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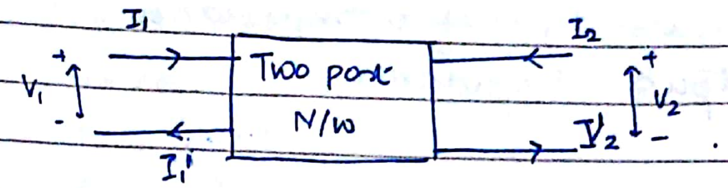
3. Two Port Network.



Amp ⇒ Zin, Zout, Vg. ratio, I ratio. ⟹ parameters

→ Z parameter (Impedance)

→ Y parameter (Admittance)

→ H parameter (Hybrid)

→ ABCD (transmission parameter).

* Z parameter: (O/C parameter)

$$V_1 = f(I_1, I_2)$$

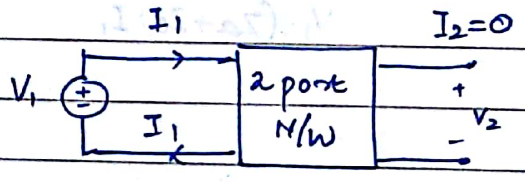
$$V_2 = f(I_1, I_2).$$

$$\Rightarrow V_1 = Z_{11}I_1 + Z_{12}I_2$$

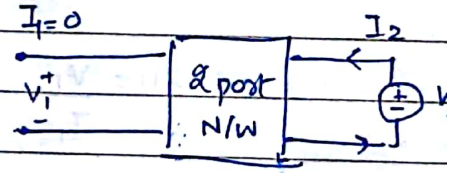
$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

- $Z_{11} = V_1/I_1 \Big|_{I_2=0}$ $\qquad Z_{21} = \dfrac{V_2}{I_1}\Big|_{I_2=0}$



- $Z_{12} = \dfrac{V_1}{I_2}\Big|_{I_1=0}$ $\qquad Z_{22} = \dfrac{V_2}{I_2}\Big|_{I_1=0}$



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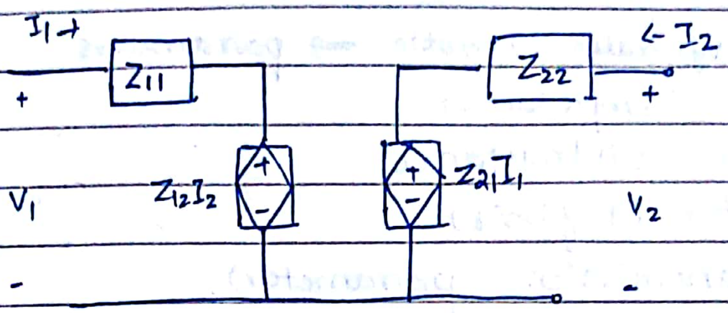
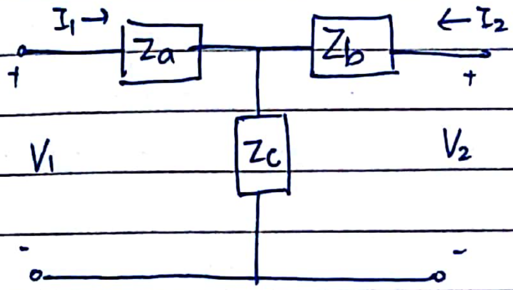
Z_{11} = open circuit input impedance
Z_{21} = open ckt forward transfer impedance
Z_{12} = o/c reverse transfer impedance
Z_{22} = o/c output impedance



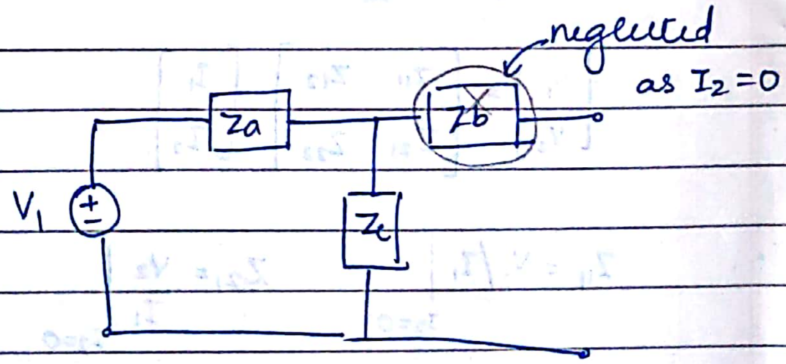
eg 1:



$\therefore Z_{11} = \left.\dfrac{V_1}{I_1}\right|_{I_2=0}$.

• To find Z_{11}

$V_1 = (Z_a + Z_c) I_1$



$\Rightarrow Z_{11} = \dfrac{V_1}{I_1} = \dfrac{(Z_a+Z_c)I_1}{I_1} = (Z_a+Z_c)\,\Omega$.

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- For $Z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0}$ (same ckt)

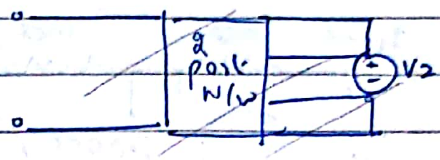
V_2 = voltage across Z_C

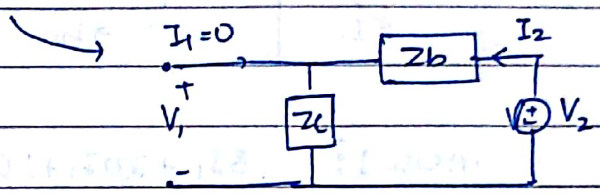
$V_2 = V_{ZC}$

$V_2 = Z_C \times I_1$.

$Z_{21} = \frac{Z_C \times I_1}{I_1} = Z_C\,\Omega$

- For ~~Z_{12}~~ $Z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0}$





$V_1 = Z_C I_2$

$Z_{12} = \frac{Z_C I_2}{I_2} = Z_C\,\Omega$

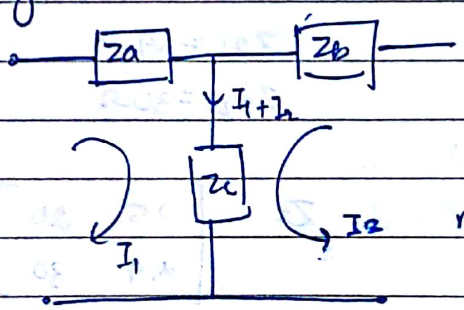
IIIly

$Z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0} = \frac{(Z_b+Z_C)\,I_2}{I_2} = (Z_b+Z_C)\,\Omega.$

$$\Rightarrow \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$Z = \begin{bmatrix} \cancel{Z_C(a+Z_b)}\,(Z_a+Z_c) & Z_c \\ Z_c & (Z_b+Z_c) \end{bmatrix} \Omega$$

* By mesh analysis.



mesh 1:

$-V_1 + Z_a I_1 + Z_C(I_1+I_2) = 0$

$V_1 = (Z_a+Z_C)\,I_1 + Z_C I_2$ — (1)

mesh 2:

$-V_2 + Z_b I_2 + Z_C\,(I_1+I_2) = 0$

$V_2 = Z_C I_1 + (Z_b+Z_C)\,I_2$ — (2)

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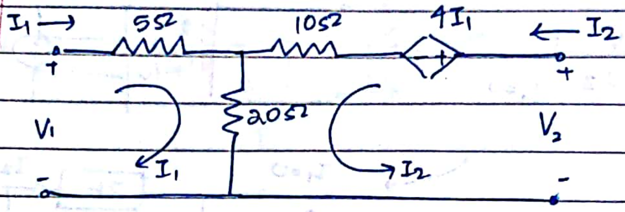
$Z_{21} = Z_{12}$ symmetric network
system is reciprocal

$Z_{11} = Z_{22} \Rightarrow$ system is symmetric

eg 2. * Given this network, find the Z parameter



mesh 1: $5I_1 + 20I_1 + 20I_2 = V_1$ — ①

mesh 2: $4I_1 + 10I_2 + 20I_2 + 20I_1 = V_2$ — ②

$25I_1 + 20I_2 = V_1$ — ①

$24I_1 + 30I_2 = V_2$ — ②

$\therefore Z_{11} \Rightarrow$

$V_1 = Z_{11}I_1 + Z_{12}I_2$

$V_2 = Z_{21}I_1 + Z_{22}I_2$

comparing eq

$Z_{11} = 25\,\Omega$

$Z_{12} = 20\,\Omega$

$Z_{21} = 24\,\Omega$

$Z_{22} = 30\,\Omega$

$$Z = \begin{bmatrix} 25 & 20 \\ 24 & 30 \end{bmatrix} = 270\,\Omega$$

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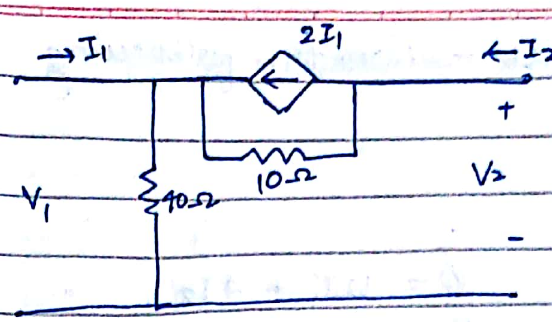
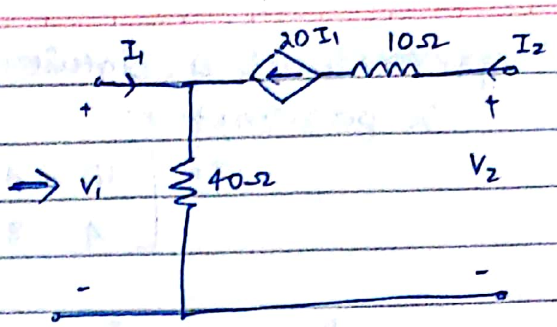
eg 3 *.





$$V_1 = 40I_1 + 40I_2 \quad -①$$

$$V_2 = -20I_1 + 50I_2 + 40I_1 \quad -②$$

comparing $Z_{11} = 40$

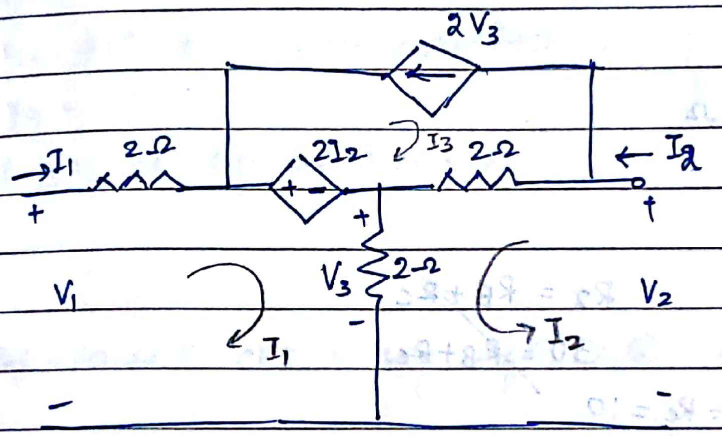
$Z_{12} = 40$

$Z_{21} = 20$

$Z_{22} = 50$.

$$Z = \begin{bmatrix} 40 & 40 \\ 20 & 50 \end{bmatrix}$$

eg 4.



$I_3 = -2V_3$

$V_3 = 2(I_1 + I_2)$

$\Rightarrow I_3 = -4I_1 - 4I_2$

mesh 1:

$$-V_1 + 2I_1 + 2I_2 + 2(I_1 + I_2) = 0$$

$$4I_1 + 4I_2 = V_1$$

mesh 2:

$$2(I_2 + I_3) + 2(I_2 + I_1) = V_2 .$$

$$2I_1 + 4I_2 + 2I_3 = V_2$$

$$2I_1 + 4I_2 - 8I_1 - 8I_2 = V_2$$

$$-6I_1 - 4I_2 = V_2$$

$Z_{11} = 4$, $Z_{12} = 4$, $Z_{21} = -6$, $Z_{22} = -4\,\Omega$

$$Z = \begin{bmatrix} 4 & 4 \\ -6 & -4 \end{bmatrix}$$

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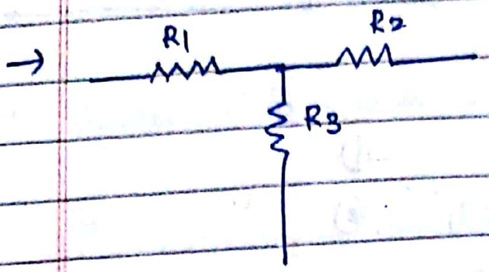
eg5. construct a circuit that realises the following Z parameter.

$$Z = \begin{bmatrix} 12 & 4 \\ 4 & 8 \end{bmatrix}$$

$V_1 = 12I_1 + 4I_2$

$V_2 = 4I_1 + 8I_2$

→ 

$Z_{11} = 12$ $\quad$ $Z_{12} = 4$

$Z_{21} = 4$ $\quad$ $Z_{22} = 8$

$Z_{11} = R_1 + R_3 = R_1 + 4 = 12$

$Z_{22} = R_2 + R_3 = 4 + R_3 = 8$

Z_{12} & Z_{21} are same

$R_1 = 8$ $\quad$ $R_2 = 4$ $\quad$ $R_3 = 4$

$R_1 = 8\Omega$

$R_3 = 4\Omega$

$R_2 = 4\Omega$.

eg6:

$$Z = \begin{bmatrix} 40 & 10 \\ 20 & 30 \end{bmatrix} \Omega$$

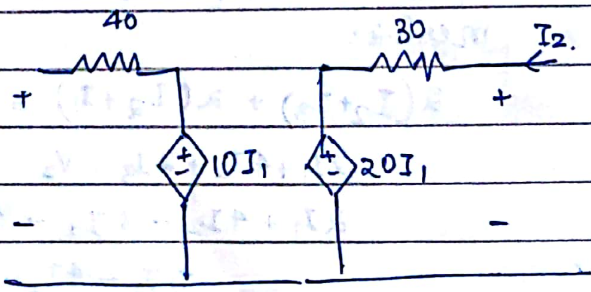
$R_1 = \cancel{40} + R_A + R_C$ $\quad$ $R_2 = R_B + R_C$

$\cancel{R_1} = 40 = R_A + R_C$ $\quad$ $30 = R_B + R_C$

$R_3 = R_C = 10$

$40 = R_A + 10$ $\quad$ $30 = R_B + 10$

$R_A = 30$ $\quad$ $R_B = 20$



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* Y parameter (admitance parrameter) / s/c parameter

$I_1 = f(V_1, V_2)$

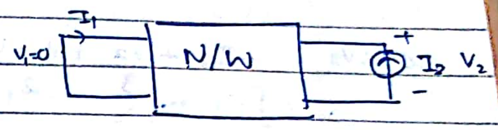
$I_2 = f(V_1, V_2)$

$I_1 = Y_{11} V_1 + Y_{12} V_2 \quad —①$

$I_2 = Y_{21} V_1 + Y_{22} V_2 \quad —②$

$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \qquad Y_{21}\Big|_{V_2=0} = \frac{I_2}{V_1}$

$Y_{21} = \left.\frac{I_1}{V_2}\right|_{V_1=0} \qquad Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$



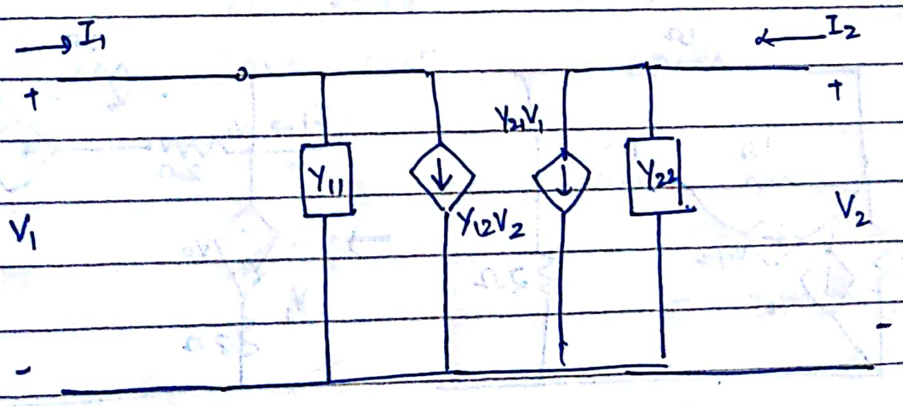
Y_{11} = s/c i/p admitance

Y_{12} = s/c transfer admitance } equal → reciprocal

Y_{21} = "   if $Y_{22} = Y_{11}$ → symmetric

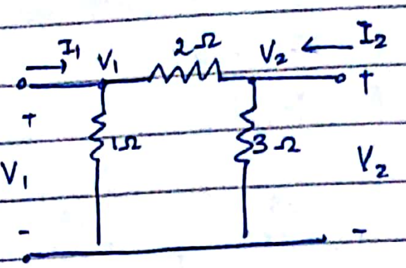
Y_{22} = S/c o/p admitance

• equivalent ckt using eq① & ②. for y parameter



for y → make in π network.

eg 1:



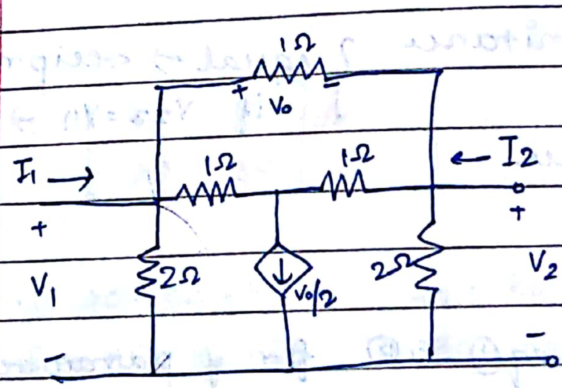
node V_1 $\quad -I_1 + \frac{V_1}{1} + \frac{V_1 - V_2}{2} = 0 \rightarrow V_1[1 + 1/2] + V_2[-1/2] = I_1$

node V_2 $\quad -I_2 + \frac{V_2}{3} + \frac{V_2 - V_1}{2} = 0 \rightarrow V_1[-1/2] + V_2[1/3 + 1/2] = I_2$

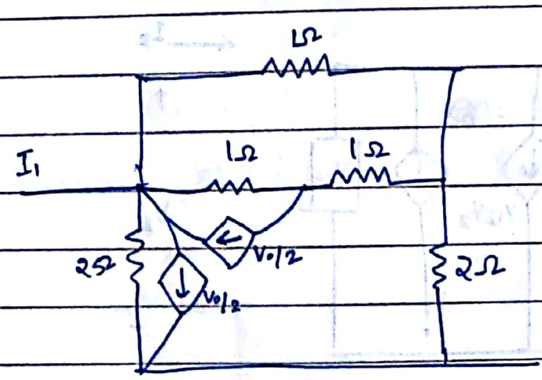
comparing $\quad Y_{11} = 3/2 \ \mho \quad Y_{12} = -1/2 \ \mho$

$\quad Y_{21} = -1/2 \ \mho \quad Y_{22} = 5/6 \ \mho$

eg 2:



→



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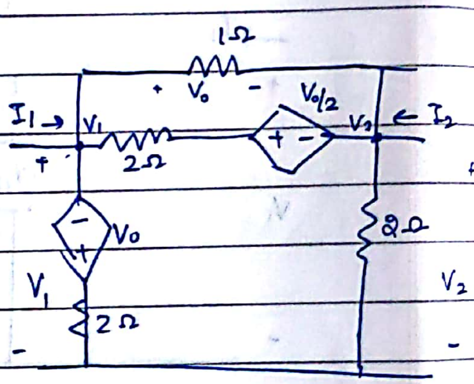

