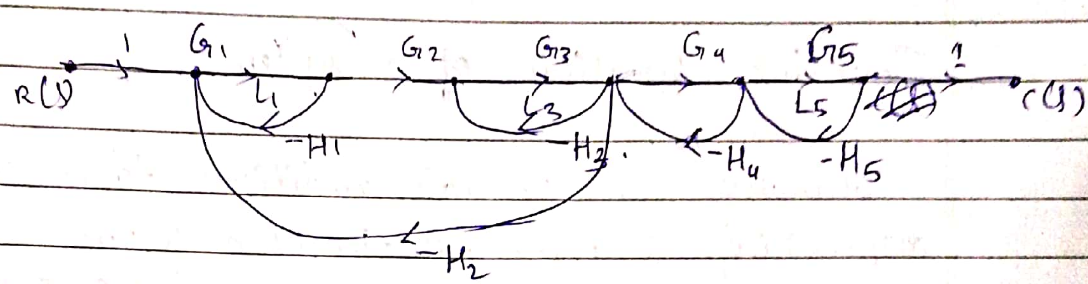
$$\frac{C(s)}{R(s)} = \frac{1}{6R_0 + 7R_1}$$

7)



i) $K = 1$.

$T_1 = G_1 G_2 G_3 G_4 G_5$

ii) No of loops = 5

$L_1 = -G_1 H_1$

$L_2 = -G_1 G_2 G_3 H_2$

$L_3 = -G_3 H_3$

$L_4 = -G_4 H_4$

$L_5 = -G_5 H_5$

iii) $L_{nT_1} = L_1 L_3 = G_1 G_3 H_1 H_3$

$L_{nT_2} = L_1 L_4 = G_1 G_4 H_1 H_4$

$L_{nT_2} = L_1 L_5 = G_1 G_5 H_1 H_5$

$L_{nT_3} = L_3 L_5 = G_3 G_5 H_3 H_5$

$L_{nT_4} = L_2 L_5 = G_1 G_2 G_3 G_5 H_2 H_5$

$L_{nT_1} = L_1 L_3 L_5 = -G_1 \cancel{H_2} G_3 G_5 H_1 H_3 H_5$.

iv) $\Delta = 1 - [-G_1H_1 - G_1G_2G_3H_2 - G_3H_3 - G_4H_4 - G_5H_5] + [G_1G_3H_1H_3 + G_1G_4H_1H_4 + G_1G_5H_1H_5 + G_3G_5H_3H_5 + G_1G_2G_3G_5H_2H_5] - [G_1G_3G_5H_1H_3H_5]$

$\Delta = 1 + G_1H_1 + G_1G_2G_3H_2 + G_3H_3 + G_4H_4 + G_5H_5 + G_1G_3H_1H_3 + G_1G_4H_1H_4 + G_1G_5H_1H_5 + G_3G_5H_3H_5 + G_1G_2G_3G_5H_2H_5 + G_1G_3G_5H_1H_3H_5$

v) $\Delta_1 = 1 - 0 = 1$

vi) $$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3G_4G_5}{\Delta}$$

$$\frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3 G_4 G_5}{1 + G_1 H_1 + G_1 G_2 G_3 H_2 + G_3 H_3 + G_4 H_4 + G_5 H_5 + G_1 G_3 H_1 H_3 + G_1 G \ldots + G_1 G_5 H_1 H_5 + G_3 G_5 H_3 H_5 + G_1 G_2 G_3 G_5 H_2 H_5 + G_1 G_3 G_5 H \ldots}$$

Transfer function of a system:

Following are the main conditions to be satisfied while determining Transfer function of a system.

i) The ~~give~~ parameters given must be in S domain.

Ex- Capacitor $C \to 1/Cs$ & $L \to Ls$;

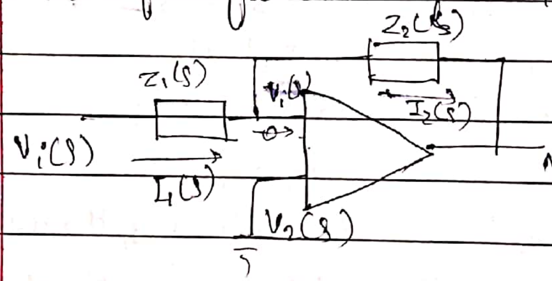
$V_i(s)$, $V_o(s)$, $I(s)$ etc.

$$H(s) = \frac{V_o(s)}{V_I(s)}$$

ii) Initial conditions are assumed to be zero.

iii) There is no loading effect.

Transfer function of an inverting amplifier using OpAmp.



KCL of node $V_1(s)$

$$I_i(s) = I_2(s) \quad \text{--- (1)}$$

$$\frac{V_i(s) - V_1(s)}{Z_1(s)} = \frac{V_1(s) - V_o(s)}{Z_2(s)} \quad \text{--- (2)}$$

$$\frac{V_i(s)}{Z_1(s)} - \frac{V_1(s)}{Z_1(s)} = \frac{V_1(s)}{Z_2(s)} - \frac{V_o(s)}{Z_2(s)} \quad (3)$$

$$V_{id} = \frac{V_o}{A} \simeq 0$$

$$V_{id} = V_2(s) - V_1(s) = -V_1(s)$$

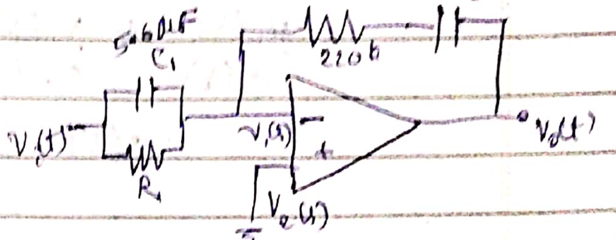
$$\frac{V_i(s)}{Z_1(s)} = -\frac{V_o(s)}{Z_2(s)}$$

$$\frac{V_0(s)}{V_i(s)} = -\frac{Z_2(s)}{Z_1(s)} = \text{T.F} = H(s) \quad \text{(A)}$$

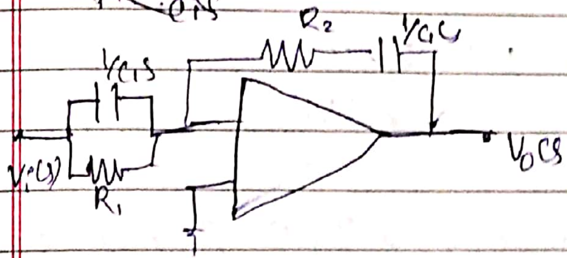
Determine the transfer function $\frac{V_0(s)}{V_i(s)}$ for the following circuit

$$C_1 \rightarrow \frac{1}{C_1 S} = \frac{1}{(5.6\times10^{-6})S}$$

$$C_2 = \frac{1}{C_2 S} = \frac{1}{(0.1\times10^{-6})S}$$



$R_1 \| \frac{1}{C_1 S}$



$$\therefore R_1 \| 1/C_1 s = \frac{R_1 \times 1/C_1 s}{\left(R_1 + \frac{1}{C_1 s}\right)} = \frac{R_1}{R_1 C_1 S + 1} \rightarrow Z_1(s)$$