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For node 1: $-I_1 + \frac{V_1 + V_0}{2} + \frac{V_1 - V_0/2 - V_2}{2} + \frac{V_1 - V_2}{1} = 0$ —①

For node 2: $-I_2 + \frac{V_2 + V_0/2 - V_1}{2} + \frac{V_2}{2} + \frac{V_2 - V_1}{1} = 0.$ —②

① → $V_1 [1/2 + 1/2 + 1] + V_2 [-1/2 - 1] + V_0 [1/2 - 1/4] = 0$

As $V_0 = V_1 - V_2$

$V_1 [1/2 + 1/2 + 1] + V_2 [-1/2 - 1] + (V_1 - V_2)(1/2 - 1/4) = 0$

$V_1 [1/2 + 1/2 + 1 + 1/2 - 1/4] + V_2 [-1/2 - 1 - 1/2 + 1/4] = 0.$ —①

② → $V_1 [-1/2 - 1] + V_2 [1/2 + 1/2] + V_0 [1/4] = 0$

As $V_0 = V_1 - V_2$

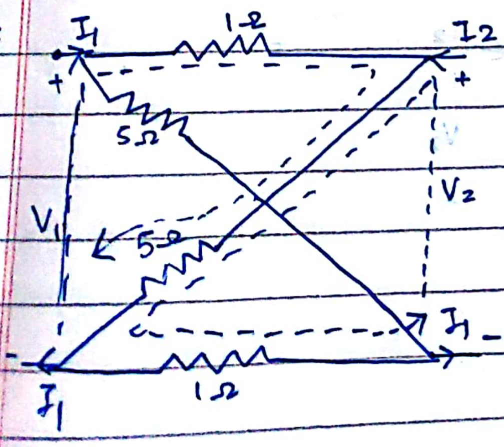
$V_1 [-1/2 - 1 + 1/4] + V_2 [1/2 + 1/2 + 1 - 1/4] = 0$ —②

comparing the eqs

$Y_{11} = 2.25$ $\quad Y_{12} = -1.75$ $\quad \rightarrow Y = \begin{bmatrix} 2.25 & -1.75 \\ -1.25 & 1.75 \end{bmatrix}$

$Y_{21} = -1.25$ $\quad Y_{22} = 10.75$

eg 3:



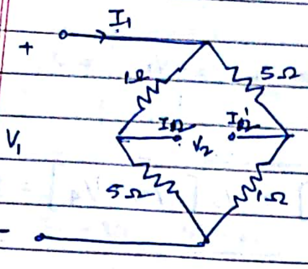
mesh 1:

$-V_1 + 1(I_1 - I_2) + 5(I_1 + I_2) = 0$

mesh 2.:

$-V_2 + 5(I_1 + I_2) + 1(I_2 - I_1) = 0.$

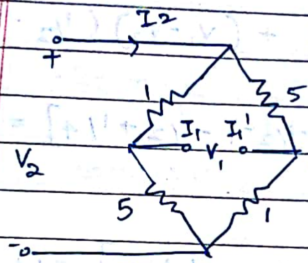
Redrawing the circuit.



mesh 1:

$-V_1 + 1(I_1 - I_2) + 5(I_1 + I_2) = 0$

$V_1 = 6I_1 + 4I_2$



mesh 2:

~~$-V_2 + 1(I_2 + I_1)$~~

$-V_2 + 1(I_2 - I_1) + 5(I_2 + I_1) = 0$

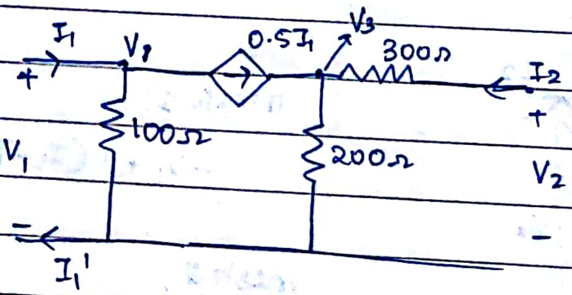
$V_2 = 6I_2 + 4I_1$

comparing $Z_{11} = 6$ $\quad Z_{21} = 4$

$Z_{12} = 4$ $\quad Z_{22} = 6$

$$Z = \begin{bmatrix} 6 & 4 \\ 4 & 6 \end{bmatrix}$$

Q.4 Find Y parameter and Z parameter



By nodal analysis.

node V_1

$$-I_1 + \frac{V_1}{100} + 0.5I_1 = 0.$$

$$\frac{V_1}{100} - 0.5I_1 = 0 \quad \text{—(1)}$$

$$V_1 = 50I_1$$

$$I_1 = \frac{\cancel{50}}{\cancel{V_1}}\frac{V_1}{50} \quad \text{—(7)}$$

$Z_{11} = 50 \qquad Z_{12} = 0$

$Z_{21} = 100 \qquad Z_{22} = 500.$

$Y_{11} = \frac{1}{50} \qquad Y_{12} = 0$

$\cancel{Y_{21} = \frac{1}{100}} \qquad Y_{22} = \frac{1}{500}$

$Y_{21} = -0.004 \mho$

node V_3

$$-0.5I_1 + \frac{V_3}{200} + \cancel{\frac{V_3 - V_2}{300}} - I_2 = 0$$

$$\frac{V_3}{200} - 0.5I_1 - I_2 = 0 \quad \text{—(2)}$$

Also =

$$\frac{V_2 - V_3}{300} = I_2$$

$$V_2 - 300I_2 = V_3 \quad \text{—(3)}$$

sub eq (3) in eq (2)

$$\frac{V_2 - 300I_2}{200} - 0.5I_1 - I_2 = 0$$

$$\frac{V_2}{200} - 0.5I_1 - 301I_2 = 0 \quad \text{—(4)}$$

$$V_2 = 100I_1 + 500I_2$$

$\cancel{I_1 = \frac{V_2}{100}} \qquad I_2 = \frac{V_2}{500} - \frac{100I_1}{500}$ —(5)

$I_1 = \frac{V_2}{100} - \frac{500I_2}{100}$ —(6) got Y_{11} & Y_{12} from eq (7)

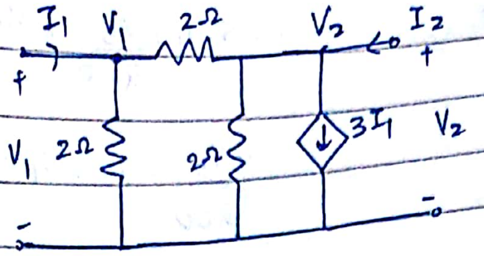
eq (5) comparing with

$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

I_1 is not reqd ∴ eliminate $\frac{-100}{500}I_1$ in eq (5)

∴ eq (5) ⇒ $I_2 = \frac{-1}{5}(0.002V_1) + \frac{V_2}{500}$

eg 5: Find y and z parameter.



Nodal analysis:

node V_1:

$$-I_1 + \frac{V_1}{2} + \frac{V_1 - V_2}{2} = 0$$

$$V_1\left[\frac{1}{2}+\frac{1}{2}\right] + V_2\left[\frac{-1}{2}\right] = I_1$$

$$V_1 - \frac{V_2}{2} = I_1 \quad -(1)$$

node 2

$$\frac{V_2}{2} + \frac{V_2 - V_1}{2} + 3I_1 - I_2 = 0$$

$$\frac{V_2}{2} + V_1\left[\frac{-1}{2}\right] + V_2\left[\frac{1}{2}\right] + 3I_1 = I_2$$

$$\frac{V_2}{2} + \frac{-V_1}{2} + \frac{V_2}{2} + \frac{3V_1 - 3V_2}{2} = I_2$$

$$\frac{5V_1}{2} - \frac{V_2}{2} = I_2 \quad -(2)$$

$Y_{11} = 1 \qquad Y_{12} = -0.5$

$Y_{21} = 2.5 \qquad Y_{22} = -0.5$

$$Y = \begin{bmatrix} 1 & -0.5 \\ 2.5 & -0.5 \end{bmatrix}.$$

from eq (1) $\Rightarrow$ $I_1 = V_1 - 0.5V_2$

$V_1 = I_1 + 0.5V_2$

$\cancel{V_1 = I_1 +}$

$I_2 = 2.5V_1 - 0.5V_2$

$V_2 = \frac{2.5V_1}{0.5} - \frac{I_2}{0.5}$

Rearrange case $\left[\; V_1 = Z_{11}I_1 + Z_{12}I_2 \qquad V_2 = Z_{21}I_1 + Z_{22}I_2 \;\right]$

$\Rightarrow V_1 = I_1 + 0.5V_2$

$V_2 = 5\left[I_1 + 0.5V_2\right] - \frac{I_2}{0.5}$

$V_1 = I_1 + 0.5(-3.3I_1 + 1.33I_2)$

$-0.5V_2 = 5I_1 - \frac{I_2}{0.5}$

$V_1 = I_1 - 1.66I_1 + 0.665I_2$

$V_1 = -0.66I_1 + 0.66I_2$

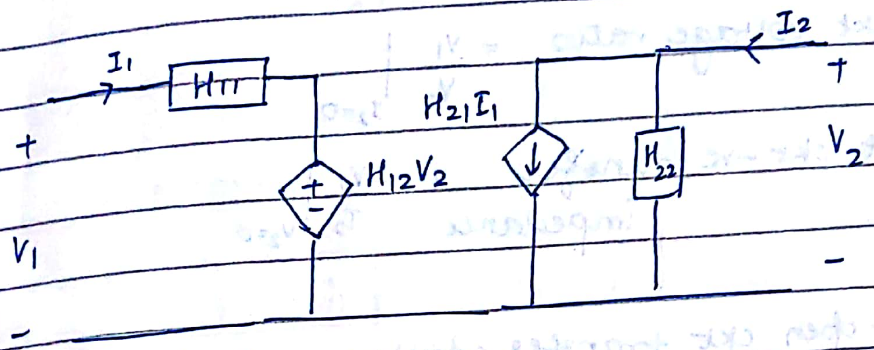
$V_2 = -3.3I_1 + 1.33I_2$

$$Z = \begin{bmatrix} -0.66 & 0.66 \\ -3.3 & 1.33 \end{bmatrix}$$

Hybrid parameter.

$V_1 = f(I_1, V_2)$
$I_2 = f(I_1, V_2)$.

$V_1 = H_{11} I_1 + H_{12} V_2$
$I_2 = H_{21} I_1 + H_{22} V_2$.



$H_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$ = s/c Input impedance

$H_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$ = o/c Reverse voltage gain

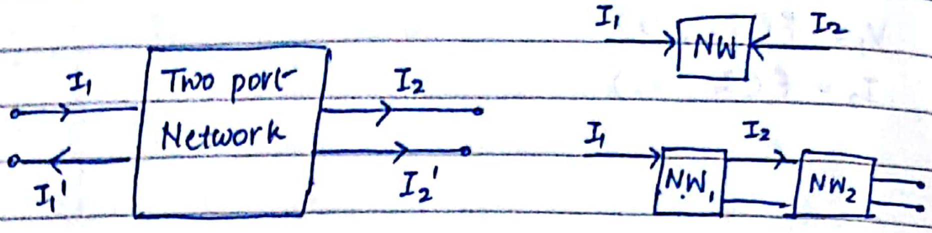
$H_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$ = s/c forward current gain

$H_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$ = o/c o/p admitance

* $H_{12} = -H_{21} \rightarrow$ Reciprocal

* $\Delta H = 1 \rightarrow$ symmetric.

* T - transmission parameter / ABCD parameter



$V_1 = AV_2 - BI_2$

$I_1 = CV_2 - DI_2$

A = open ckt voltage ratio $= \left.\frac{V_1}{V_2}\right|_{I_2=0}$

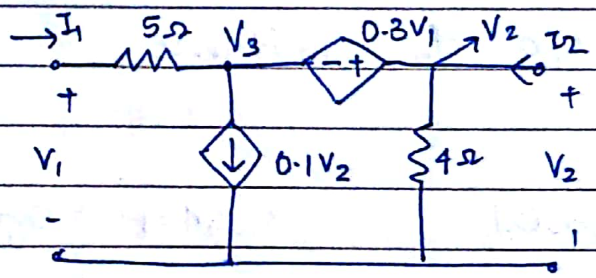
B = short ckt -ve transfer impedance $= \left.\frac{-V_1}{I_2}\right|_{V_2=0}$

C = ~~sho~~ open ckt transfer admitance $= \left.\frac{I_1}{V_2}\right|_{I_2=0}$

D = negative short circuit current ratio $= \left.\frac{-I_1}{I_2}\right|_{V_2=0}$

* $AD - BC = 1 \rightarrow$ Reciprocal
* $A = D \rightarrow$ symmetry

eg 1: Find all the parameters.



Supernode

store 67

source eq : $V_2 - V_3 = 0.3V_1$ — ①

~~$V_2 - 0.3V_1 = V_3$ — ①~~

node(V_3, V_2)

$$-I_1 + 0.1V_2 + \frac{V_2}{4} - I_2 = 0 \quad — ②$$

$$V_2 = 2.85 I_1 + 2.85 I_2 \quad — ③$$

$$I_1 = \frac{V_1 - V_3}{5}$$

$$\boxed{V_3 = V_1 - 5I_1}$$

sub in eq ①

$$① \Rightarrow \quad V_2 - V_1 + 5I_1 = 0.3V_1$$

$$1.3V_1 = 5I_1 + V_2.$$

$$I_1 = 0.26V_1 - 0.2V_2 \quad — ④$$

$I_1 = Y_{11}V_1 + Y_{12}V_2$ $\qquad$ $V_1 = Z_{11}V_1 + Z_{12}V_2$

$I_2 = Y_{21}V_1 + Y_{22}V_2$ $\qquad$ $V_2 = Z_{21}V_1 + Z_{22}V_2.$

$Y_{11} = 0.26 \quad Y_{12} = -0.2 \qquad Z_{21} = 2.85 \quad Z_{22} = 2.85.$

$$③ \Rightarrow \quad V_2 = 2.85(0.26V_1 - 0.2V_2) + 2.85I_2$$

$$V_2 = 0.741V_1 - 0.57V_2 + 2.85I_2.$$

$$I_2 = -0.26V_1 + 0.550V_2 \quad — ⑤$$

$Y_{21} = -0.26 \qquad Y_{22} = 0.55$

$$Y = \begin{bmatrix} 0.26 & -0.2 \\ -0.26 & 0.55 \end{bmatrix}$$

$I_1 = 0.26V_1 - 0.2V_2$

$= 0.26V_1 - 0.2(2.85I_1 + 2.85I_2)$

$I_1 = 0.26V_1 - 0.57I_1 - 0.57I_2$

$V_1 = 6.038I_1 + 2.192I_2$ — ⑥

$z_{11} = 6.03 \quad z_{12} = 2.19$

$$z = \begin{bmatrix} 6.03 & 2.19 \\ 2.85 & 2.85 \end{bmatrix}$$

put eq ⑤ in eq ⑥

$V_1 = 6.038I_1 + 2.192(-0.26V_1 + 0.550V_2)$

$V_1 = 6.038I_1 - 0.569V_1 + 1.2056V_2$

$1.569V_1 = 6.038I_1 + 1.2056V_2$

$V_1 = 3.848I_1 + 0.768V_2$

$I_2 = -0.26V_1 + 0.550V_2$ put eq ⑥ in eq ⑤

$I_2 = -0.26(6.038I_1 + 2.192I_2) + 0.550V_2$

$I_2 = -1.56I_1 - 0.569I_2 + 0.550V_2$

$I_2 = -0.994I_1 + 0.350V_2$ — ⑦

$h_{11} = 3.848 \quad h_{12} = 0.768$

$h_{21} = -0.994 \quad h_{22} = 0.350 \qquad H = \begin{bmatrix} 3.848 & 0.76 \\ -0.99 & 0.355 \end{bmatrix}$

store
67

transission

$V_1 = AV_2 - BI_2$

$I_1 = CV_2 - DI_2$

eq ⑥ → $V_1 = 6.038 I_1 + 2.19 I_2$ (put eq ④)

$V_1 = 6.036 (0.26 V_1 - 0.26 V_2) + 2.19 I_2$

$V_1 = 1.569 V_1 - 1.569 V_2 + 2.19 I_2$

$V_1 = 2.75 V_2 - 3.84 I_2$

~~eq ④ →~~ $\cancel{I_1 = 0.26 V_1 - 0.26 V_2}$

$\cancel{I_1 = 0.26 (2.75 V_2 - 3}$

eq ⑦

$I_1 = \frac{+0.350 V_2}{0.994} - \frac{I_2}{0.994}$

$I_1 = 0.352 V_2 - 1.066 I_2$

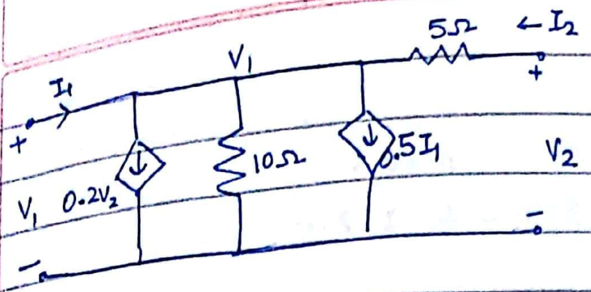
$A = 2.75$ $B = 3.84$

~~Tcett~~ $C = 0.35$ $D = 1.066$

$\therefore T = \begin{bmatrix} 2.75 & 3.84 \\ 0.35 & 1.066 \end{bmatrix}$

	Z	Y
Z	$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$	$\begin{bmatrix} Y_{22}/\Delta Y & -Y_{12}/\Delta Y \\ -Y_{21}/\Delta Y & Y_{11}/\Delta Y \end{bmatrix}$
Y.	$\begin{bmatrix} Z_{22}/\Delta Z & -Z_{12}/\Delta Z \\ -Z_{21}/\Delta Z & Z_{11}/\Delta Z \end{bmatrix}$	$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$

eg 7.



By nodal V_1 :

$$-I_1 + \frac{V_1}{10} + 0.2V_2 + 0.5I_1 + \frac{V_1 - V_2}{5} = 0$$

$$V_1\left[1/10 + 1/5\right] + V_2\left[-1/5 + 0.2\right] + I_1\left[0.5 - 1\right] = 0$$

node V_2 =

$$\frac{V_2 - V_1}{5} - I_2 = 0$$

$$\frac{V_2 - V_1}{5} = I_2$$

eq① $I_1 = +0.6V_1 + 0$ $\qquad$ $V_1 = 1.667 I_1 + 0$ — ③

eq② $I_2 = -0.2V_1 + 0.2V_2$ $\qquad$ $V_2 = \frac{I_2}{0.2} + \frac{0.2V_1}{0.2}$

$Y_{11} = 0.6$ $\quad Y_{12} = 0$ $\qquad$ $V_2 = 5I_2 + V_1$ — ④

$Y_{21} = -0.2$ $\quad Y_{22} = 0.2$

$V_2 = 5I_2 + 1.66 I_1$ — ⑤

$Z_{11} = 1.66$ $\quad Z_{12} = 0$

~~$Z_{21} = 5$~~ $\quad$ ~~$X_{22} = 1.66$~~

$Z_{21} = 1.66$ $\quad Z_{22} = 5$

$V_1 = H_{11} I_1 + H_{12} V_2$

$I_2 = H_{21} \cdot I_1 + H_{22} V_2$

Date
67

eq(3) → $V_1 = 1.667 I_1 + 0.$ $H_{11} = 1.66$ $H_{12} = 0$

$H_{21} = -0.33$ $H_{22} = 0.2.$

eq(5) → $I_2 = \frac{V_2}{5} - \frac{I_1(1.66)}{5}$

$= 0.2V_2 - 0.332 I_1$

$$H \begin{bmatrix} 1.667 & 0 \\ \cancel{0.2} & \cancel{0.332} \\ -0.332 & 0.2 \end{bmatrix}$$

T parameter

eq(4) → $V_1 = A V_2 - B I_2$

eq(5) → $I_1 = C V_2 - D I_2$

$V_1 = V_2 - 5I_2$

$I_1 = \frac{V_2}{1.66} - \frac{5I_2}{1.66}$

$A = 1$ $B = +5$

$C = 0.602$ $D = 3.01$

$$T = \begin{bmatrix} 1 & 5 \\ 0.602 & 3.01 \end{bmatrix}$$

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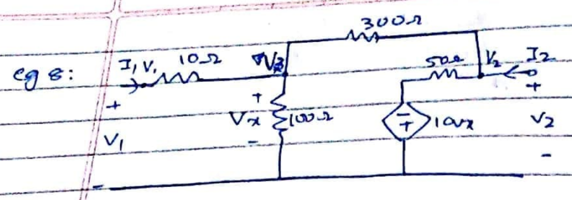
eg 8:



node V_3

$$\frac{V_3 - V_1}{10} + \frac{V_3}{100} + \frac{V_3 - V_2}{300} = 0$$

$$① \rightarrow V_1\left[-1/10\right] + V_2\left[-1/300\right] + V_3\left[1/10 + 1/100 + 1/300\right] = 0$$

node V_2

$$\frac{V_2 - V_3}{300} + \frac{V_2 + 10V_x}{50} - I_2 = 0$$

$$\frac{V_2 - V_3}{300} + \frac{V_2 + 10V_3}{50} - I_2 = 0$$

$$② \rightarrow V_2\left[1/300 + 1/50\right] + V_3\left[-1/300 + 10/50\right] = I_2$$

node V_1

$$-I_1 + \frac{V_1 - V_3}{10} = 0$$

$$I_1 = \frac{V_1 - V_3}{10} \qquad \boxed{-10 I_1 + V_1 = V_3} \rightarrow ⊛$$

$\cancel{10 I_1 =}$ ~~or~~

$\cancel{I_1 = 0.1V_1 - 0.1V_3}$

$\cancel{V_3 =}$

$$\cancel{① \rightarrow V_1\left[-1/10\right] + V_2\left[-1/300\right] + (0.1V_1 - 0.1V_3)(0.113) = 0}$$

store 67

eq ① $-0.1V_1 - 0.003V_2 + \underbrace{(V_1 - 10I_1)}_{\text{due to eq } ⊛}(0.113) = 0$

$$-0.1V_1 - 0.003V_2 + 0.113V_1 - 1.13I_1 = 0$$

$$I_1 = 0.0115V_1 \mp 0.0026V_2 \; -$$

$$Y_{11} = 0.0115 \quad Y_{12} = -0.0026$$

eq ② $V_2(0.023) + 0.196V_3 = I_2$ ← due to eq ⊛.

$$V_2(0.023) + (0.196)(\cancel{0.1V_1} - \cancel{0.1} - 10I_1 + V_1) = I_2$$

$$0.0023V_2 - 1.96I_1 + 0.196V_1 = I_2$$

sub eq ③ in this

$$0.0023V_2 - 0.0225V_1 - 0.0050V_2 + 0.196V_1 = I_2$$

$$I_2 = 0.1735V_1 + 0.0027V_2$$

$$Y_{21} = 0.1735 \quad Y_{22} = 0.0027$$

$$Y \begin{bmatrix} 0.0115 & -0.0026 \\ 0.1735 & 0.0027 \end{bmatrix}$$

$$\Delta Y = 0.73$$

$$3.105 \times 10^{-5} + 4.511 \times 10^{-9}$$

$$(0.3105 + 4.511) \times 10^{-9}$$

$$4.82 \times 10^{-4}$$

$$\boxed{0.482 \times 10^{-3}}$$

rearranging eq

$$V_1 = \frac{I_1}{0.0115} + \frac{0.0026V_2}{0.0115}$$

$$= 8.69I_1 + 0.0226V_2$$

$$= 8.69I_1 + 0.0226\left(\frac{I_2}{0.0027} - \frac{0.1735V_1}{0.0027}\right)$$

$$V_1 = 8.69I_1 + 8.37I_2 - 1.45V_1$$

$$V_1 = 3.54I_1 + 3.29I_2$$

$$V_2 = \frac{I_2}{0.0027} - \frac{0.1735V_1}{0.0027}$$

$$V_2 = 370.37I_2 - 64.25V_1$$

$$V_2 = 370.37I_2 - 64.25(3.54I_1 + 3.29I_2)$$

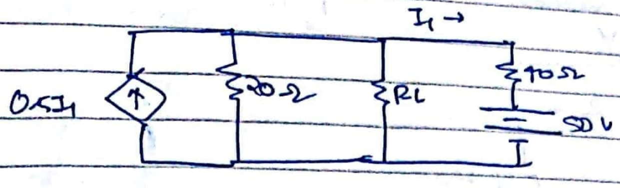
$$V_2 = 370.37I_2 - 227.445I_1 - 211.38I_2$$

$$V_2 =$$

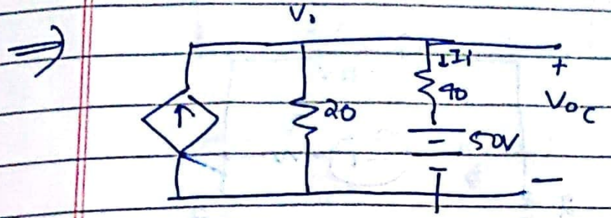
$$Z_{11} = \frac{Y_{22}}{\Delta y} = \cancel{0.7} \frac{-0.0026}{0.73m} = \frac{\cdot 0.002}{\cancel{0.073}} \frac{0.0026}{0.73m} = 2.7$$

store
67

1a. Determine max. power that could be dissipated in R_L



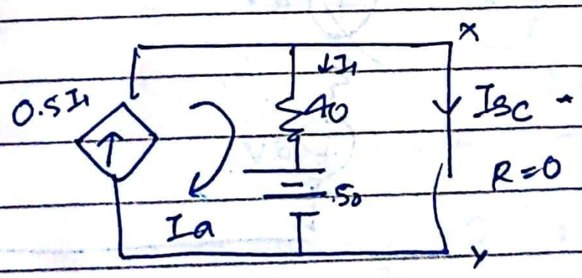
$\Rightarrow$



$$-0.5I_1 + \frac{V_{oc}}{20} + \frac{V_{oc}-50}{40} = 0$$

$$-0.5\left(\frac{V_{oc}-50}{40}\right) + \frac{V_{oc}}{20} + \frac{V_a-50}{40} = 0$$

$$\boxed{V_{oc} = 10V}$$



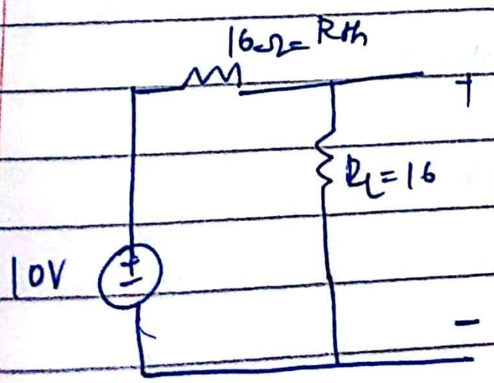
$I_a = 0.5 I_1 = 0.5(I_a - I_b)$

$= 40(I_b - I_a) = 50$

$I_a =$

$I_b = 0.625A = I_{SC}$

~~R_1~~



$$R_{th} = \frac{10}{0.625} = 16\Omega = R_L$$

$$P_{max} = \frac{5^2}{16} = 1.5625$$

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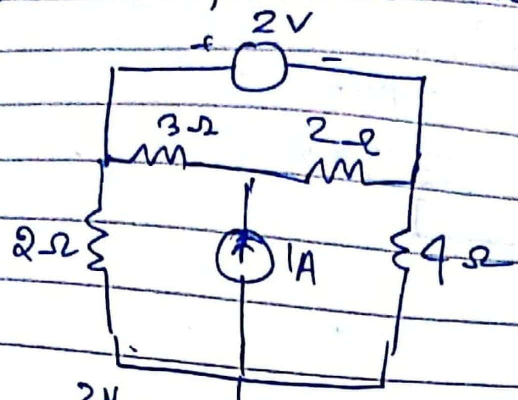
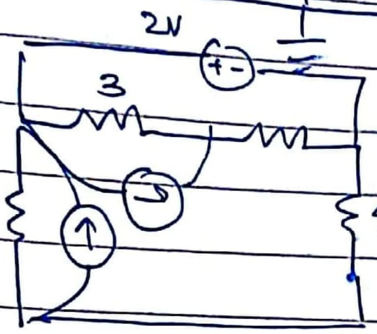
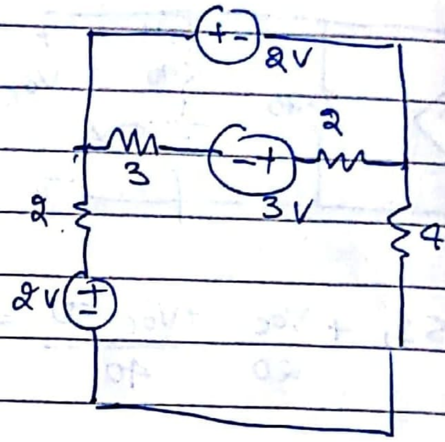
1b) Find the current through 4Ω using source shifting/ source transformation technique

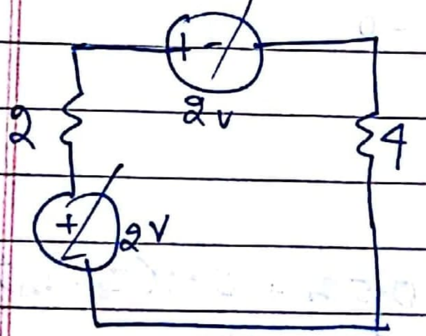


⇒



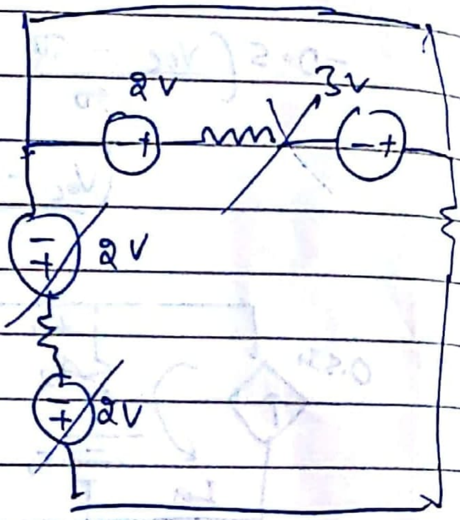
⇒





Or

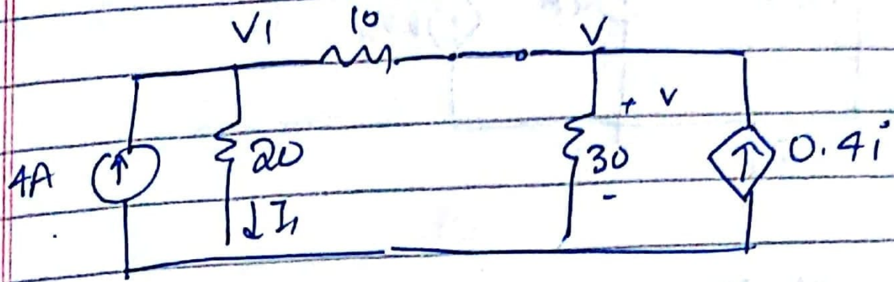
I=0



I=0

8

1c) Find the voltage V using superposition theorem for a network shown below.



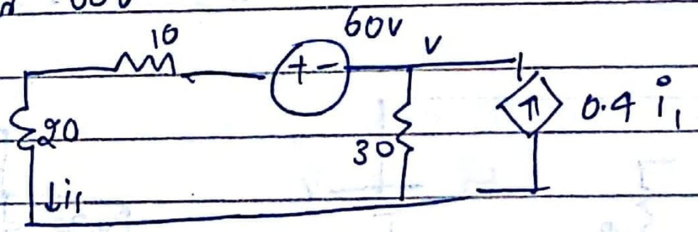
$$-4 + \frac{V_1}{20} + \frac{V_1 - V}{10} = 0$$

$$\frac{V}{30} - 0.4 i_1 + \frac{V - V_1}{10} = 0$$

where $i_1 = \frac{V_1}{20}$

$\underline{V = 60 \text{ volts}}$

Consider 60V



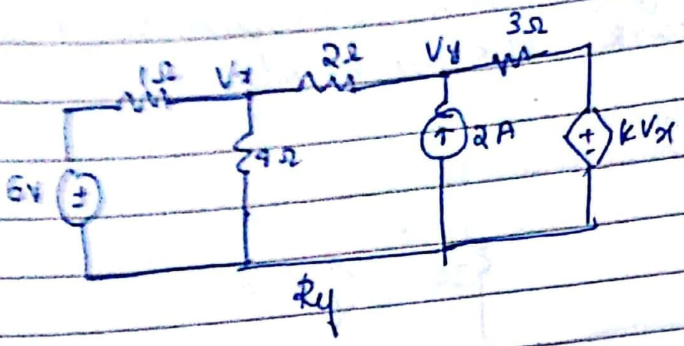
$$\frac{V+60}{30} + \frac{V}{30} - 0.4 i_1 = 0 \quad \text{—①}$$

where $i_1 = \frac{V+60}{30}$ sub in ① $\frac{V+60}{30} + \frac{V}{30} - \frac{0.4(V+60)}{30} = 0$

$V = 22.5$

$V = 60 + 22.5 = 82.5V$

⇒ Find the value of k that will cause V_y to be zero



Apply KCL at V_x

$$\frac{V_x}{4} + \frac{V_x - V_y}{2} + \frac{V_x - 6}{1} = 0$$

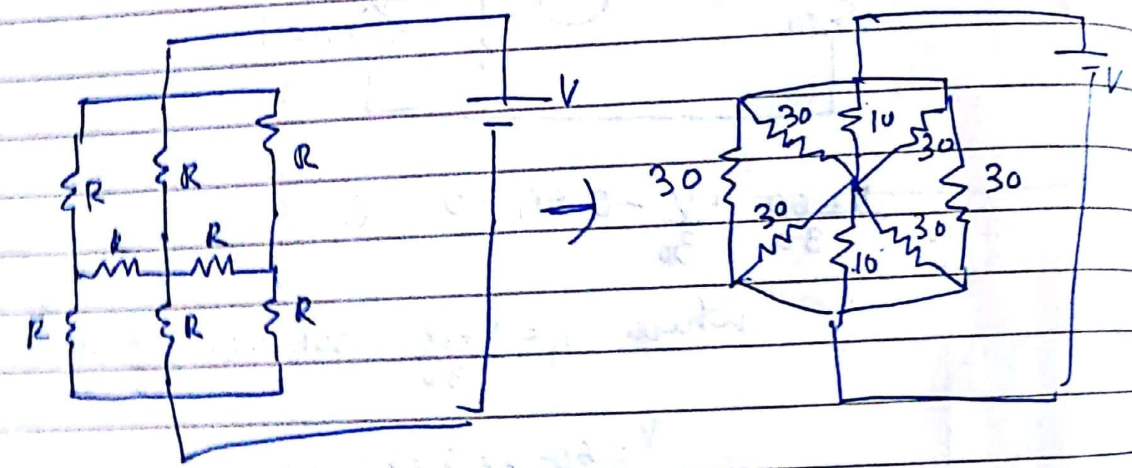
Apply KCL at V_y

$$\frac{V_y - V_x}{2} - 2 + \frac{V_y - kV_x}{3} = 0$$

Make $V_y = 0 \Rightarrow$ find k

$k = -3.25$

b)



store
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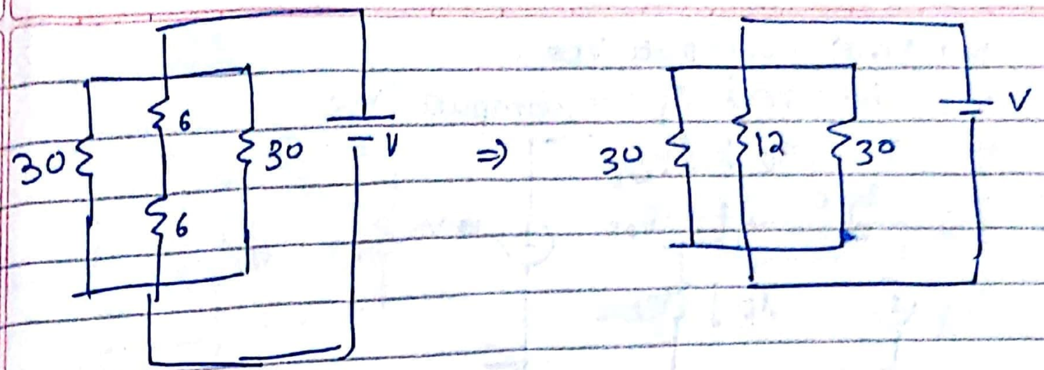
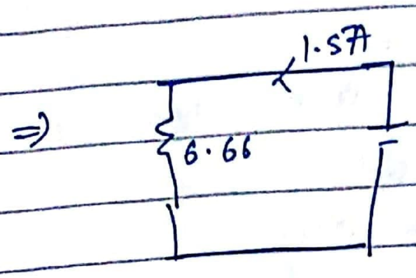
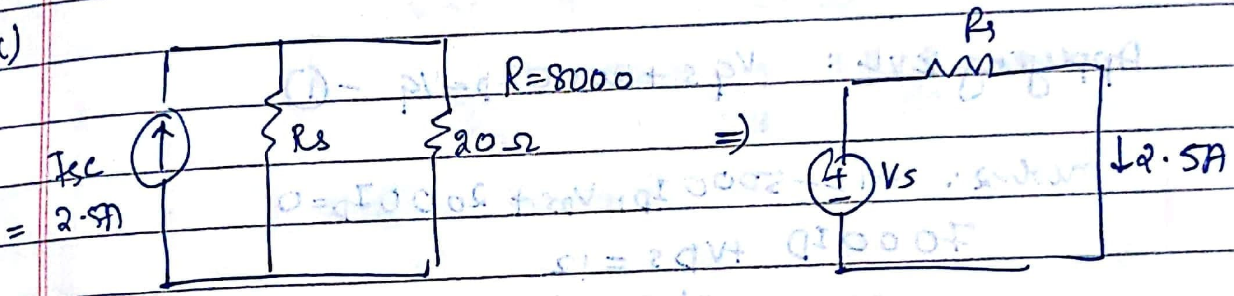
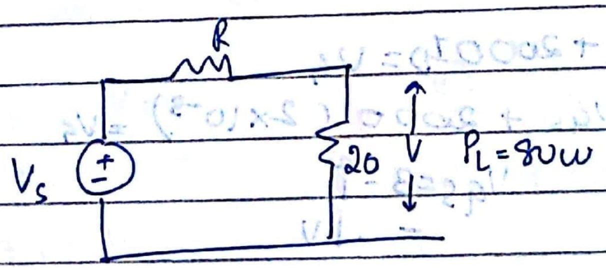




2) c)



$2.5 = \frac{V}{R_s}$ or $V = 2.5R_s$ — (1)



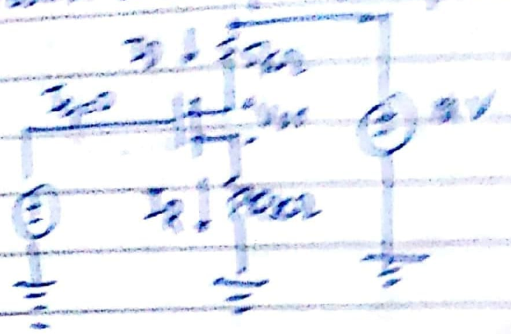
$V = \frac{20}{20+R} V$.