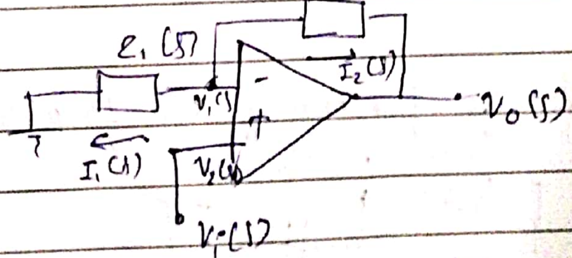
$$R_2 + 1/C_2 s = \frac{R_2 C_2 S + 1}{C_2 S} \rightarrow Z_2(s)$$

$$H(s) = \frac{V_0(s)}{V_i(s)} = -\frac{R_2 C_2 S + 1}{C_1(S)} \times \frac{R_1 C_1 S + 1}{R_1}$$

$$= \frac{1 + R_2 C_2 S + R_1 C_1 S + R_1 R_2 C_1 C_2 S^2}{R_1 C_2 S}$$

$$= \frac{(1.7057S^2 + 78.39S + 38.46)}{S}$$

Transfer function of a non inverting amplifier using



We know that, as the differential gain A is too large

$V_{id} \simeq 0 \quad \text{--- (1)}$

$\therefore V_{id} = V_2(s) - V_1(s) = 0 \quad \text{--- (2)}$

$V_2(s) = V_1(s) = V_i(s) \quad \text{--- (3)}$

KCL of $V_1(s)$

$$I_1(s) + I_2(s) = 0 \quad -(4)$$

$$I_1(s) = -I_2(s) \quad -(5)$$

$$\frac{V_1(s)}{Z_1(s)} = -\frac{(V_1(s) - V_0(s))}{Z_2(s)} \quad -(6)$$

$$\frac{V_1(s)}{Z_1(s)} + \frac{V_1(s)}{Z_2(s)} = \frac{V_0(s)}{Z_2(s)}$$

$$\cancel{V_i(s)} \frac{Z_2(s)}{Z_1(s) + \cancel{Z_2(s)}} = \cancel{\frac{V_0(s)}{V_i(s)}}$$

$$\frac{V_0(s)}{V_i(s)} = Z_2(s)\left(\frac{1}{Z_1(s)} + \frac{1}{Z_2(s)}\right)$$

$$H(s) = 1 + \frac{Z_2(s)}{Z_1(s)}$$

Q If $Z_1(s)$ is impedance of $10\mu F$ C & $Z_2(s)$ is impedance of $100k\Omega$ resistor. Determine the transfer function $V_0(s)/V_i(s)$ if these components are used with a) an inverting amplifier b) an non inverting amplifier.

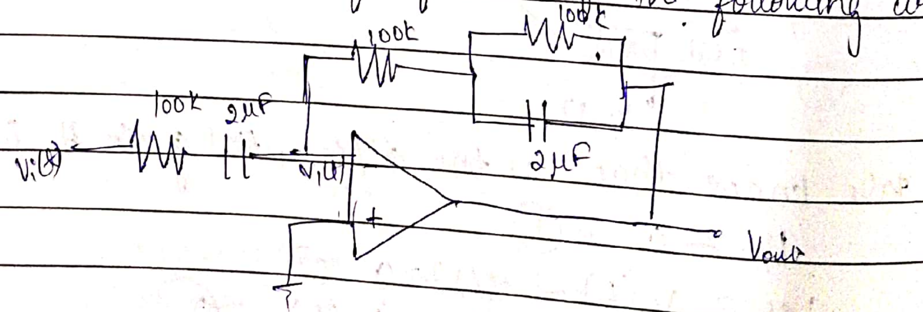
a) $$\frac{V_0(s)}{V_i(s)} = -\frac{Z_2(s)}{Z_1(s)} = -C_1(s) \times 100\times10^3$$

$$= 10\times10^{-6}\times10^5$$

$$= s$$

b) $$\frac{V_0(s)}{V_i(s)} = \frac{Z_2(s)}{Z_1(s)} = s\times10\times10^{-6}\times10^5 = s$$

Q Determine the transfer function of the following circuit.

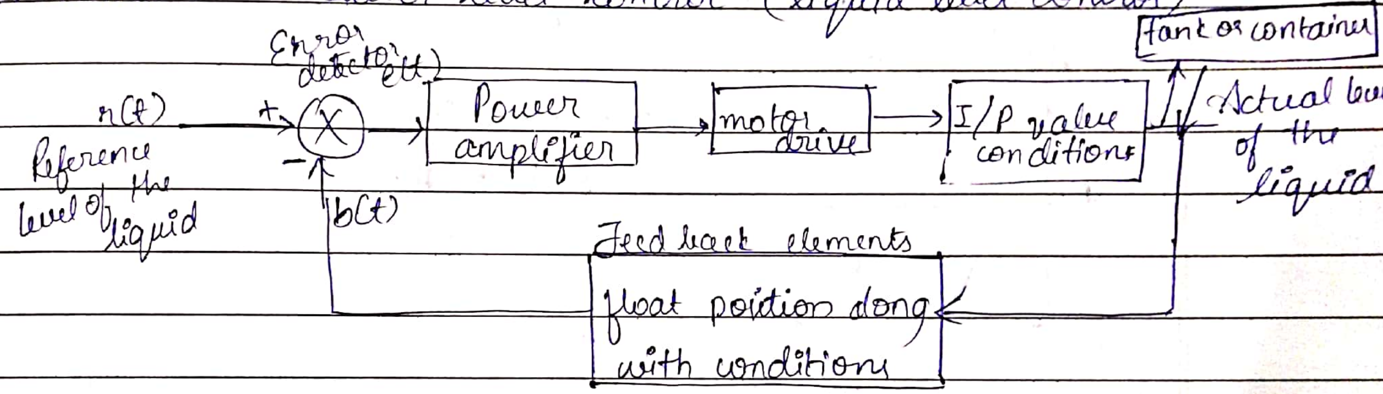


$$\frac{V_o(s)}{V_i(s)} = \frac{-s(s+10)}{(s+5)^2}$$

$$Z_1(s) = 10^5 + \frac{1}{2\times10^{-6} s} = \frac{20+1}{2\times10^{-6}} = 1.05\times10^{3}$$

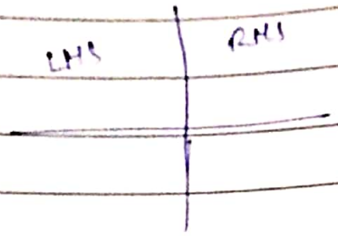
$$=$$

Automatic water level control (liquid level control)



Time Response Analysis:

• Time response
  - Transient response
  - Steady state response



$S_T = -1/T_2$

$T_2 < T_1$

$T_2 \to$ more stable

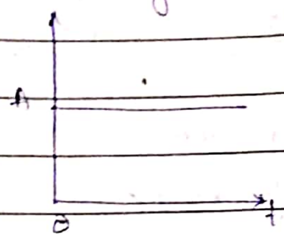
LHS → stable RHS → unstable.

Test signals for control system:

i) Step signal

$r(t) = A\,u(t)$

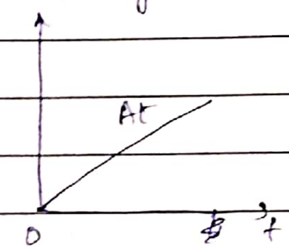
$R(s) = A/s$



ii) Ramp signal

$r(t) = At$

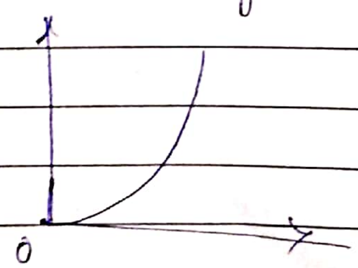
$R(s) = \frac{A}{s^2}$.



iii) Parabolic signal

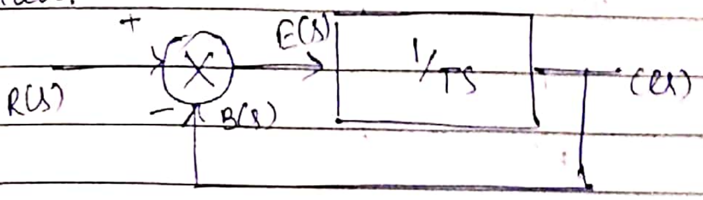
$r(t) = \frac{At^2}{2}$.

$R(s) = \frac{A}{s^3}$.



Time response Analysis of first order system:

Let us consider the general block diagram for a order system with unity feedback as shown in below:



$$C(s) = R(s)\cdot\frac{1/Ts}{1+1/Ts} = \frac{1}{1+Ts} \rightarrow (1)$$

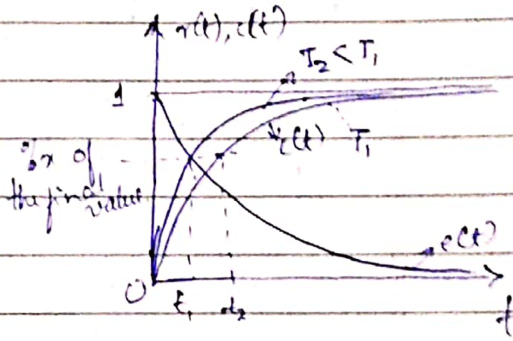
where T → time constraint of the system. Let us study the response of this system for an unit step input, so that

Let $R(s) = \frac{1}{s}$ — (2)

$$C(s) = \frac{1}{s(1+Ts)} \quad — (3)$$

$$C(s) = \frac{1}{s} - \frac{T}{1+Ts} \quad — (4).$$

$$c(t) = 1 - e^{-t/T}$$



$$e(t) = r(t) - c(t)$$
$$= 1 - (1 - e^{-t/T})$$
$$= e^{-t/T}$$

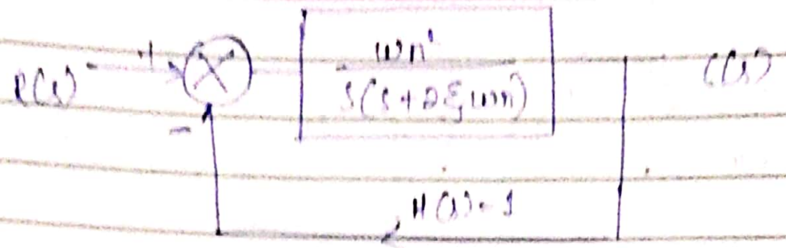
The system with the lesser T (time constraint) reaches steady state easily from ~~standard~~ transient state.

Steady state error $= e_{ss} = \lim_{t\to\infty} e(t) = \lim_{t\to\infty} e^{-t/T} = 0$

Above eqn. shows that the system tracks the unit step input with zero steady state error.

Time response of 2nd order system:

Let us consider the following block diagram for general eqn ~~for~~ of a second order system.



$$T.F = \frac{G(s)}{1+G(s)H(s)} = \frac{\omega_n^2/s(s+2\xi\omega_n)}{1+\frac{\omega_n^2}{s(s+2\xi\omega_n)}} = \frac{\omega_n^2}{s^2+2\xi_1\omega_n s+\omega_n^2} \quad \text{--- (A)}$$

~~ω_n = ω_n~~

ω_n = undamped natural frequency of system.

ξ_1 = damping factor or damping ratio.

which is defined as

$$\xi_1 = \frac{\text{Exponential decay freq.}}{\text{Natural freq. (rad/sec)}}$$

Let us consider the system response for unit step input.

$R(s) = \frac{1}{s}$ from eqn. (A) we can write.

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi_1\omega_n s+\omega_n^2}$$

$$C(s) = \frac{R(s)\times\omega_n^2}{s^2+2\xi_1\omega_n s+\omega_n^2} \quad \text{--- (B)}$$

$$C(s) = \frac{1}{s}\times\frac{\omega_n^2}{s^2+2\xi_1\omega_n s+\omega_n^2}$$

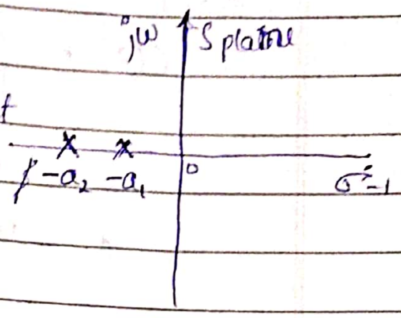
The system response for the unit step i/p depends upon value of ξ and the roots of the characteristic eqn. or say the nature of the roots will depend upon the value of damping ratio ξ_1.

Hence it is studied under 4 headings.