$P_L = 80 = \frac{V^2}{20} = \frac{\left(\frac{20 \; V_s}{20+R}\right)^2}{20}$ — (2)

$V_{oc} = 200V$, $P_{max} = 125W$

$R_L = 80\Omega$

3) If $I_D = 1.5mA$, compute V_{DS}.
If $I_D = 2mA$ and $V_G = 3V$ compute V_{GS}



$I_D = 1.5mA$

$V_{DS} = ?$

If: $I_D = 2mA$; $V_G = 3V$

Find V_{GS}.

Applying KVL : $V_{GS} + 2000 I_D = V_G$ — (1)

mesh 2 : $-12 + 5000 I_D + V_{DS} + 2000 I_D = 0$

$7000 I_D + V_{DS} = 12$

Put $I_D = 1.5 mA$

$\underline{V_{DS} = 1.5V}$

sub in eq (1)

$V_{GS} + 2000 I_D = V_G$

$V_{GS} + 2000 (2 \times 10^{-3}) = V_G$

$V_{GS} = 3 - 4$

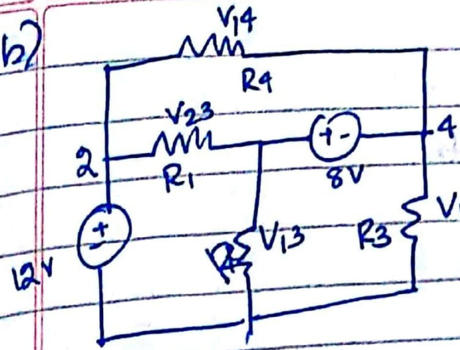
$= -1V$

$\therefore V_{GS} = -1 V$

$V_{DS} = 1.5V$; $V_{GS} = -1V$

store
67

b)



$V_{23} = V_3 - V_2$ — ①

$V_{13} = V_3 - V_1 = V_3$ — ②

$V_{14} = V_4 - V_1 = V_4$ — ③

$V_{24} = V_4 - V_2$ — ④

$V_{34} = V_4 - V_3 = -8V$

$V_2 = 12V$.

node 3 & 4 → supernode:

$V_{43} = V_3 - V_4 = 8$ — ⑤

$$\frac{V_3 - V_2}{R_1} + \frac{V_3}{R_2} + \frac{V_4}{R_3} + \frac{V_4 - V_2}{R_4} = 0$$

using eq ①, ②, ③, ④

$$\frac{V_{23}}{R_1} + \frac{V_{13}}{R_2} + \frac{V_{14}}{R_3} + \frac{V_{24}}{R_4} = 0$$

eq⑤ $V_3 - V_4 = 8$

$V_3 - V_4 = 8$ $\qquad V_4 = V_{14}$

$V_{13} - V_{14} = 8$ $\qquad V_3 = V_{13}$

$V_{13} = 8V$. $\qquad V_{14} = 0V$.

$V_3 = 8$ — ⑥

- $V_{23} = V_3 - V_2$

$V_{23} = 8 - 12 = -4V$

- $V_{24} = V_4 - V_2$

$= 0 - (+12) = -12$

ii) when $V_{14} = 6V$

eg 5 ⇒ $V_3 - V_4 = 8$

$V_{13} - V_{14} = 8$

$V_{13} = 8 + 6$

$V_{13} = 14 = V_3$

$V_{23} = V_3 - V_2$

$= 14 - 12$

$V_{23} = 2V$

$V_{24} = V_4 - V_2$

$= 6 - 12$

$= -6V$

iv) when $V_{14} = 6V$

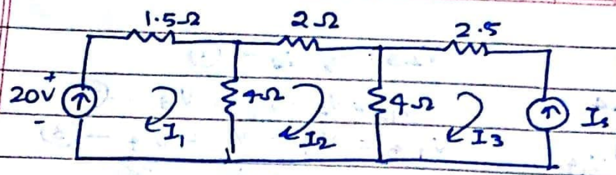
eq⑤ ⇒ $V_3 - V_4 = 8$

$V_{13} = 8 - 6 = 2 = V_3$

$V_{23} = V_3 - V_2 = 2 - 12 = -10$

$V_{24} = V_4 - V_2 = -6 - 12 = -18V$

c)



$I_1 = 4A$ $\quad I_3 = -I_s$

Loop 2 :

$$-2I_2 - 4(I_2 - I_3) - 14(I_2 - I_1) = 0$$

$$-2I_2 - 4I_2 + 4I_3 - 14I_2 + 14I_1 = 0$$

$$-20I_2 + 4I_3 = -14(4)$$

$$-20I_2 + 4I_3 = -56 \quad \text{—} \; (1)$$

Loop 1 :

$$20 - 1.5I_1 - 14(I_1 - I_2) = 0$$

$$20 - 1.5(4) - 14(4) + 14I_2 = 0$$

$$14I_2 = -20 + 6 + 56$$

$$I_2 = 3A$$

sub in eq (1)

$$-20(3) + 4I_3 = -56$$

$$4I_3 = -56 + 60$$

$$4I_3 = 4$$

$$I_3 = 1$$

$I_s = -I_3$

$I_s = -1A$ //

$$P_{1.5} = (1.5) \times (4)^2$$

$$= 24W$$

store
67

$P_{2\Omega} = I^2R$
$= 3^2 \times 2 = 18\,W$

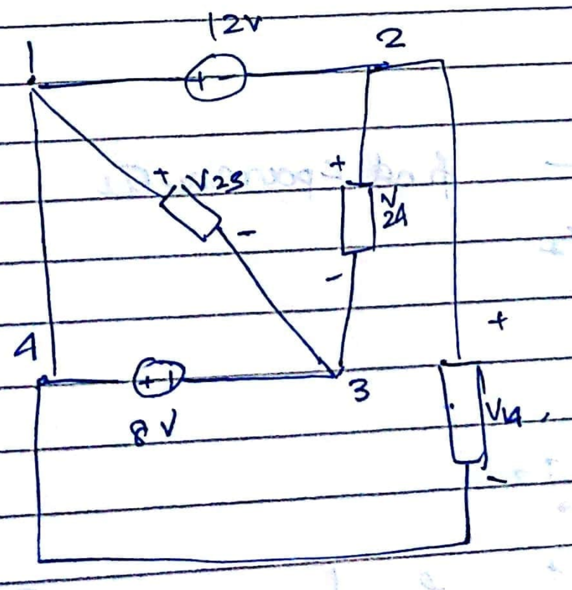
$P_{1,4} = (I_1 - I_2)^2 R$
$= (4-3)^2 \, 14 \quad = 14\,W$

$P_{25} = I_3^2 R$
$= 1^2 . 2.5 = 2.5\,W$

$P_4 = (I_2 - I_3)^2 R$
$= 4 \times 4 = 16\,W$

Q. 3b) A certain circuit consists 6 elements and 4 nodes numbered 1,2,3 and 4. Each circuit element is connected between a different pair of nodes, the voltage V_{12} (positive reference at first named node) is 12V and $V_{34} = -8V$. Apply suitable & find V_{13}, V_{23} and V_{24}, if V_{14} =
i) 0
ii) 6V
iii) -6V

→



$-8 + 12 + V_{23} = 0$
$V_{23} = -4$

~~$V_{24} =$~~

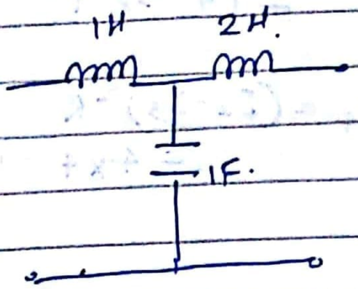
$12 + V_{24} - V_{23} = 0$
$12 + V_{24} + 4 = 0$
$V_{24} = -16$

$8 - V_{24} + V_{14} = 0$
$8 - V_{24} + V_{14} = 0$
$8 + 16 + V_{14} = 0$

0034

~~V24~~

1)



find z parameter

$X_C = 1/j\omega C$

$X_L = j\omega L$

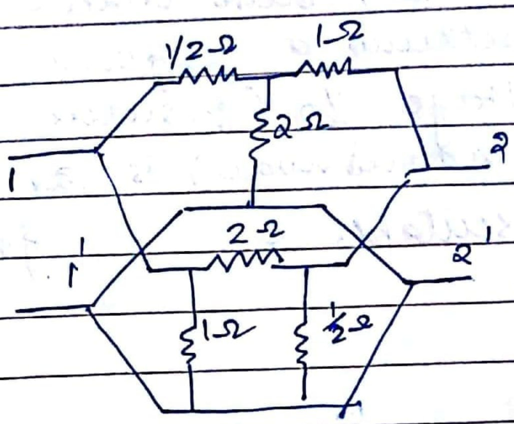
$S = j\omega$

$V_1 = (1 + \frac{1}{s}) I_1 + \frac{1}{s} I_2$

$V_2 = (\frac{1}{s}) I_1 + (2s + \frac{1}{s}) I_2$

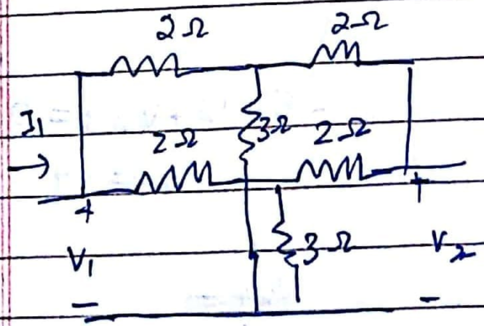
$Z = \frac{1}{s} + \frac{i}{\ }$

2)



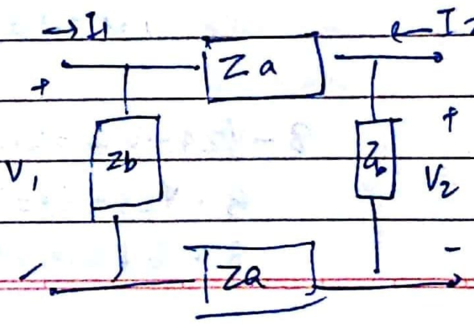
find the y parameter

3)



find z parameter

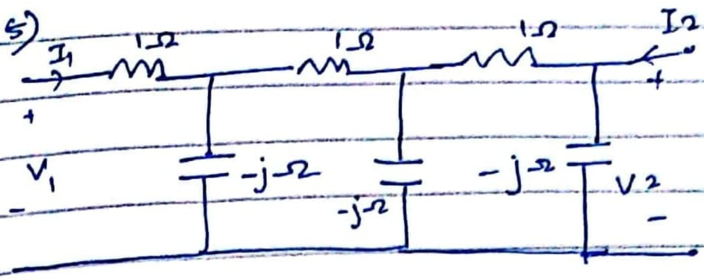
4)



find z parameter

store
67

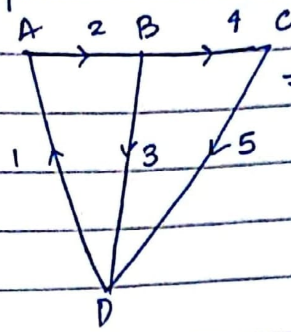
5)



find T parameter.

Network Topology

Graph

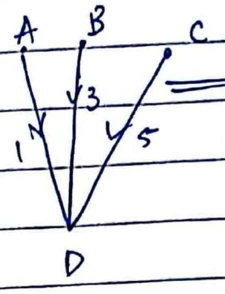


$\Longrightarrow$ oriented graph. (closed)

$n = 4$

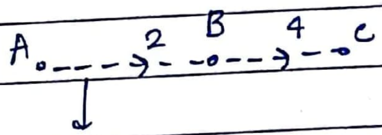
$b = 5$

Tree (not closed)



$\Longrightarrow$ twigs (1,3,5) $(n-1)$

co-tree – branches which are not present in the tree



Branches = links

$l = b - (n-1)$

- Graph: The connection of network topology shown by replacing all physical elements by lines is called as graph.
- Oriented graph: A graph with directions represented for each branch is called as oriented graph

- Tree: A tree of a graph is any set of branches that connects every other node directly or indirectly without forming closed loop.

- Twigs: the branches of a tree is called as twigs. twig is represented by thick lines

- no of branches in a tree is $n-1$.

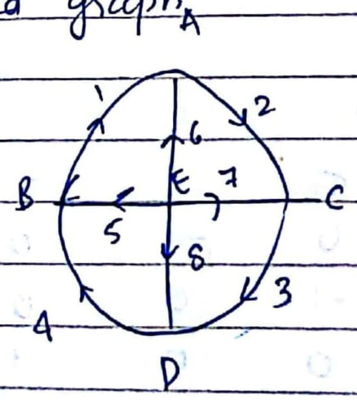
- co-tree - (complementary tree): The set of branches of the connected graph not included in the ~~tre~~ tree, is called co-tree

- links - the branches of the co-tree are called links and is represented by dotted lines & the no of links in given by '$b-(n-1)$'.

oriented graph

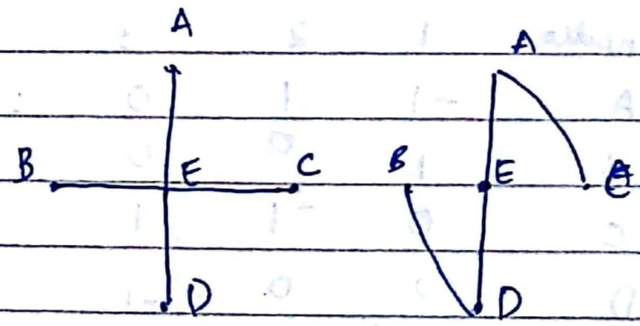
$n=5$

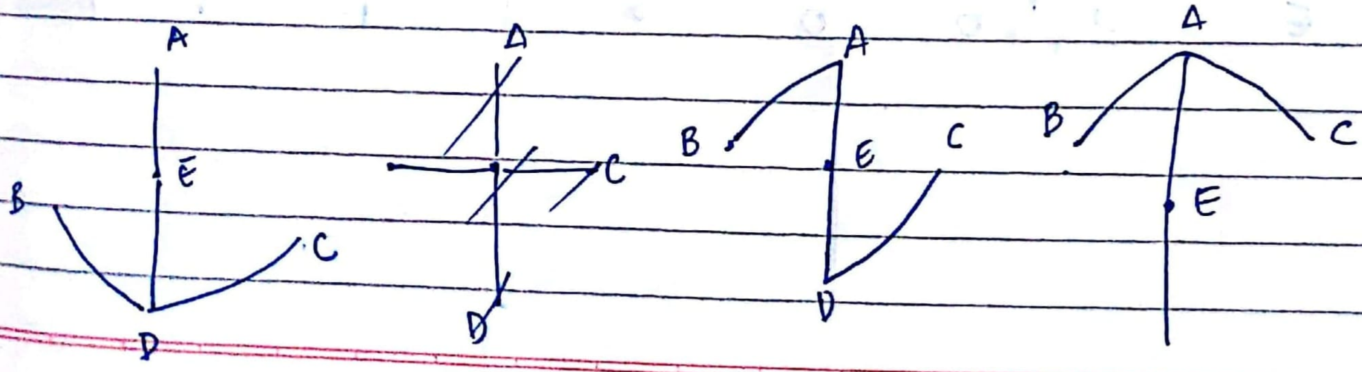
$b=8$

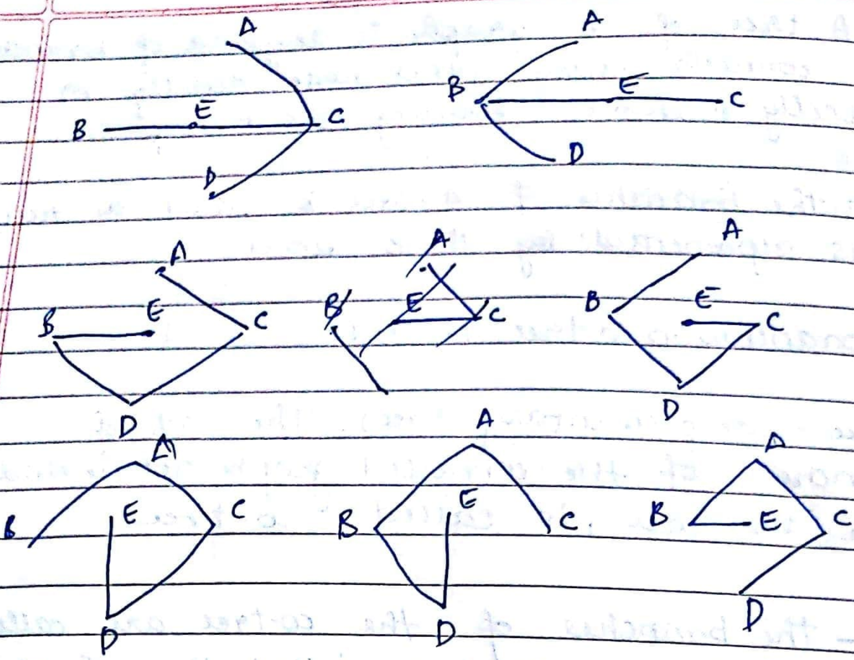


tree $= n-1 = 4$. branches

branches







Entry I ⇒ -ve

leaving I ⇒ +ve

not connected ⇒ 0

	Branch			no				
no of nodes	1	2	3	4	5	6	7	8
A	-1	1	0	0	0	-1	0	0
B	1	0	0	-1	-1	0	0	0
C	0	-1	1	0	0	0	-1	0
D	0	0	-1	1	0	0	0	-1
E	0	0	0	0	1	1	1	1

Incidence matrix

Its a mathematical model that represents a given network & its graph with all information available in the network. This matrix shows which branch is connected to which node & its orientation

$$A = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & -1 & -1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Adding columns = 0.