1. $1 < \xi_1 < \infty \rightarrow$ overdamped condition
2. $\xi_1 = 1 \rightarrow$ critically damped condition
3. $\xi_1 < 1 \rightarrow$ underdamped condition
4. $\xi_1 = 0 \Rightarrow$ undamped condition.

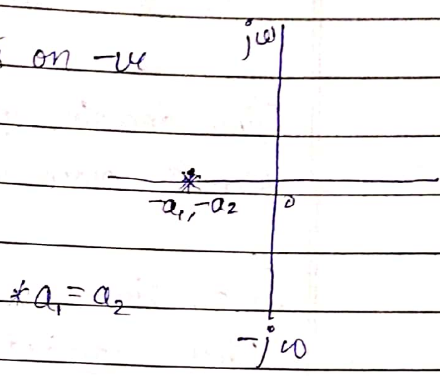
Overdamped condition:

Here there are 2 roots with –ve real part which are located as shown in the below fig



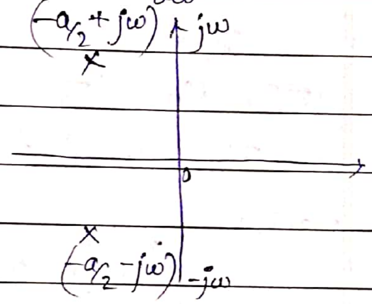
Critically damped condition:

Here the roots are repeated roots on –ve real axis



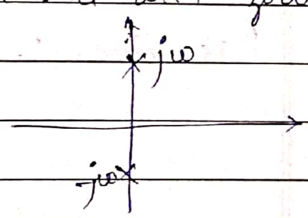
Under damped condition:

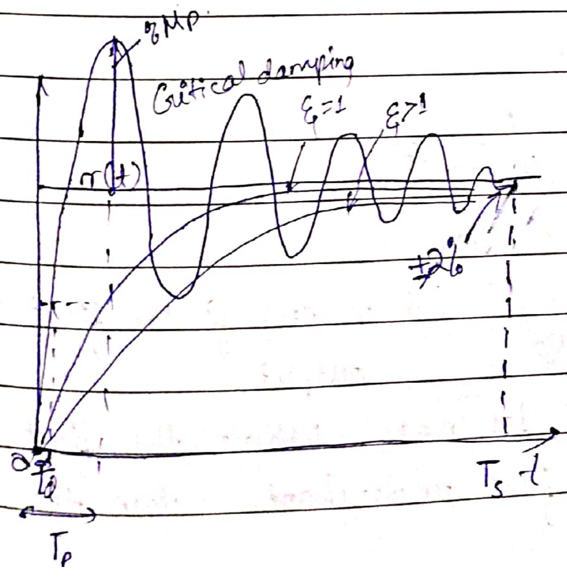
Here the roots are complex conjugate roots but with –ve real part

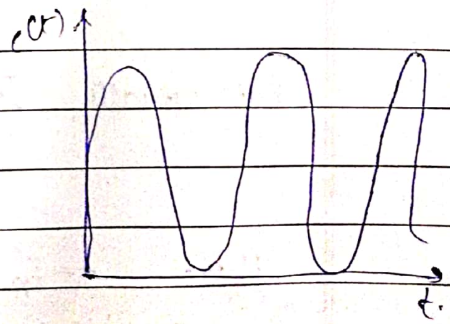


Undamped condition:

Here the roots are complex conjugate but with zero real part.







$\xi < 1$

$$C(t) = \left[1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta)\right]$$

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)$$

Time domain specifications OR transient response specification of second order system for ($\xi<1$) i.e., underdamped condition:

Delay time (T_d)

Time required for the response to reach 50% of the final value

$$\boxed{T_d = \frac{1+0.7\xi}{\omega_n}}$$

~~T_r =~~

Rise time (T_r)

Time required for the response to move from 10% to 90% of the final value:

$$\boxed{T_r = \frac{\pi-\theta}{\omega_d} \text{ sec.}}$$

Peak time (T_p)

$$T_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$$

But actually is derived from

$t = \frac{n\pi}{\omega_d}$ where $n=1$ for the first overshift of the output.

Hence $t = T_p = \frac{\pi}{\omega_d}$ for $n=1$. Where the first overshoot occurs & $n=2$ for the first undershoot, then the corresponding

$t = \frac{2\pi}{\omega_d}$

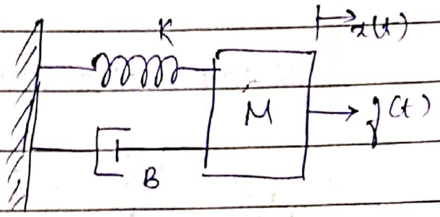
Peak overshoot (M_p)

$M_p \rightarrow \% M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100$

Settling time (T_s)

$T_s = \frac{4}{\xi\omega_n}$ for a tolerance band of ±2%

Q. For the following mech. system determine the transfer function, values of ξ, ω_n, $\% M_p$, T_s & T_p given that $K = 33$ N/m, $B = 15$ N/m/sec; $M = 3$ kg.



$\rightarrow f(t) = M\frac{d^2x(t)}{dt^2} + K\,x(t) + B\frac{dx(t)}{dt}$ — ①

$F(s) = MS^2X(s) + BS\,X(s) + KX(s)$ — ②

$\frac{X(s)}{F(s)} = T.F = \frac{1}{MS^2 + BS + K}$ — ③

$= \frac{1/M}{S^2 + \frac{B}{M}S + \frac{K}{M}}$ — ④.

$\left[\because TF = \frac{\omega_n^2}{S^2 + 2\xi\omega_n S + \omega_n^2} \right]$

$2\xi\omega_n = \frac{B}{M} = \frac{15}{3} = 5$

$2\xi\omega_n = 5$.