⇒ particular node I leaving = I entering

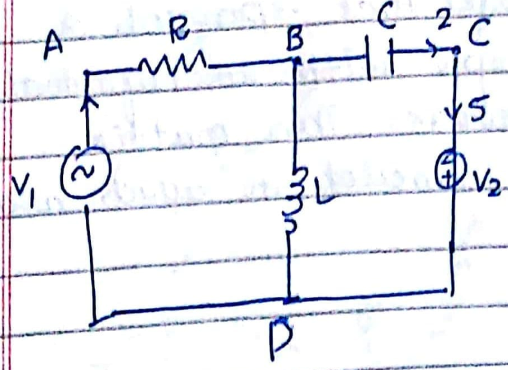
deleting one node row ⇒ reduced incidence matrix (A_R)

$$[A_r] = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & -1 & -1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & -1 \end{bmatrix}$$

Total number of possible trees $[A_r \cdot A_r^T]$

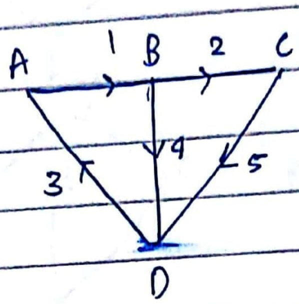
$= \cancel{\text{det}}\ \det [A_r \cdot A_r^T]$

Q.1. The incidence matrix for the network is given draw the corresponding graph.



nodes = 4

branches = 5



$\Rightarrow I_M =$ ~~A=~~

~~b = n - 1 = a~~

$$A = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ -1 & 1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 1 & -1 & -1 \end{bmatrix} \end{array}$$

$$A_R = \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ -1 & 1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

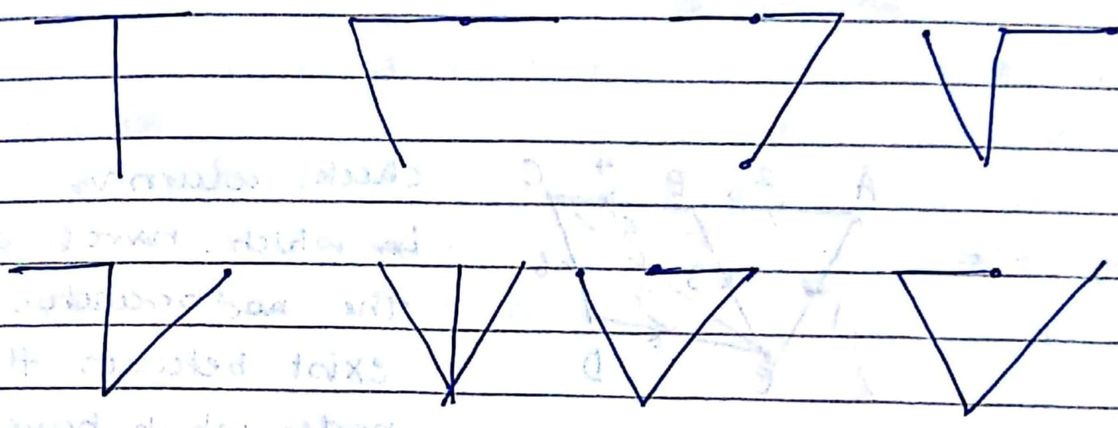
$$\cancel{A \cdot A_R} \; [A_R]^T = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow A_R \cdot [A_R]^T \Rightarrow$$

$$\begin{bmatrix} 1+1 & -1 & 0 \\ -1 & 1+1+1 & -1 \\ 0 & -1 & 1+1 \end{bmatrix} = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

$$\text{Det}\left[A_R \cdot A_R^T\right] = 2(6-1) + 1(-2-0) + 0$$

$$= 5\times 2 + 1(-2) = 10-2 = 8.$$

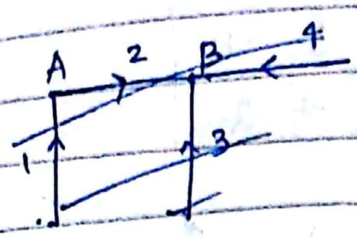
possible trees

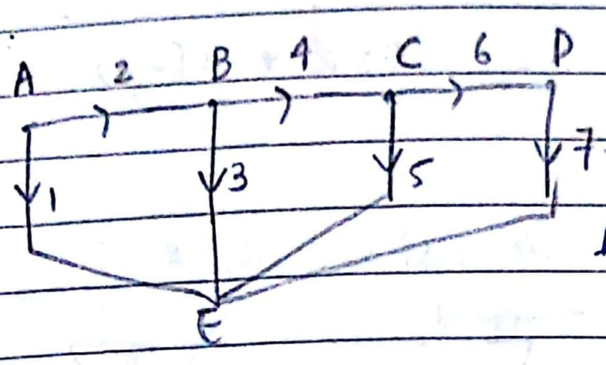


Q2) the incidence matrix for the net ---.

$$I_m = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix} \leftarrow \text{Reduced.}$$

$$I_m = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 \\ -1 & 0 & -1 & 0 & -1 & 0 & -1 \end{bmatrix} \end{array}$$

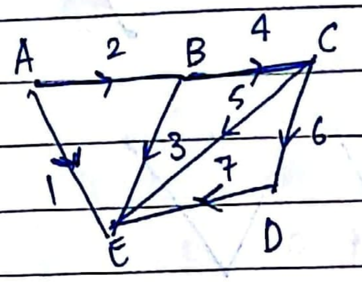




⇒ ✓

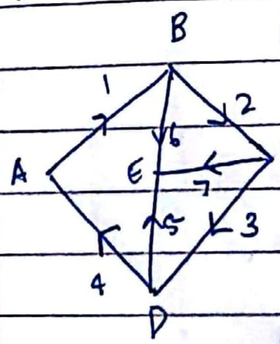
Also correct

=



check columns in which have (1 or -1) the rod branches exist between those nodes which have 1 & -1 the put direction

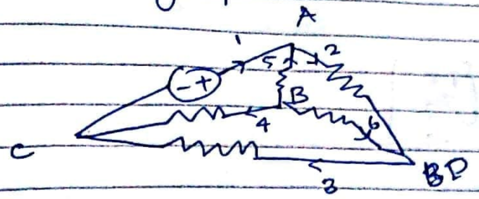
Q. Draw the incidence matrix



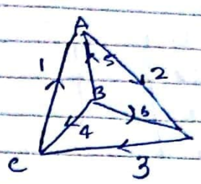
	1	2	3	4	5	6	7
A	1	0	0	-1	0	0	0
B	-1	1	0	0	0	1	0
C	0	-1	1	0	0	0	1
D	0	0	-1	1	1	0	0
E	0	0	0	0	-1	-1	-1

store 67

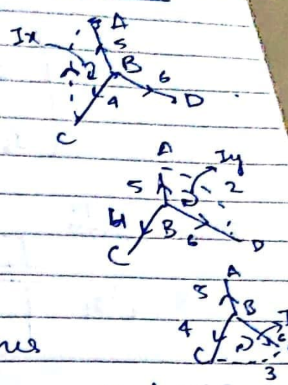
Q. Draw the graph



o.graph



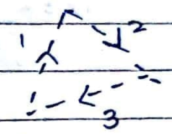
tree



twigs
(5, 4, 6)

n−1
=3 branches

co tree



loop currents

$$
\begin{array}{c|c}
 & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \\
\begin{matrix} I_x \\ I_y \\ I_z \end{matrix} & \begin{bmatrix} 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{bmatrix} \Rightarrow B
\end{array}
$$

$V_1 + V_4 - V_5 = 0$

$V_2 + V_5 - V_6 = 0$

see columns

$I_1 = I_x$ $\quad$ $I_4 = I_x - I_z$

$I_2 = I_y$ $\quad$ $I_5 = I_y - I_x$

$I_3 = I_z$ $\quad$ $I_6 = I_z - I_y$

[Tie-set matrix]

$$B V_B = 0 \qquad \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix}$$

KVL- I_x $\quad V_1 + V_4 - V_5$

I_y $\quad V_2 + V_5 - V_6$

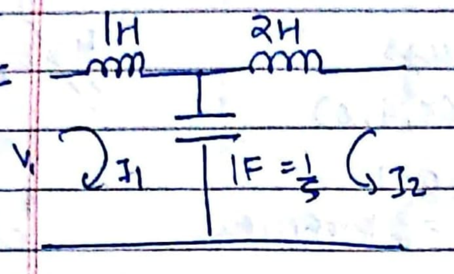
I_z $\quad V_3 - V_4 + V_6$

see rows

$I_B = B^T I_L$

$$\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_6 \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix} \begin{bmatrix} I_x \\ I_y \\ I_z \end{bmatrix}$$

H|w



Find z parameter

$X_C = \frac{1}{j\omega C}$ $\quad X_L = j\omega L$

$$V_1 = \left(1+\frac{1}{s}\right) I_c + \frac{1}{s} I_2$$

$$= \left(1+\frac{1}{j\omega}\right) I_1 + \frac{1}{j\omega} I_2$$

$$V_1 = \left(\frac{1+j\omega}{j\omega}\right) I_1 + \left(\frac{1}{j\omega}\right) I_2$$

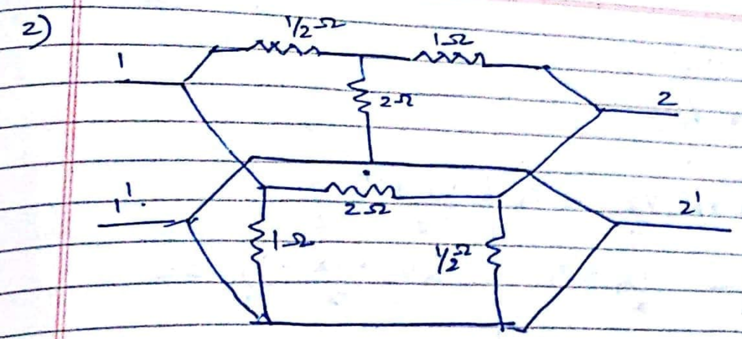
$$V_2 = \left(\frac{1}{2}+\frac{1}{s}\right) I_1 + \left(2s+\frac{1}{s}\right) I_2$$

$$= \left(\frac{1}{2}+\frac{1}{j\omega}\right) I_1 + \left(2j\omega+\frac{1}{j\omega}\right) I_2$$

$$= \left(\frac{2+j\omega}{2j\omega}\right) I_1 + \left(\frac{2j\omega^2+1}{j\omega}\right) I_2$$

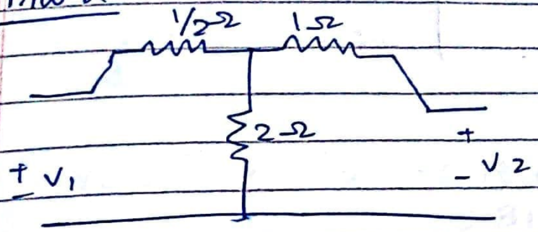
$$Z = \begin{bmatrix} 1+j\omega/j\omega & 1/j\omega \\ 2+j\omega/2j\omega & 2(j\omega)^2+1/j\omega \end{bmatrix}$$

2)

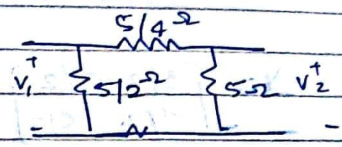


Find y parameter.

n/w a



$\Rightarrow$



$Y \rightarrow \Delta$

$$\frac{1/2+2+1}{2} = 5/2 \times 1/2 = 5/4$$

$$\frac{5/2}{1/2} = \frac{5}{1} \times \frac{2}{1}$$

Applying nodal

$$\frac{V_1}{5/2} + \frac{V_1 - V_2}{5/4} = I_1$$

$$I_1 = \frac{2}{5}V_1 + \frac{4}{5}V_1 - \frac{4}{5}V_2$$

$$I_2 = \frac{6}{5}V_1 - \frac{4V_2}{5} \quad -(1)$$

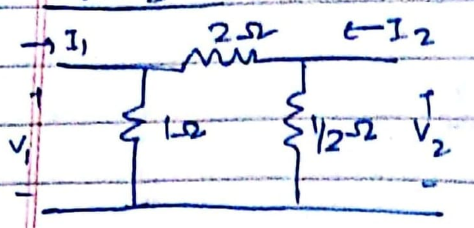
$$\frac{V_2}{5} + \frac{V_2 - V_1}{5/4} = I_2$$

$$\frac{V_2}{5} + \frac{4}{5}V_2 - \frac{4}{5}V_1 = I_2$$

$$I_2 = -\frac{4}{5}V_1 + V_2$$

$$Y_a = \begin{bmatrix} 6/5 & -4/5 \\ -4/5 & 1 \end{bmatrix}$$

N/w b



nodal $\frac{V_1}{1} + \frac{V_1 - V_2}{2} = I_1$

$V_1 + \frac{V_1}{2} - \frac{V_2}{2} = I_1$

$\Rightarrow I_1 = \frac{3}{2}V_1 - \frac{V_2}{2}$

$\frac{V_2}{1/2} + \frac{V_2 - V_1}{2} = I_2$

$2V_2 + \frac{V_2}{2} - \frac{V_1}{2} = I_2$

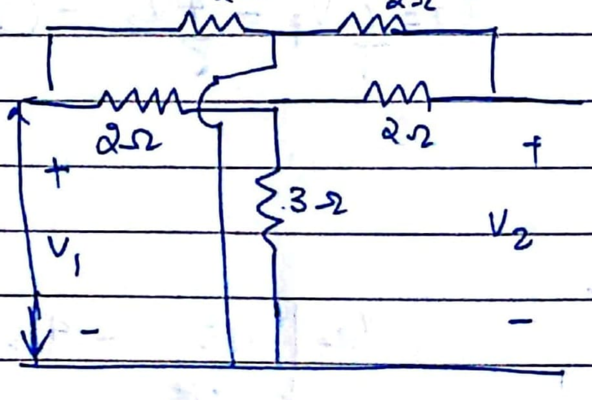
$I_2 = \frac{-V_1}{2} + \frac{5V_2}{2}$

$$Y_b = \begin{bmatrix} 3/2 & -1/2 \\ -1/2 & 5/2 \end{bmatrix}$$

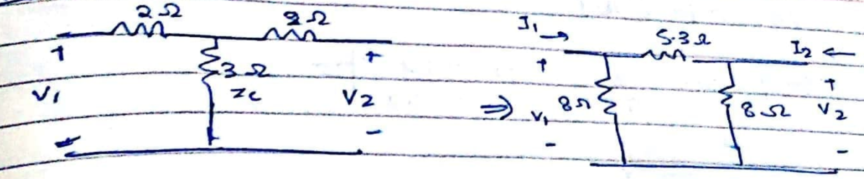
$$Y = \begin{bmatrix} 6/5 & -4/5 \\ -4/5 & 1 \end{bmatrix} + \begin{bmatrix} 3/2 & -1/2 \\ -1/2 & 5/2 \end{bmatrix}$$

$$Y = \begin{bmatrix} 2.7 & -1.3 \\ -1.3 & 3.5 \end{bmatrix} \mho$$

3)



N/W a



nodal

$$\frac{V_1}{8} + \frac{V_1 - V_2}{5.33} = I_1$$

$$I_1 = 0.3126V_1 - 0.1876V_2$$

$$I_2 = -0.1876V_2 + 0.3126V_2$$

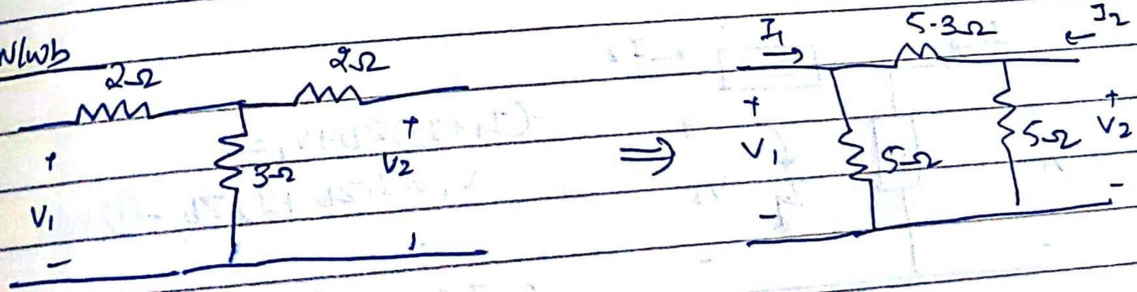
$$\frac{V_2}{8} + \frac{V_2 - V_1}{5.3} = I_2$$

$$Ya = \begin{bmatrix} 0.3126 & -0.1876 \\ -0.1876 & 0.3126 \end{bmatrix} \mho \qquad Za = \begin{bmatrix} 5 & 3 \\ 3 & 5 \end{bmatrix} \Omega$$

$$Z_{1} = Za + Zc = 5\Omega \qquad Z_{21} = Zc = 3\Omega$$

$$Z_{12} = Zc = 3\Omega \qquad Z_{22} = Zb + Zc = 5\Omega$$

N/wb



$$I_1 = 0.3126V_1 - 0.1876V_2$$

$$I_2 = -0.1876V_1 + 0.3126V_2$$

$$Yb = \begin{bmatrix} 0.3126 & -0.1876 \\ -0.1876 & 0.3126 \end{bmatrix} \mho$$

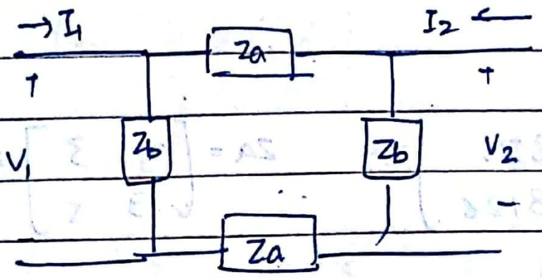
$Y = [Y_a] + [Y_b]$

$$Y = \begin{bmatrix} 0.6252 & -0.3752 \\ -0.3752 & 0.6252 \end{bmatrix} \mho$$

$Z = [Z_a] + [Z_b]$

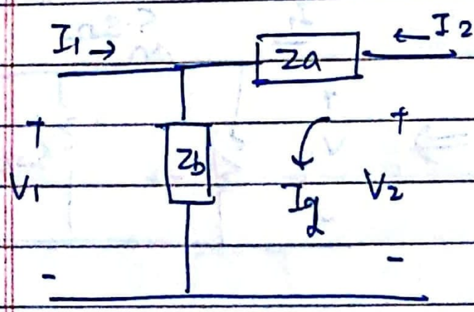
$$Z = \begin{bmatrix} 10 & 6 \\ 6 & 10 \end{bmatrix} \Omega$$

4)



find z parameter

n/w a



$-(I_1+I_2)Z_b + V_1 = 0$

$V_1 = I_1 Z_b + I_2 Z_b$ — (1)

$(-I_1 Z_a - (I_2+I_1)Z_b + V_2 = 0$

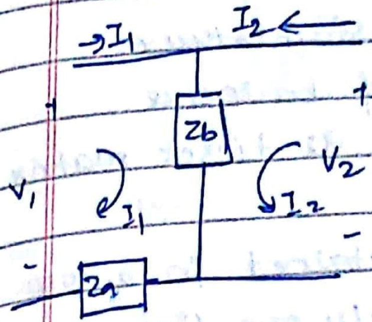
$V_2 = I_2 Z_a + I_2 Z_b + I_1 Z_b$

$V_2 = I_1 Z_b + (Z_a + Z_b) I_2$ — (2)

$$Z_a = \begin{bmatrix} Z_b & Z_b \\ Z_b & Z_a + Z_b \end{bmatrix} \Omega$$

store
67

n/w b



$-(I_1+I_2)Z_b - I_1 Z_a + V_1 = 0$

$V_1 = I_1 Z_b + I_2 Z_b + I_1 Z_a$

$V_1 = (Z_a+Z_b) I_1 + Z_b I_2$ — (1)

$-(I_1+I_2)Z_b + V_2 = 0$

$V_2 = I_1 Z_b + I_2 Z_b$ — (2)

$$Z_b = \begin{bmatrix} Z_a+Z_b & Z_b \\ Z_b & Z_b \end{bmatrix} \Omega$$

$$Z = [Z_a] + [Z_b]$$

$$Z = \begin{bmatrix} Z_a+Z_b & 2Z_b \\ 2Z_b & Z_a+2Z_b \end{bmatrix} \Omega$$

(5) n/w a) $= T_a \begin{bmatrix} -(1-jw/jw) & -(1-2jw/jw) \\ -1/jw & -(1-jw/jw) \end{bmatrix}$

n/w b) $T = -\frac{1}{j^2} \begin{bmatrix} -4j^3+7j^2-6j+2 & -2j^2+2j \\ -2j^3+4j^2-4j+2 & -j^2+2j \end{bmatrix}$

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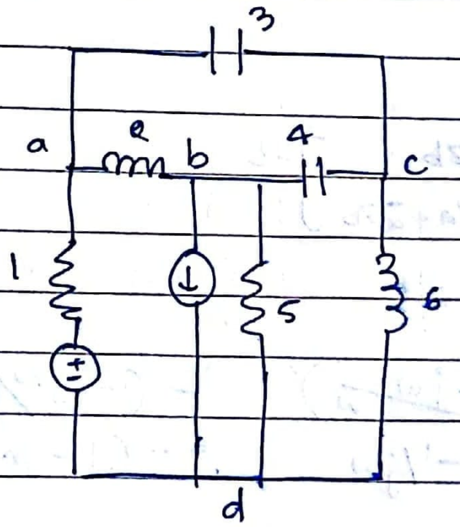
Network Topology continued :

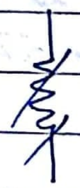
- Tieset matrix

A matrix representing the relationship between no of loop/link currents and no of branches in the directed graph is known as tieset matrix

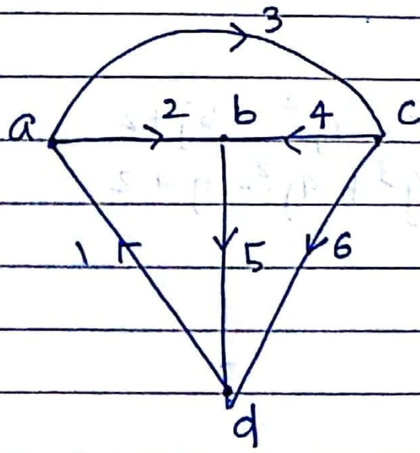
- Tieset is a set of branches contained in a loop such that each loop contains only one link and rest are the branches

Q. Draw the tieset schedule for the given network considering 2,3,6 as tree





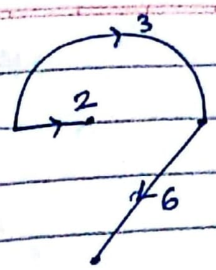
O. graph

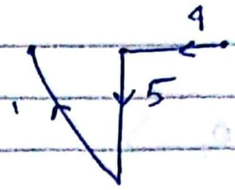


nodes = 4

twigs = 4-1=3 twigs

⇒ 2,3,6.