$\omega_n^2 = \frac{K}{M} = \frac{33}{3} = 11$

$\boxed{\omega_n = \sqrt{11}} = 3.316$

$\boxed{\xi = \frac{5}{2\sqrt{11}}} = 0.7537$

$$\% M_p = e^{-\pi \xi / \sqrt{1-\xi_1^2}} \times 100$$

$$= e^{-3.607} \times 100$$

$$= \underline{2.727\%}$$

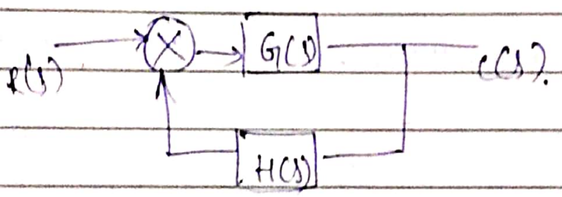
$$T_s = \frac{4}{\xi_1 \times \omega_n}$$

$$T_s = \underline{1.6\ s}$$

$$T_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n \sqrt{1-\xi_1^2}}.$$

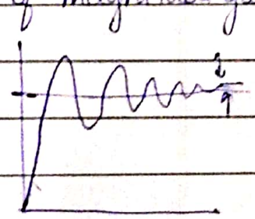
$$T_p = \underline{1.4415\ s}$$

Steady state error & error constant



1. Steady state error constant for step i/p of magnitude point.

$r(t) = A\, u(t)$ $\quad R(s) = \frac{A}{s}$



considering error eqn.

$$E(s) = \frac{R(s)}{1 + G(s)H(s)}$$

$$= \frac{A/s}{1 + G(s)H(s)}$$

Then the corresponding steady state error

$$e_{ss} = \lim_{s \to 0} s \cdot E(s) = \lim_{s \to 0} s \cdot \frac{A/s}{1 + G(s)H(s)}$$

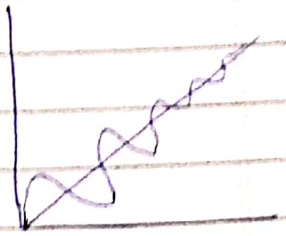
$$= \frac{A}{1+\lim_{s\to 0} G(s)H(s)}$$

where $\lim_{s\to 0} G(s)H(s)$ is denoted by K_p which is called as positional error constant.

where $G(s)\cdot H(s)$ is called as open loop transfer function (OLTF)

then, $\boxed{e_{ss} = \frac{A}{1+K_p}}$ — (A)

2) Ramp i/p of magnitude A.



$r(t) = At$ $R(s) = A/s^2$

$$E(s) = \frac{R(s)}{1+G(s)H(s)} \quad \text{— (1)}$$

$$E(s) = \frac{A/s^2}{1+G(s)H(s)} \quad \text{— (2)}$$

$$e_{ss} = \lim_{s\to 0} s.E(s) = \lim_{s\to 0} \frac{s.\ A/s^2}{1+G(s)H(s)}$$

$$= \frac{A}{\lim_{s\to 0} s + \lim_{s\to 0} sG(s)H(s)} \quad \text{— (3)}$$

$$= \frac{A}{\lim_{s\to 0} s.G(s)H(s)} \quad \text{— 3'}$$

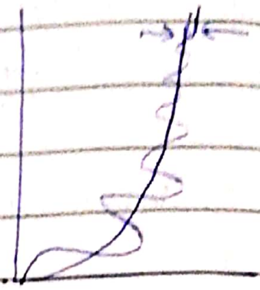
$$\lim_{s\to 0} sG(s)H(s) = K_v \quad \text{— (4)}$$

$K_v \rightarrow$ velocity error constant.

$\therefore$ $\boxed{e_{ss} = \frac{A}{K_v}}$ — (B)

3) Parabolic i/p of magnitude A.

$r(t) = \frac{A}{2}t^2$; $R(s) = A/s^3$



$$E(s) = \frac{R(s)}{1+G(s)H(s)} \quad \text{--- (1)}$$

$$E(s) = \frac{A/s^3}{1+G(s)H(s)} \quad \text{--- (2)}$$

$$e_{ss} = \lim_{s\to 0} s.E(s) = \lim_{s\to 0} \frac{s \cdot A/s^{3}}{1+G(s)H(s)}$$

$$= \frac{A}{\lim_{s\to 0} s^2 + \lim_{s\to 0} s^2 G(s)H(s)} \quad \text{--- (3)}$$

$$= \frac{A}{\lim_{s\to 0} s^2 G(s)H(s)} \quad \text{--- 3'}$$

$$\lim_{s\to 0} s^2 G(s)H(s) = K_a \quad \text{--- (4)}$$

$K_a \to$ acceleration error constant

$$\boxed{e_{ss} = \frac{A}{K_a}} \quad \text{--- (C)}$$

Types of the system.

$$G(s)H(s) = \frac{K(1+T_1 s)(1+T_2 s)\dots}{s^n(1+T_a s)(1+T_b s)\dots} \quad \text{or} \quad \frac{K'(s+z_1)(s+z_2)}{s^n(s+P_1)(s+P_2)}$$

Time constant form   pole zero form

Study of steady state error & error constant of type 0, type 1, type 2 system

I) Type '0' system: $G(s)H(s) = K\frac{(1+T_1(s))(1+T_2(s))}{(1+T_a(s))(1+T_b(s))}$

a) For a step I/p of 'A'

$R(s) = \frac{A}{s}$; $K_p = \lim_{s\to 0} G(s)H(s) = \lim_{s\to 0} \frac{K(1+T_1 s)(1+T_2 s)}{(1+T_a s)(1+T_b s)}$

classmate

Date

Page

$$\therefore e_{ss} = \frac{A}{1+K_p} = \frac{A}{1+K}$$

b) For a ramp i/p of A ; $R(s) = A/s^2$.

$$K_v = \lim_{s\to 0} s.G(s)H(s) = \lim_{s\to 0} s\frac{K(1+T_1 s)(1+T_2 s)}{(1+T_a s)(1+T_b s)} = 0$$

$$\therefore e_{ss} = \frac{A}{K_v} = \infty$$

c) For a parabolic signal A.

$R(s) = A/s^3$. $K_a = \lim_{s\to 0} s^2 G(s)H(s) = \lim_{s\to 0} s^2 \frac{K(1+T_1 s)(1+T_2 s)}{(1+T_a s)(1+T_b s)}$

$$\therefore e_{ss} = \frac{A}{K_a} = \frac{A}{0} = \infty$$

II Type '1' system.

i) For a step i/p of 'A'

$R(s) = A/s$; $K_p = \lim_{s\to 0} G(s)H(s) = \lim_{s\to 0} \frac{K(1+T_1 s)(1+T_2 s)}{s(1+T_a s)(1+T_b s)} = \infty$

$$\therefore e_{ss} = \frac{A}{1+K_p} = \frac{A}{\infty} = 0.$$

ii) For a ramp i/p of A ; $R(s) = A/s^2$.

$K_v = \lim_{s\to 0} s.G(s)H(s) = \lim_{s\to 0} = \lim \frac{sK(1+T_1 s)(1+T_2 s)}{s(1+T_a s)(1+T_b s)} = \cancel{\infty}\ K$

iii) For a parabolic signal A.