[1, 5, 4]

Tree.

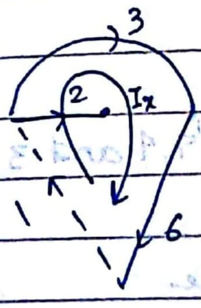
co-tree. (make in dotted lines)

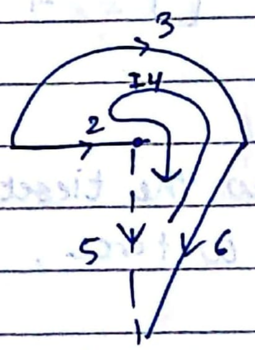
~~Tieset matrix.~~ Tie set matrix

	1	2	3	4	5	6
A	-1	1	1	0	0	0
B	0	-1	0	-1	1	0
C	0	0	-1	1	0	1
D	1	0	0	0	-1	-1

1st case:
add 1 to the tree

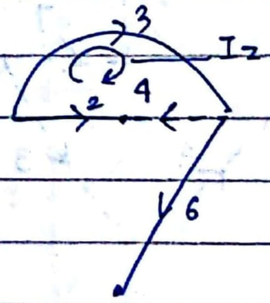
Case 2:
add 5 to the tree





Tie set schedule

Case 3: add 4 to the tree



	1	2	3	4	5	6
Ix	1	0	1	0	0	1
Iy	0	1	-1	0	1	-1
Iz	0	-1	1	1	0	0

$$= B \cdot V_B = 0 \quad \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 & 1 & -1 \\ 0 & -1 & 1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix} = 0$$

KVL equations

$$V_1 + V_3 + V_6 = 0$$
$$V_2 - V_3 + V_5 - V_6 = 0$$
$$-V_2 + V_3 + V_4 = 0$$

$I_1 = I_x$ $\quad\quad$ $I_4 = I_z$

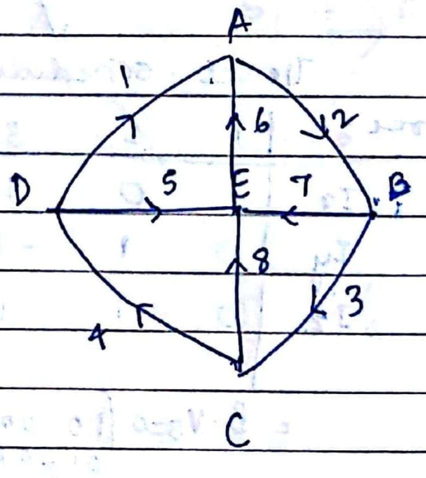
$I_2 = I_y - I_z$ $\quad\quad$ $I_5 = I_y$

$I_3 = I_x - I_y + I_z$ $\quad\quad$ $I_6 = I_x - I_y$

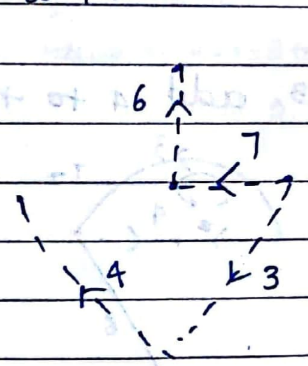
$$I_B = B^T . I_L$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 1 & -1 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} = \begin{bmatrix} I_x \\ I_y \\ I_z \end{bmatrix}$$

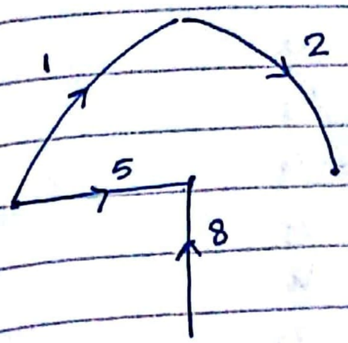
Q. Draw the tieset matrix considering 6, 7, 4 and 3 as co-tree.



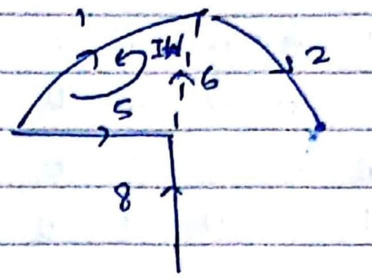
co-tree



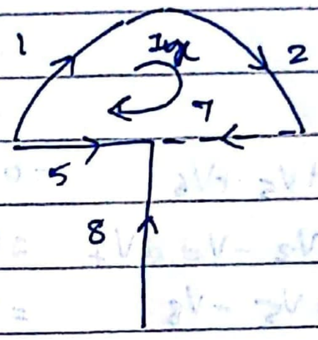
tree



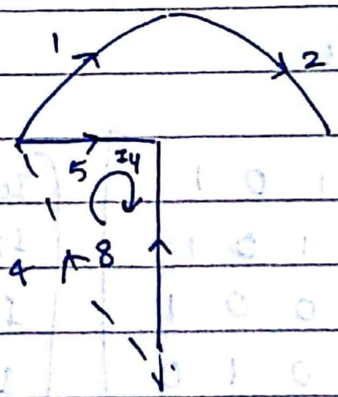
case1: add 6 to the tree



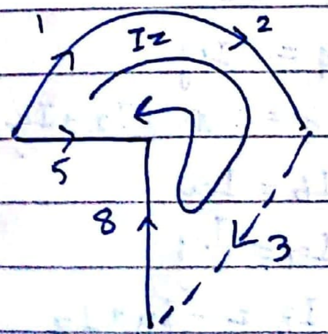
case2: add 7 to the tree



case3: add 4 to the tree



case 4: add 3 to the tree



Tieset schedule.

	1	2	3	4	5	6	7	8
I_w	-1	0	0	0	1	1	0	0
I_x	1	1	0	0	-1	0	0	0
I_y	0	0	0	1	1	0	0	-1
I_z	1	1	1	0	-1	0	0	1

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$B \cdot V_B = 0.$

$$\begin{bmatrix} -1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & -1 \\ 1 & 1 & 1 & 0 & -1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{bmatrix}$$

$$\Rightarrow \begin{aligned} -V_1 + V_5 + V_6 &= 0 \\ V_1 + V_2 - V_5 + V_7 &= 0 \\ V_4 + V_5 - V_8 &= 0 \\ V_1 + V_2 + V_3 - V_5 + V_8 &= 0 \end{aligned}$$

$I_B = B^T I_L$

$$\begin{bmatrix} -1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & -1 & 1 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} I_\omega \\ I_x \\ I_y \\ I_z \end{bmatrix} \Rightarrow$$

$$\begin{aligned} I_1 &= -I_\omega + I_x + I_z \\ I_2 &= I_x + I_z \\ I_3 &= I_z \\ I_4 &= I_y \\ I_5 &= I_\omega - I_x + I_y + I_z \\ I_6 &= I_\omega \\ I_7 &= I_y \\ I_8 &= -I_y + I_z. \end{aligned}$$

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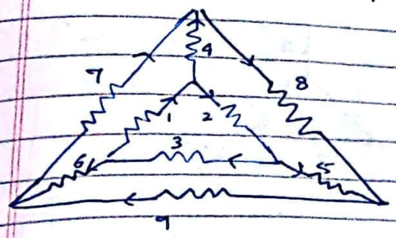
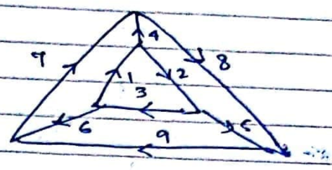
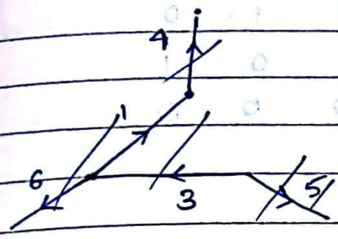
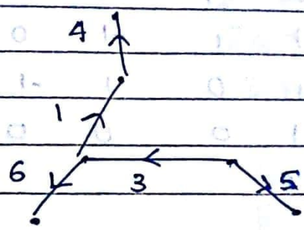
Q. 1, 4, 3, 6, 5 as tree



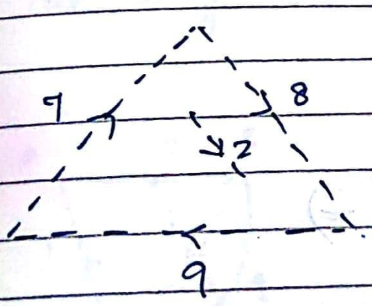


→ tree

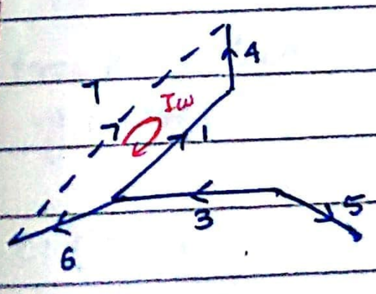


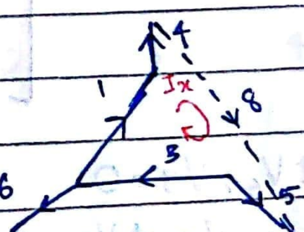


co-tree



case 1 : add 7 to tree case 2: add 8 to tree





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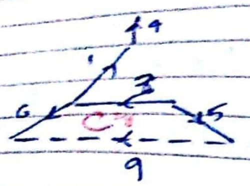
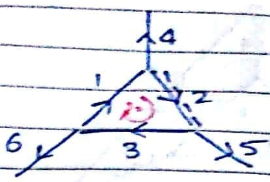
case 3. add 9 to tree



case 4: add 2 to tree



Tieset schedule.

	1	2	3	4	5	6	7	8	9
I_w	-1	0	0	-1	0	1	1	0	0
I_x	1	0	1	1	-1	0	0	1	0
I_y	0	0	-1	0	1	-1	0	0	1
I_z	1	1	1	0	0	0	0	0	0

$B \cdot V_B$

$$\begin{bmatrix} -1 & 0 & 0 & -1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 1 & -1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \\ V_9 \end{bmatrix} \Rightarrow$$

$V_z =$

$$-V_1 - V_4 + V_6 + V_7 = 0$$

$$V_1 + V_3 + V_4 - V_5 + V_8 = 0$$

$$-V_3 + V_5 - V_6 + V_9 = 0$$

$$V_1 + V_2 + V_3 = 0$$

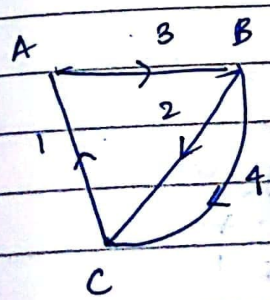
$B^T \cdot I_L$

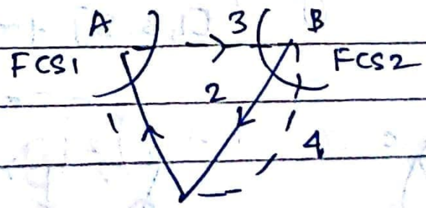
$$\begin{bmatrix} -1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & -1 & 1 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 1 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} I_w \\ I_x \\ I_y \\ I_z \end{bmatrix} \Rightarrow$$

$I_1 = -I_w + I_x + I_z$
$I_2 = I_z$
$I_3 = I_x - I_y + I_z$
$I_4 = -I_w + I_x$
$I_5 = -I_x + I_y$
$I_6 = I_w - I_y$
$I_7 = I_w$
$I_8 = I_x$
$I_9 = I_y$

* Fundamental cutset:
→ divide the graph in 2 parts.
→ cut only twig & any no of link.

eg:





tree branch or vg cutset	1	2	3	4
e_1 (FCS1)	1	0	-1	0
e_2 (FCS2)	0	1	-1	1

row wise (KCL)
$I_1 - I_3 = 0$
$I_2 - I_3 + I_4 = 0$

column wise (KVL)
$V_1 = e_1$, $V_2 = e_2$
$V_3 = -e_1 - e_2$
$V_4 = e_2$

$$Q\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 1 \end{bmatrix}$$

$V_B = Q^T E_T$

$QI_B = 0$

$$\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = 0.$$

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & -1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$$

$I_1 - I_3 = 0$

$I_2 - I_3 + I_4 = 0$

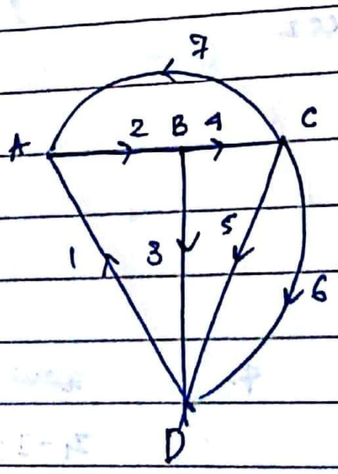
$V_1 = e_1 \quad V_3 = -e_1 - e_2$

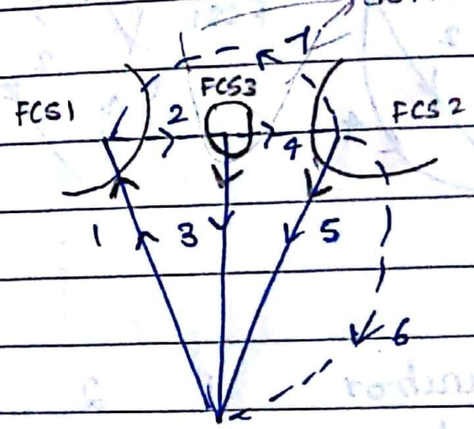
$V_2 = e_2 \quad V_4 = e_2$

* no of twigs = no of cutsets.

Q. Determine the fundamental cutset for the directed graph.

[1, 3, 5] tree





(1,3,5) – 3 cut sets , 1 cutset – 1 twig, rest links

- cut twig 1, then twig 3, & twig 5

	1	2	3	4	5	6	7
FCS1(e_1)	1	-1	0	0	0	0	1
FCS2(e_2)	0	0	0	-1	1	1	1
FCS3(e_3)	0	-1	1	1	0	0	0

$$Q\begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 & 1 & 1 & 1 \\ 0 & -1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$QI_B = 0$

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 & 1 & 1 & 1 \\ 0 & -1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \\ I_7 \end{bmatrix} \Rightarrow \begin{matrix} I_1 - I_2 + I_7 = 0 \\ -I_4 + I_5 + I_6 + I_7 = 0 \\ -I_2 + I_3 + I_4 = 0. \end{matrix}$$

$V_B = Q^T E_T$

$$\begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & -1 \\ 0 & 0 & 1 \\ 0 & -1 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} e_1 \\ e_2 \end{bmatrix} \Rightarrow$$

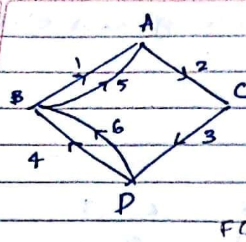
$V_1 = e_1$, $V_3 = e_3$

$V_2 = -e_1 - e_3$, $V_4 = -e_2 + e_3$

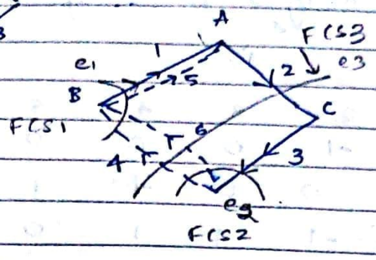
$V_5 = e_2$, $V_6 = e_2$

$V_7 = e_1 + e_2$.

Q. [5, 6, 4]



cotree.



3-twigs - 3 cuts.

	1	2	3	4	5	6
e_1	1	0	0	-1	1	-1
e_2	0	0	1	-1	0	-1
e_3	0	1	0	-1	0	-1

$$Q \begin{bmatrix} 1 & 0 & 0 & -1 & 1 & -1 \\ 0 & 0 & 1 & -1 & 0 & -1 \\ 0 & 1 & 0 & -1 & 0 & -1 \end{bmatrix}$$

$V_1 = e_1 \quad V_2 = e_2 \quad V_3 = e_2 \quad V_4 = -e_1 - e_2 - e_3 = Q^T E^T$

$V_5 = e_1 \quad V_6 = -e_1 - e_2 - e_3$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & -1 & -1 \\ 1 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}$$

$Q I_B = 0$

$I_1 - I_4 + I_5 - I_6 = 0$

$I_3 - I_4 - I_6 = 0$

$I_2 - I_4 - I_6 = 0$

$$\Leftarrow \begin{bmatrix} 1 & 0 & 0 & -1 & 1 & -1 \\ 0 & 0 & 1 & -1 & 0 & -1 \\ 0 & 1 & 0 & -1 & 0 & -1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix}$$

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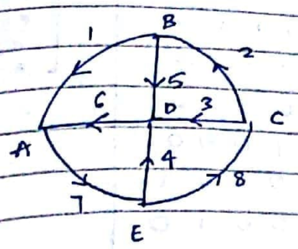
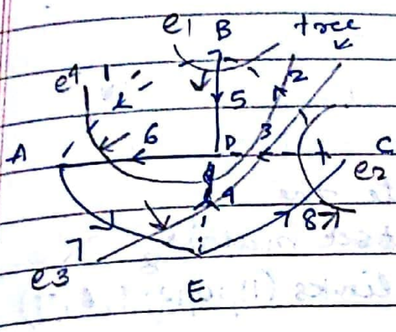
Q. write the cut set matrix twigs = {6, 5, 7, 8}.