$R(s) = A/s^3$ - $K_a = \lim_{s\to 0} s$

III Type '2' system

i) For a step I/P of 'A'

$R(s) = \frac{A}{s}$; $K_p = \lim_{s\to 0} G(s)H(s) = \lim_{s\to 0} \frac{K(1+T_1 s)(1+T_2 s)}{s^2(1+T_a s)(1+T_b s)} = \infty$

$$e_{ss} = \frac{A}{1+K_p} = \frac{A}{\infty} = 0.$$

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ii) For a ramp i/p of A $R(s) = A/s^2$

$$K_v = \lim_{s \to 0} s\,G(s)\,H(s) = \lim_{s \to 0} \frac{s \cdot K(1+T_1 s)(1+T_2 s)}{s^3(1+T_a s)(1+T_b s)} = \infty$$

$$e_{ss} = \frac{A}{K_v} = 0$$

iii) For a parabolic signal A,

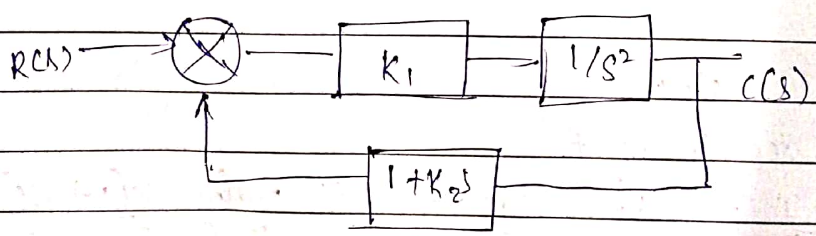
$$R(s) = \frac{A}{s^3} \quad K_a = \lim_{s \to 0} s^2 G(s)\,H(s) = \lim_{s \to 0} \frac{s^2\,K(1+T_1 s)(1+T_2 s)}{s^2\,(1+T_a s)(1+T_b s)} = K$$

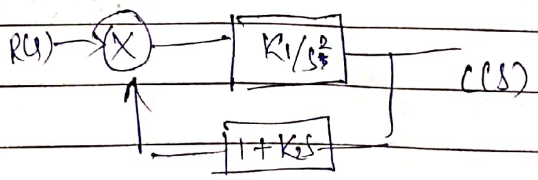
$$e_{ss} = \frac{A}{K_a} = \frac{A}{K}.$$

Type of i/p signal	Type 0	Type 1	Type 2
i) Step i/p	$\frac{A}{1+K}$	0	0
ii) Ramp i/p	∞	A/K	0
iii) Parabolic i/p	∞	∞	A/K

Error constants K_p, K_v & K_a describe ability of system to reduce or eliminate steady state errors. As the type of system becomes higher, progressively more steady state errors are eliminated.

For the system below determine the values of K_1 & K_2 so that M_p is 25% & T_p = 4 sec, assume unit step input.





$G(s) = K_1/s^2 \qquad H(s) = 1 + K_2 s$

$G(s)H(s) = \dfrac{K_1}{s^2(1+K_2 s)}$

$\dfrac{C(s)}{R(s)} = \dfrac{K_1}{s^2 + K_1K_2 s + K_1} \rightarrow ①$

Std form $\dfrac{C(s)}{R(s)} = \dfrac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$

$2\xi_1\omega_n = K_1K_2$

$\omega_n = \dfrac{K_1K_2}{2\xi_1} \qquad \omega_n^2 = K_1$

$\omega_n = \sqrt{K_1}$

$M_p = 25\% = 0.25 = e^{-\pi\xi/\sqrt{1-\xi_1^2}}$

$\xi_1 = 0.4037$

$T_p = 4\,sec = \dfrac{\pi}{\omega_d} = \dfrac{\pi}{\omega_n\sqrt{1-\xi_1^2}}$

$\omega_n = 0.8584$ rad/sec

$K_1 = 0.7369$

$K_2 = 0.9405$

A system is given by differential equation $\dfrac{d^2y}{dt^2} + 4\dfrac{dy}{dt} + 8y = 8x$ where y is the output & x is the input. & Determine all time domain specification for unit step input.

$\dfrac{d^2y}{dt^2} + 4\dfrac{dy}{dt} + 8y = 8x$

→ $s^2Y(s) + 4sY(s) + 8Y(s) = 8X(s)$

$Y(s)[s^2 + 4s + 8] = 8X(s)$

$\dfrac{Y(s)}{X(s)} = TF = \dfrac{8}{s^2+4s+8} = \dfrac{\omega_n^2}{s^2 + 2\xi_1\omega_n s + \omega_n^2}$

$2\xi_1\omega_n = 4 \qquad \omega_n^2 = 8$

$\xi_1 = 0.707 \qquad \omega_n = \sqrt{8} = 2.828$

$\cancel{M_p = e^{-\pi\xi/\sqrt{1-\xi^2}}}$

$\omega_d = \omega_n\sqrt{1-\xi_1^2} = \cancel{0.1716}$ 2.002 rad/sec.

$\% M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100 = 4.325\%$

$T_d = \frac{1+0.7\xi}{\omega} = 0.5286\,s.$

$T_s = \frac{4}{\xi\omega_n} = 2\,s.$

$T_p = \frac{\pi}{\xi\omega_d} = 1.571\,sec.$

$T_r = \frac{\pi-\theta}{\omega_d} = 1.18\,sec.$ } 0–100% (time taken)

Q A system has $G(s) = \frac{15}{(s+1)(s+3)}$ & $H(s) = 1$. determine the characteristic eqn., ω & ξ values, time at which the first undershoot will occur & time period of oscillations.

→ $TF = \frac{G(s)}{1+G(s)H(s)} = \frac{15/(s+1)(s+3)}{1+\frac{15}{(s+1)(s+3)}} = \frac{15}{s^2+4s+18}$

The characteristic equation is given by $(1+G(s)H(s)) = 0$

$s^2+4s+18 = 0.$

$2\xi\omega_n = 4$ $\quad$ $\omega_n^2 = 18$

$\xi = 0.471$ $\quad$ $\omega_n = 4.242$ $\quad$ $\omega_d = \omega_n\sqrt{1-\xi^2} = 3.742\,sec.$

$t = \frac{2\pi}{\omega_d} = \frac{2\pi}{\omega_n\sqrt{1-\xi^2}} = \frac{2\pi}{3.742} = 1.679\,sec.$

$T = \frac{2\pi}{\omega_d} = 1.679\,sec$

$T_s = \frac{4}{\xi\omega_n} = 2.002\,sec$

We can also determine no. of cycles at the output before it settles down

1 cycle → 1.679 sec

? → 2 sec

1.19 cycles

Q. For a system with OLTF $= \frac{K}{s^2(s+2)(s+3)}$. Determine the value of K to limit the steady state error to 10 ($e_{ss} = 10$) when input to the system is $r(t) = 1 + 10t + \frac{40}{2}t^2$. Also find type & order of the system.

→ $$G(s)H(s) = \frac{K}{s^2(s+2)(s+3)}$$

From the given signal determine the range of step i/p, ramp & parabolic i/p.

step i/p → $A_1 = 1$.

ramp i/p → $A_2 = 10$

parabolic i/p → $A_3 = 40$

$$K_p = \lim_{s \to 0} G(s)H(s)$$

$$= \lim_{s \to 0} \frac{K}{s^2(s+2)(s+3)} = \infty$$

$$K_v = \lim_{s \to 0} sG(s)H(s) = \lim_{s \to 0} \cancel{s}\left(\frac{K}{s^{\cancel{2}}(s+2)(s+3)}\right) = \infty$$

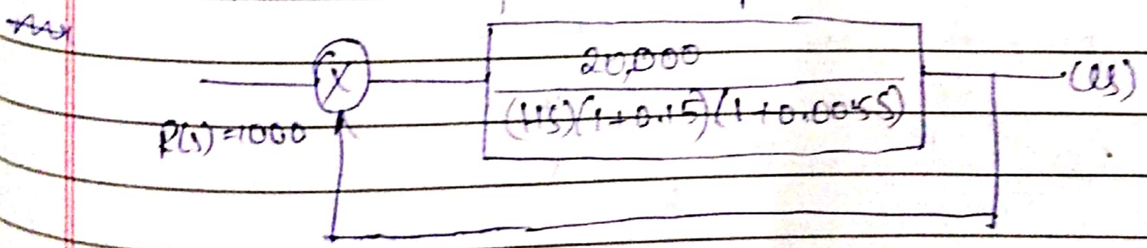
$$K_a = \lim_{s \to 0} s^2 G(s)H(s) = \lim_{s \to 0} \frac{\cancel{s^2} K}{\cancel{s^2}(s+2)(s+3)} = \frac{K}{6}$$

$$e_{ss} = \frac{A}{K_a} = \frac{6A}{K}.$$

$$K = \frac{6 \times 40}{10}$$

$K = 24$ [K → system gain]

Q. Following block diagram shows an oven the set point is at 1000°C what is the steady state temperature.



where $R(s) = 1000/s$

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$\cancel{1+}G(s)H(s) = \cancel{1+}\dfrac{20,000}{(\cancel{1+s})(1+0.1s)(1+0.005s)}$

$K_p = \lim_{s\to 0} G(s)H(s) = \dfrac{20,000}{1\times 1\times 1} = 20000$

$K_v = \lim_{s\to 0} sG(s)H(s) = 0$

$K_a = \lim_{s\to 0} s^2 G(s)H(s) = 0$

$e_{ss} = \dfrac{A}{1+K_p} = \dfrac{1000}{1+20000} = 0.0499.$

Actual o/p = desired o/p − error = $1000 - 0.0499 = 999.95^\circ C$

steady state temp.

Q. Negative feedback system has $G(s) = \dfrac{K}{s(s+2)(s^2+2s+5)}$, what is the type & order of the system?

For ~~a~~ unit ramp i/p it is required that $e_{ss} \le 0.2$, $K = ?$

Determine the steady state error if the i/p is $r(t) = 2+4t+\frac{t^2}{2}$.

→ $A_1 = 2$

$A_2 = 4$

$A_3 = 1.$

$K_p = \lim_{s\to 0} G(s)H(s) = \lim_{s\to 0} \dfrac{K}{s(s+2)(s^2+2s+5)} = \infty$

$K_v = \lim_{s\to 0} sG(s)H(s) = \lim_{s\to 0} \dfrac{\cancel{s}\,K}{\cancel{s}(s+2)(s^2+2s+5)} = \dfrac{K}{10}.$

$K_a = \lim_{s\to 0} sG(s)H(s) = \lim_{s\to 0} \dfrac{s^{\cancel{2}}\,K}{\cancel{s}(s+2)(s^2+2s+5)} = 0.$

$e_{ss} = \dfrac{A}{K_v} \Rightarrow 0.2 = \dfrac{A}{K} \times 10$

For unit ramp $A=1$.

$K = \dfrac{10}{0.2} = 50$

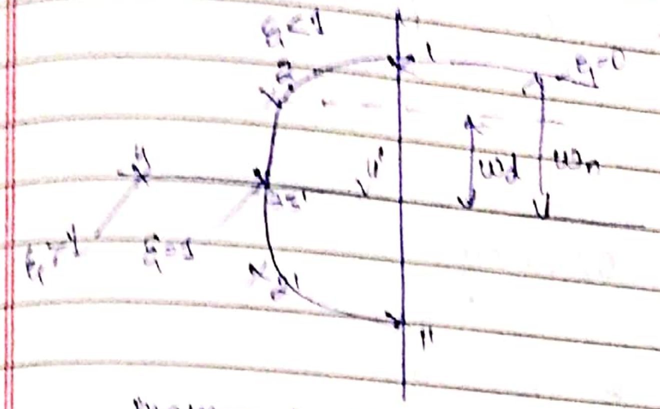
$K \ge 50$ $\quad$ $50 \le K \le \infty$

classmate

Date ______
Page ______

$e_{ss} = \frac{A}{K_v} \sim \frac{A \times 10}{K} = \frac{A \times 10}{50}$

$e_{ss} = 0 + \frac{10}{50} + \infty = \infty$

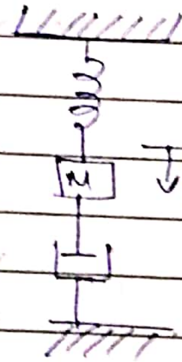


movement of poles as a function of ξ for a second order system

Q. A spin mass damper is given below. An experiment was conducted by applying force of 2N to the mass. The response was recorded using an xy plotter and the experimental result is as shown below. Determine the values of M, K & B.

→ $f(t) = 2N$ ⇒ $F(s) = 2/s$

M, K & B = ?