→ 4 twigs = 4 cuts   links = (1, 2, 3, 4)

~~add.~~



	1	2	3	4	5	6	7	8
e1	1	-1	0	0	1	0	0	0
e2	0	-1	-1	0	0	0	0	1
e3	0	-1	-1	-1	0	0	1	0
e4	1	-1	-1	-1	0	1	0	0

$$Q \begin{bmatrix} 1 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 & 0 & 0 & 0 & 1 \\ 0 & -1 & -1 & -1 & 0 & 0 & 1 & 0 \\ 1 & -1 & -1 & -1 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \\ I_7 \\ I_8 \end{bmatrix} \Rightarrow \begin{array}{l} I_1 - I_2 + I_5 = 0 \\ -I_2 - I_3 + I_8 = 0 \\ -I_2 - I_3 - I_4 + I_7 = 0 \\ I_1 - I_2 - I_3 - I_4 + I_6 = 0 \end{array}$$

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$Q^T[e] =$

$V_1 = e_1 + e_4$ $\quad V_5 = e_1$

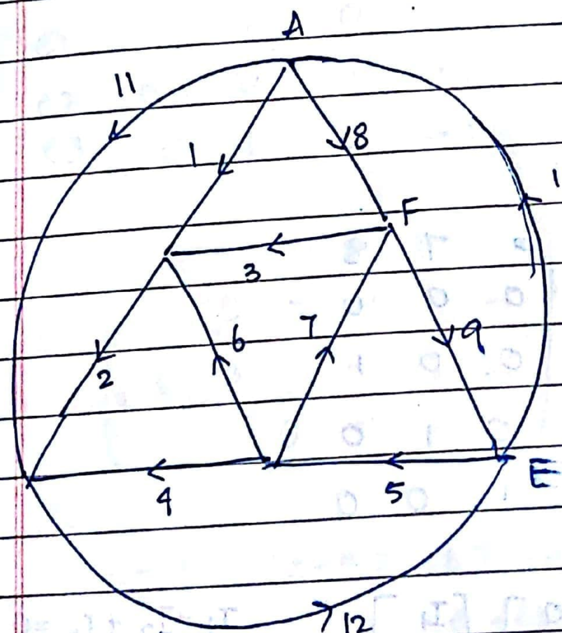
$V_2 = -e_1 - e_2 - e_3 - e_4$ $\quad V_6 = e_4$

$V_3 = -e_2 - e_3 - e_4$ $\quad V_7 = e_3$

$V_4 = -e_3 - e_4$ $\quad V_8 = e_2$

$$\Leftarrow \begin{bmatrix} 1 & 0 & 0 & 1 \\ -1 & -1 & -1 & -1 \\ 0 & -1 & -1 & -1 \\ 0 & 0 & -1 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix}$$

Q.



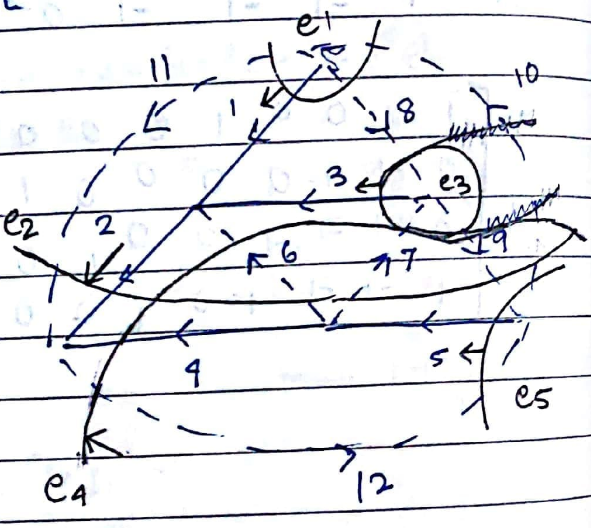
write the cutset matrix

links (11, 10, 12, 8, 9, 6, 7)

$\rightarrow$ twigs (1, 2, 3, 4, 5)

= cuts = 5

$\rightarrow$



store 67

Q)

	1	2	3	4	5	6	7	8	9	10	11	12
e1	1	0	0	0	0	0	0	1	0	-1	1	0
e2	0	1	0	0	0	-1	-1	0	1	-1	1	0
e3	0	0	1	0	0	0	-1	-1	1	0	0	0
e4	0	0	0	1	0	1	1	0	-1	1	0	-1
e5	0	0	0	0	1	0	0	0	-1	1	0	-1

$$Q\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_{12} \end{bmatrix} \Rightarrow \quad I_1 + I_8 - I_{10} + I_{11} = 0$$

$$I_2 - I_6 - I_7 + I_9 - I_{10} + I_{11} = 0$$

$$I_3 - I_7 - I_8 + I_9 = 0$$

$$I_4 + I_6 + I_7 - I_9 + I_{10} - I_{12} = 0$$

$$I_5 - I_9 + I_{10} - I_{12} = 0$$

$$Q^T \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \end{bmatrix} = $$

$V_1 = e_1$ | $V_7 = -e_2 - e_3 + e_4$

$V_2 = e_2$ | $V_8 = e_1 - e_3 \cancel{+e_4}$

$V_3 = e_3$ | $V_9 = e_2 + e_3 - e_4 - e_5$

$V_4 = e_4$ | $e_{10} = -e_1 - e_2 + e_4 + e_5$

$V_5 = e_5$ | $e_{11} = e_1 + e_2$

$V_6 = -e_2 + e_4$ | $e_{12} = -e_4 - e_5$.

Tie-set

→ graph
→ tree
→ add link one by one
→ loop direction based on link direction
→ no of loops = no of links
→ Row wise = ~~KVL~~ voltage terms
$V_B . B = 0$.
(B matrix)
→ Column wise = KVL current terms

Cut-set

→ graph
→ tree & co-tree
→ cut twigs (1 twig at a time)
→ no of cuts = no of twig.
→ direction of ~~twig~~ FCS is based on direction of twig
→ Row wise = current terms
column wise = voltage terms

* In a graph → n = nodes
b = branch
twig = (n-1)
links = b-n+1.

FOURIER SERIES

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + \sum_{n=1}^{\infty} b_n \sin n\omega_0 t \quad -①$$

$$\int_0^T x(t) = \int_0^T a_0 + a_1 \cos \omega_0 t + a_2 \cos 2\omega_0 t + a_3 \cos 3\omega_0 t + \cdots$$
$$+ b_1 \sin \omega_0 t + b_2 \sin \omega_0 t + b_3 \cos 3\omega_0 t + \cdots.$$

$$= \int_0^T x(t)dt = \int_0^T a_0 dt.$$

these harmonics are sinusoidal and orthagonal to each other.

$$= \int_0^T x(t)dt = a_0 T$$

$$a_0 = \frac{1}{T}\int_0^T x(t)dt$$

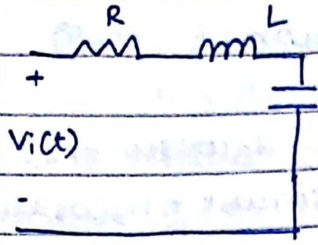
$$a_0 = \frac{1}{T}\int_0^T x(t)dt = a_n = \frac{2}{T}\int_0^T x(t)\cos n\omega_0 t\, dt \qquad b_n = \frac{2}{T}\int_0^T x(t)\sin n\omega_0 t\, dt.$$

Polar form

TIME DOMAIN AND FREQUENCY DOMAIN

→ system representation
→ system governing equation
→ characteristic equation
→ order of the system
→ poles and zeros
→ initial condition
→ Transfer function

* Integro-differential equation



$$V_i(t) = Ri(t) + L\frac{di}{dt} + \frac{1}{C}\int_{-\infty}^{t} i(t)dt$$

this is in time domain.

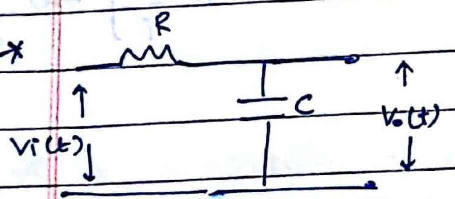
↳ convert this to Laplace transform

↳ After algebraic manipulation

↳ take inverse Laplace

↳ result [time domain]

*



KVL 1: $V_i(t) = Ri(t) + \frac{1}{C}\int_0^t i(t)dt$

$V_o(t) = \frac{1}{C}\int_0^t i(t)dt$

} System governing equation

taking Laplace transform

$$L\int_0^t F(t) = \frac{F(s)}{s} - \frac{f(0^-)}{s}.$$

$$V_c(s) = \frac{1}{C}\left[\frac{I(s)}{s} - \frac{q(0^-)}{s}\right]$$

initial charge on capacitor = 0

~~= 0~~.

$V_L = L\frac{di}{dt}$

$V_L(s) = L\left[sI(s) - I(0)\right]$

Take laplace transform

$V_i(s) = RI(s) + \frac{1}{Cs}\left[I(s) - \cancel{q(0)}^{\,0}\right]$

$V_i(s) = RI(s) + \frac{I(s)}{Cs}$

$V_o(s) = \frac{I(s)}{Cs}$

Transfer function $= H(s) = \frac{V_o(s)}{V_i(s)}$

$= \frac{\frac{\cancel{I(s)}}{\cancel{Cs}}}{\frac{\cancel{I(s)}}{\cancel{Cs}}\left[\frac{RI(s)}{I(s)} \times Cs + 1\right]} = \frac{\cancel{I(s)}}{I(s) + R\cancel{I(s)} \times \cancel{Cs}}$.

$\Rightarrow \frac{\frac{I(s)}{Cs}}{\left(R + \frac{1}{Cs}\right)\cancel{I(s)}} \Rightarrow \frac{\cancel{Cs}}{\frac{RCs+1}{\cancel{Cs}}} \Rightarrow \frac{1}{RCs+1}$.

General form

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{Q(s)}{P(s)} = \frac{\text{zero factors}}{\text{pole factors}} = \frac{(1+T_1 s)(1+T_2 s)}{(1+T_3 s)(1+T_4 s)}$$

$Q(s)=0 \qquad s = \frac{-1}{T_1}, \; s = \frac{-1}{T_2}$

$p(s)=0 \qquad s = \frac{-1}{T_3}, \; s = \frac{-1}{T_4}.$

- 2 critical frequency poles ,4 zeros.

* $\boxed{RCs+1=0}$

↳ characteristic equation.

$s = \frac{-1}{RC} \rightarrow$ pole

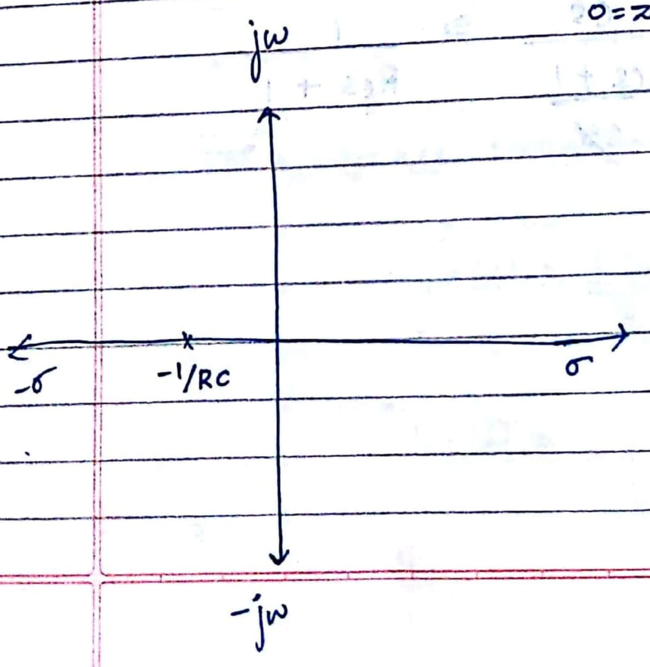
* T function $H(s) = \frac{1}{RCs+1}.$

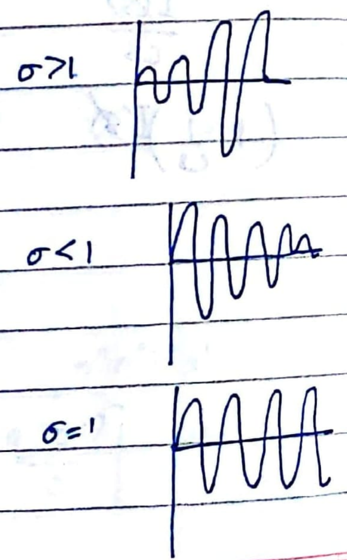
damping factor

$s = \sigma + j\omega.$

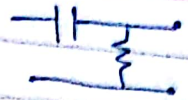
S-plane

x = poles
o = zeros

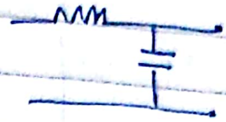




→ Time Response Analysis.

$T = \frac{RCs}{RCs+1}$ 

→ Transient Response
→ steady state response.

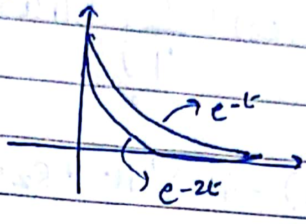
$\frac{1}{RCs+1}$ 

For any system, transient time should be less

ex: $\frac{1}{s+1}$ $\frac{1}{s+2}$

$s_1 = -1$ $s_2 = -2$

$\Rightarrow e^{-t}$ $\Rightarrow e^{-2t}$

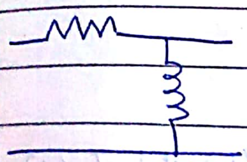


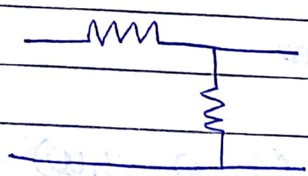
• pole that goes away from the origin, it is more stable

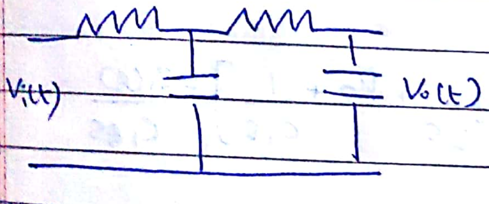
__Order of the system:__

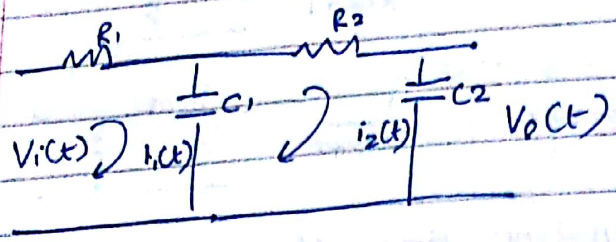
• order of a system = no of storage elements

H/w Derive transfer function for







Q. 

Mesh 1: $$V_i(t) = R_1 i_1(t) + \frac{1}{C_1}\int_0^t [i_1(t) - i_2(t)]dt \quad -(1)$$

Mesh 2: $$\frac{1}{C_1}\int_0^t [i_2(t) - i_1(t)]dt + R_2 i_2(t) + \frac{1}{C_2}\int_0^t i_2(t)dt$$

$$V_0(t) = \frac{1}{C_2}\int_0^t i_2(t)dt.$$

taking ratio $\frac{V_0(s)}{V_i(s)} = \frac{1}{RCs}$

Laplace transform of eq (1)

$$V_i(s) = RI_1(s) + \frac{1}{C_1}\left[\frac{I_1(s)}{s} - \frac{I_2(s)}{s}\right]$$

$$V_i(s) = RI_1(s) + \frac{I_1(s)}{C_1 s} - \frac{I_2(s)}{C_1 s} \quad -(1)$$

Laplace transform of eq (2)

$$\frac{1}{C_1}\left(\frac{I_2(s)}{s} - \frac{I_1(s)}{s}\right) + R_2 I_2(s) + \frac{1}{C_2}\frac{I_2(s)}{s}.$$

$$I_2(s)\left[\frac{1}{C_2 s} + R_2 + \frac{1}{C_1 s}\right] = \frac{I_1(s)}{C_1 s}.$$

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$$I_2(s)\left[\frac{C_1}{C_2}+C_1SR_2+1\right] = I_1(s) \quad \text{--- } \cancel{(2)}\ (3)$$

sub eq (3) in eq (1)

$$V_i(s) = R_1 I_2(s)\left[\frac{C_1}{C_2}+C_1SR_2+1\right]+\frac{1}{C_1S}\left[\frac{C_1}{C_2}+C_1SR_2+1\right]I_2(s)$$

$$-\frac{I_2(s)}{C_1S}.$$

$$V_i(s) = \cancel{R_1 I_2 C_1 + C_1SR_2R_1}$$

$$= \left[R_1+\frac{R_1C_1}{C_2}+R_1R_2C_1S+\frac{\cancel{1}}{\cancel{S}C_1}+\frac{1}{SC_2}+R_2-\frac{\cancel{1}}{C_1S}\right]I_2(s)$$

$$V_i(s) = \left[\frac{R_1SC_2+R_1C_1S+R_1R_2C_1C_2S^2+1+R_2C_2S}{SC_2}\right]I_2(s).$$

$R_1=R_2=R$

$C_1=C_2=C$

$$\frac{V_o(s)}{\cancel{V_i(s)}} = \frac{\cancel{I_2(s)}}{\cancel{C(s)}}$$

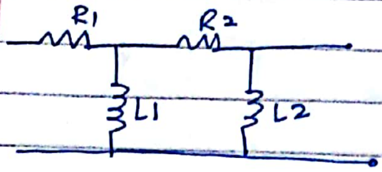
$$V_o(t)=\frac{1}{C_2}\int^t i_2(t)\,dt$$

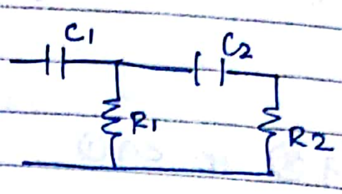
$$V_o(s)=\frac{I_2(s)}{C_2S} \quad \text{--- } (4)$$

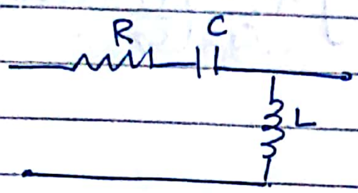
$$\frac{V_o(s)}{V_i(s)} = \frac{\cancel{I_2(s)}\times\cancel{SC}}{C\cancel{(s)}\times RSC+RCS+R^2C^2S^2+1+R\cancel{C}S]}\times\frac{1}{\cancel{I_2(s)}}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{R^2C^2S^2+3RCS+1}$$

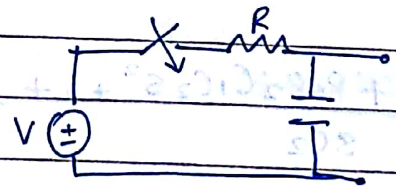
Q. Derive the transfer function.







* For the circuit



$$V_i(t) = Ri(t) + \frac{1}{C}\int_0^t i(t)\,dt$$

$V_i(t) \Rightarrow$ step i/p

$$\frac{V}{S} = RI(s) + \frac{I(s)}{CS}$$

$$\frac{V}{S} = I(s)\left[R + \frac{1}{CS}\right]$$

$$V = I(s)\left[Rs + \frac{1}{C}\right]$$

$$I(s) = \frac{V}{\left(s + \frac{1}{RC}\right)R}$$

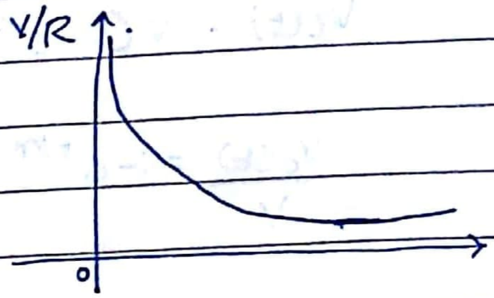
$$I(s) = \frac{V}{R} \times \frac{1}{\cancel{s}\; s + \frac{1}{RC}}$$

$L^{-1}T$

$$i(t) = \frac{V}{R} e^{-1/RC \times t}.$$

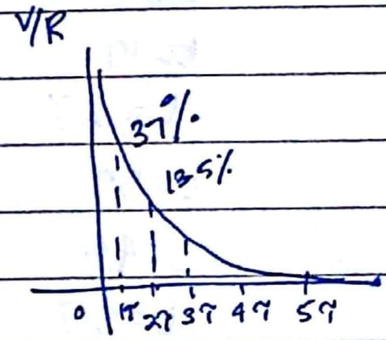
$RC = \tau =$ Time constant.

$$\boxed{i(t) = \frac{V}{R} e^{-t/\tau}.}$$



$i(t) = I_0 e^{-t/\tau}$ $\qquad \frac{i(t)}{I_0} = e^{-t/\tau}.$

t	$i(t)/I_0$		
$t=0\tau$	e^0	$=1$	
$t=1\tau$	e^{-1}	$=0.367$	
$t=2\tau$	e^{-2}	$=0.13$	
$t=3\tau$	e^{-3}	$=0.04$	
$t=4\tau$	e^{-4}	$=0.018$	
$t=5\tau$	e^{-5}	$=6.73m$	$=0.006$
$t=6\tau$	e^{-6}	$=2.47m$	$=0.0024$



when $t=\tau$

$i(t) = \tau_0$

* I_0 is the max current value

so the current decreases to 37% of its initial value.