$\% M_p = 18\%$ $T_p = 2 sec.$

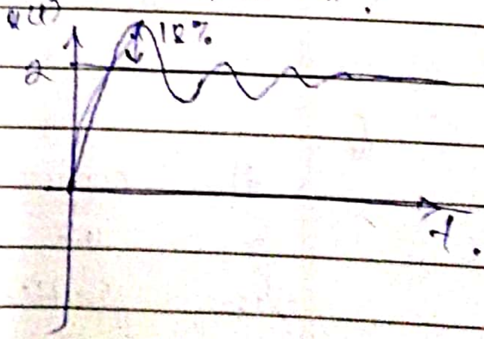
$f(t) = M\frac{d^2x(t)}{dt^2} + B\frac{dx(t)}{dt} + Kx(t)$

$TF = \frac{X(s)}{F(s)} = \frac{1}{MS^2 + BS + K} = \frac{1/M}{S^2 + B/M\, S + K/M}$

$\left(\frac{\omega_n^2}{S^2 + 2\xi\omega_n S + \omega_n^2}\right)$

$\omega_n^2 = \frac{K}{M}$ $\omega_n = \sqrt{\frac{K}{M}}$

$2\xi\omega_n = B/M$



Steady state value:

$X_{ss} = \lim_{S \to 0} SX(s) = S \cdot \frac{2}{S} \cdot \frac{1/M}{S^2 + \frac{B}{M}S + K/M}$

$2 = \frac{2/M}{K/M}$

$\therefore K = 1 N/m$

$\xi_1 = 0.5458$; $\dfrac{\xi_1}{\sqrt{1-\xi_1^2}} = 0.5458$

* $M_p = 0.18 = e^{-\pi\xi_1/\sqrt{1-\xi_1^2}}$

$\rightarrow \xi_1 = 0.4791$

$T_p = 2\,sec = \dfrac{\pi}{\omega_d} = \dfrac{\pi}{\omega_n\sqrt{1-\xi_1^2}}$

$\omega_n = 1.7895$ rad/sec.

$\omega_n = \sqrt{\dfrac{K}{M}}$

$M = \dfrac{K}{\omega_n^2} = 0.3122$ kg.

$2\xi_1\omega_n = \dfrac{B}{M} \Longrightarrow B = 2\xi_1\omega_n M.$

$B = 0.535$ N/m/sec

Q. Servomechanism is represented by an equation $\dfrac{d^2\theta}{dt^2} + 10\dfrac{d\theta}{dt} = 150E$ where $E = R - \theta$ is the actuating signal. Calculate the value of damping ratio, undamped & damped ~~ratio~~ frequency of oscillations.

$\rightarrow$ $\dfrac{d^2\theta(t)}{dt^2} + \dfrac{10\,d\theta(t)}{dt} = 150E = 150(r(t) - \theta(t))$

$s^2 \cdot \theta(s) + 10 s\,\theta(s) = 150\left[R(s) - \theta(s)\right]$

$\dfrac{\theta(s)}{R(s)} = TF = \dfrac{150}{s^2+10s+150} \rightarrow \dfrac{\omega_n^2}{s^2+2\xi_1\omega_n s+\omega_n^2}$

$2\xi_1\omega_n = 10$ $\qquad \omega_n^2 = 150$

$\xi_1 = 0.4082$ $\qquad \omega_n = 12.247$ rad/s.

$\omega_d = \omega_n\sqrt{1-\xi_1^2} = 10.2$ rad/s.

Q

The output response $c(t) = 1 + 0.2e^{-60t} - 1.2e^{-10t}$ when subjected to an unit step input, obtain the expression for closed loop transfer function & determine the value of damping ratio & undamped natural frequency of oscillations.

$c(t) = 1 + 0.2e^{-60t} - 1.2e^{-10t}$

$C(s) =$

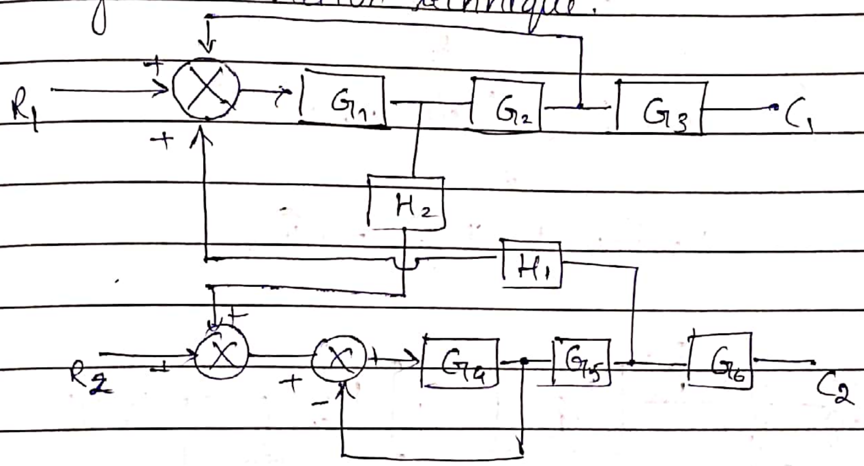
$$\frac{C(s)}{R(s)} = \frac{600}{s^2 + 70s + 600}$$

$\omega = 24.49 \text{ rad/s}$

$\xi = 0.43$

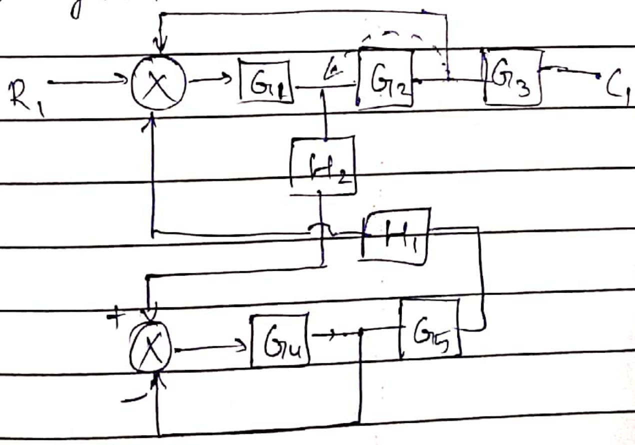
MIMO → multi-input multi output

Evaluate C_1/R_1 & C_2/R_1 for the following block diagram using block diagram reduction technique.



→ Assume R_2 to be zero

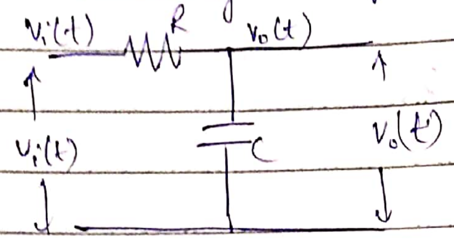
For C_1 ~~stuy~~ $/R_1$, assume $C_2 = 0$.



$$\frac{C_1}{R_1} = \frac{G_1 G_2 G_3 (1 + G_4)}{(1 + G_4)(1 + G_1 G_2) - G_1 G_4 G_5 H_1 H_2}$$

$$\frac{C_2}{R_1} = \frac{G_1 G_4 G_5 G_6 H_2}{(1+G_4 G_2)(1+G_4) - G_1 G_4 G_5 H_1 H_2}$$

Block diagram representation.



where $i(t) = \frac{v_i(t) - v_o(t)}{R}$ — ①

$I(s) = \frac{V_i(s) - V_o(s)}{R}$ — ②

& $v_o(t) = \frac{1}{C}\int_0^t i(t)\, dt$ — ③

$V_o(s) = \frac{1}{Cs} I(s)$ — ④