* the current reaches to 63% of its final value

$V_c(t) = V_i(t) - V_R(t)$

$V_c(t) = V - I(t) \cdot R$

$V_c(t) = V - R\left(\frac{V}{R} e^{-t/\tau}\right)$

$V_c(t) = V - Ve^{-t/\tau}$

$V_c(t) = V(1-e^{-t/\tau})$

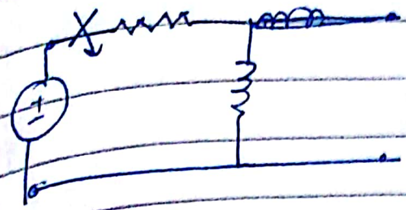
$\frac{V_c(t)}{V} = 1-e^{-t/\tau}$

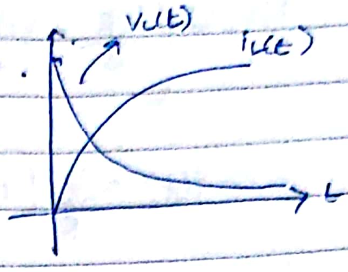
t	$V_c(t)/V$
$t=0\tau$	1
$t=1\tau$	0.632
$t=2\tau$	0.864
$t=3\tau$	0.95
$t=4\tau$	0.98
$t=5\tau$	0.99

Derive

Scanned by CamScanner

* Derive the current equation for RL circuit





$$Ri(t) + L\frac{di}{dt}$$

Laplace transform of eq

$$\frac{V}{S} = RI(S) + LSI(S)$$

$$I(s) = \frac{V/s}{(R+LS)}$$

$$I(S) = \frac{V/S}{L(S+R/L)}$$

$$I(s) = \frac{V}{L} \times \frac{1}{S(S+R/L)}$$

taking partial fraction.

$$\frac{V/L}{S(S+R/L)} = \frac{A}{S} + \frac{B}{S+R/L}$$

$$\frac{V}{L} = A\left(S+\frac{R}{L}\right) + BS$$

put $S=0$ $\qquad A = V/R$

put $S=-R/L$ $\qquad B = -V/R$

$$L^{-1}$$

$$I(s) \Rightarrow \left[\frac{V/R}{s} - \frac{V/R}{s+R/L}\right] = \frac{V}{R}\left[1 - e^{-R/Lt}\right]$$

$$i(t) = \frac{V}{R}\left[1 - e^{-t/(L/R)}\right]$$

$$i_L(t) = \frac{V}{R}\left(1 - e^{-t/\tau}\right)$$

$$V_L(t) = -Ve^{-t/\tau}$$

* $V_C(t)$ or $i_L(t)$ = [final value] + [initial value − final value] ~~(1 − e^{-t/τ})~~ $\times e^{-t/\tau}$.

* $V_L(t)$ OR $i_C(t)$ = [final value] + [initial value − final value] $(1 - e^{-t/\tau})$.

$$V_C(t) = V(1 - e^{-t/\tau}).$$

store
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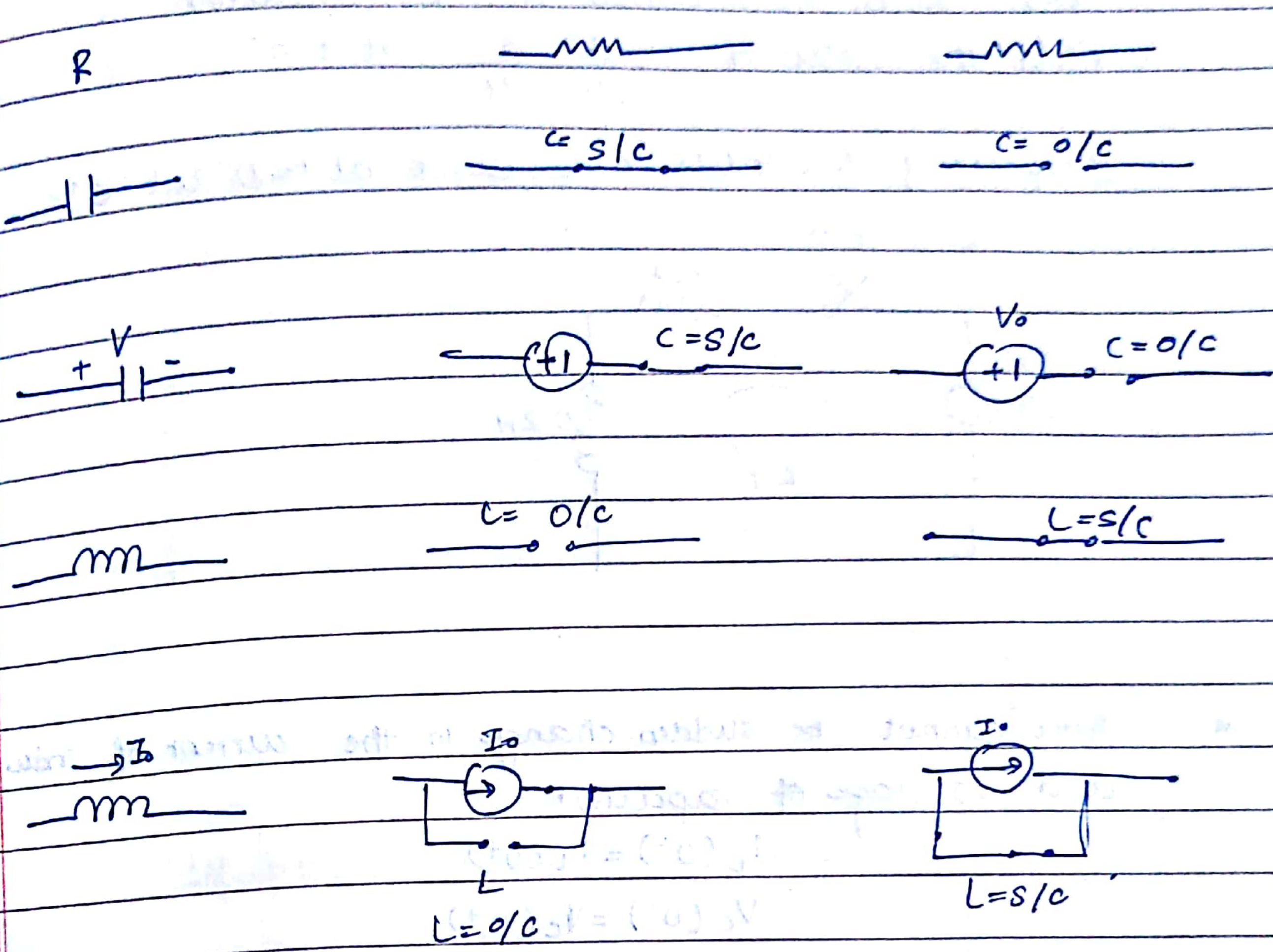
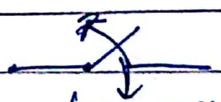
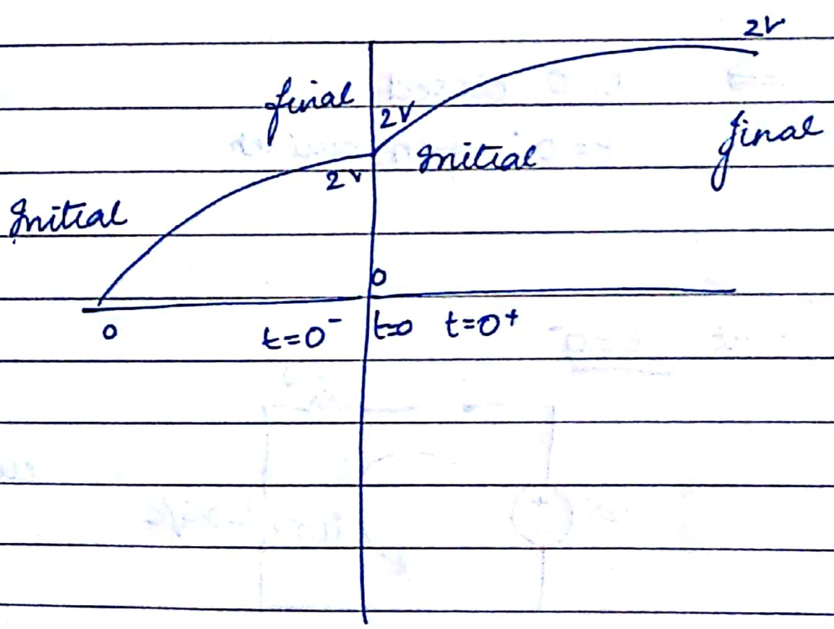
elements t=0 indial condition t=∞/0⁻ final condition.





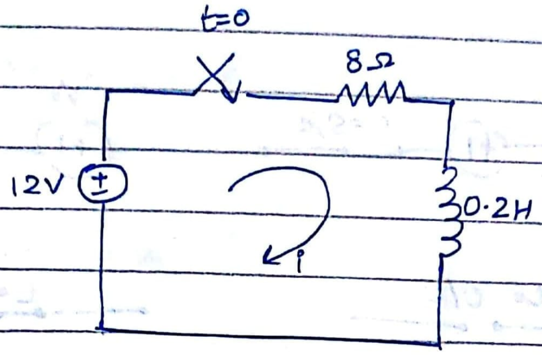
t=0 → s/w open
t=0⁻ ⇒ s/w closed
t=0⁺ → s/w open



Problems

1. In the given network, switch k is closed at $t=0$ with zero current in the inductor. Find the value of i, $\frac{di}{dt}$, $\frac{d^2i}{dt^2}$ at $t=0^+$

(referring the table take cases at $t=0^-$ & $t=0^+$)



* there cannot be sudden change in the current of inductor and volltage of capacitor

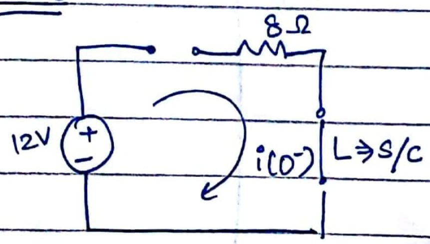
$\therefore i_L(0^-) = i_L(0^+)$

$V_c(0^-) = V_c(0^+)$

⇒ $t=0^+$ closed

$t=0^-$ open switch

∘ at $t=0^-$



$i(0^-)$ is same as $i_L(0)$

current flowing is zero

$\therefore i_L(0^-)=0$

$\boxed{i_L(0^-)=i_L(0^+)}$

$\Rightarrow i(0^+) = i(0^-)$

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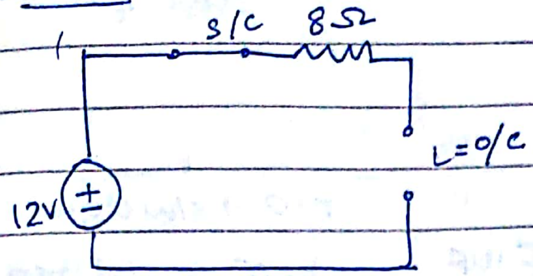
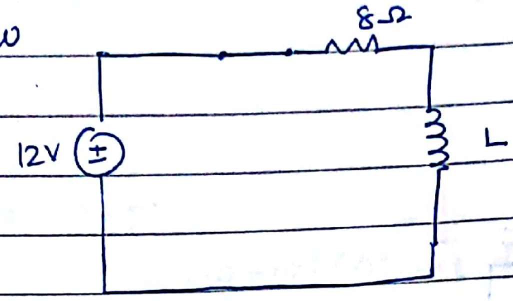
• at $t=0^+$



replace with original comp

* now



KVL

$$-12+8i(t)+L\frac{di}{dt}$$

at $t=0^+$

$$-12+8\cancel{i(0^+)}^{\,0}+L\frac{di}{dt}(0^+)=0 \quad (1)$$

$$-12+L\frac{di(0^+)}{dt}=0$$

dif eq(1)

$$L\frac{di(0^+)}{dt}=12$$

$$\frac{di(0^+)}{dt}=\frac{12}{0.2}$$

$$=60\ A/sec$$

to find $\frac{d^2i}{dt^2}$ dif eq(1)

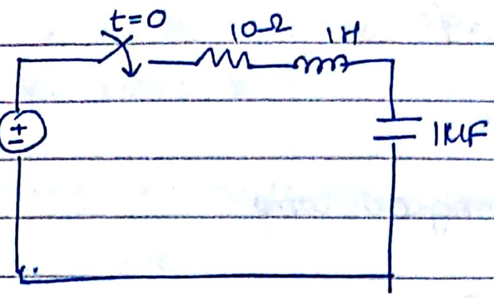
$$0+8\frac{di(0^+)}{dt}+L\frac{di^2(0^+)}{dt}=0$$

$$8\times 60+0.2\times\frac{di^2(0^+)}{dt}=0$$

$$\frac{d^2(0^+)}{dt}=\frac{-480}{0.2}=\frac{-48\cancel{0}\times 10}{2}=-2400\ A/sec^2$$

2. The switch is closed at $t=0$, determine $i, \frac{di}{dt}, \frac{d^2i}{dt^2}$

at $t=0^+$

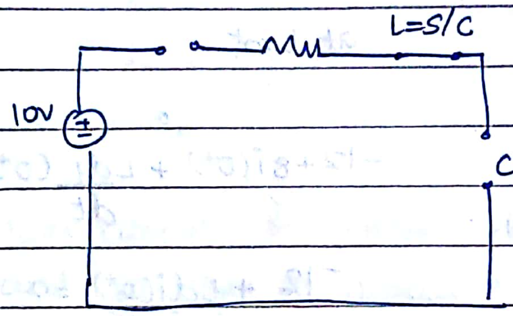


$t=0 \rightarrow$ s/w closed

$t=0^- \rightarrow$ s/w open

$t=0^+ \rightarrow$ s/w closed

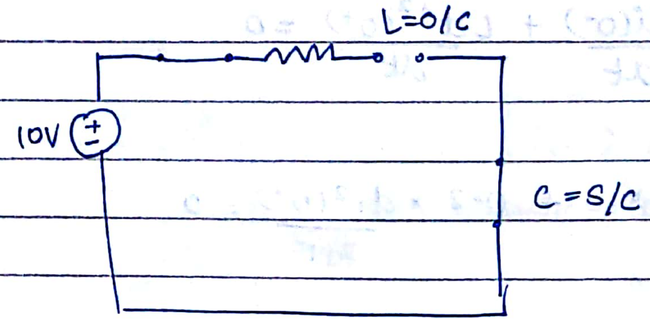
• at $t=0^-$



$i_L(0^-) = 0 = i_L(0^+)$

$V_c(0^-) = 0. = V_c(0^+)$

• at $t=0^+$



store
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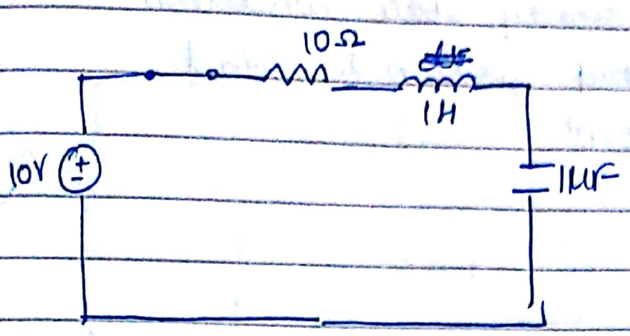
To find $\frac{di}{dt}$



KVL

$$-10 + 10\,i(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int_0^t i(t)\,dt = 0 \quad \text{—(1)}$$

at $t = 0^+$

$$-10 + 10\,\cancelto{0}{i(0^+)} + L\frac{di(0^+)}{dt} + \frac{1}{C}\int_0^t \cancelto{0}{V_C(0^+)} = 0.$$

$$-10 + L\frac{di(0^+)}{dt} = 0$$

$$\frac{di(0^+)}{dt} = \frac{10}{1} = 10\ A/sec$$

To find $\frac{d^2i}{dt^2}$.

dif the KVL eq:

$$10\frac{di(t)}{dt} + L\frac{d^2i(t)}{dt^2} + \frac{1}{C}\, i(t)\cancel{dt} = 0$$

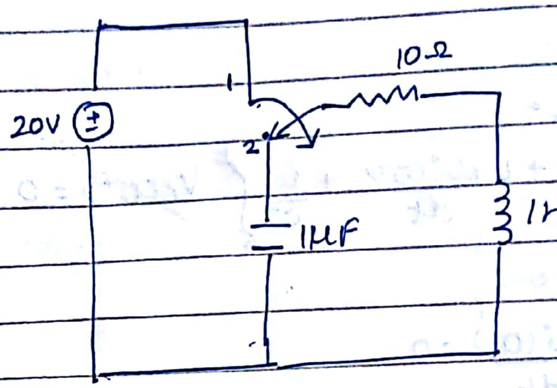
$$10\,(10) + 1\,\frac{d^2i(t)}{dt^2} + \frac{1}{1} = 0$$

$$100 + \frac{d^2i(t)}{dt^2} = 0$$

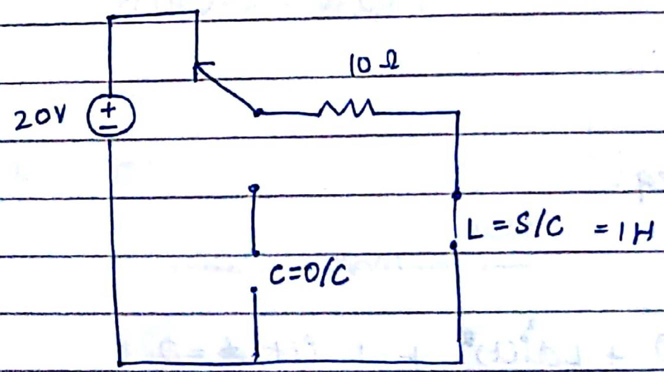
$$\frac{d^2i(t)}{dt^2} = -100\ A/sec$$

3. The switch K is changed from position 1 to position 2 at $t=0$. Steady state condition having being reached position 1. Find $i, \frac{di}{dt}, \frac{d^2i}{dt^2}$ at $t=0^+$.

→



It was in pos 1 at $t=0^-$.



By ohm's law

$I = V/R$

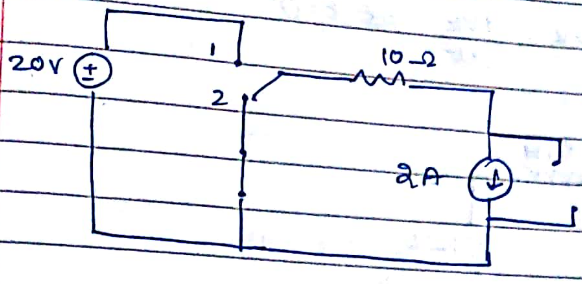
$= 20/10$

$= 2A$

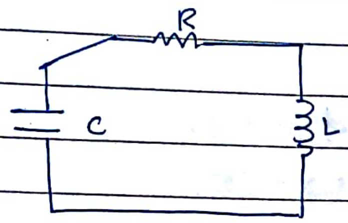
$i_L(0^-) = i_L(0^+) = 2A$

$V_c(0^-) = 0 = V_c(0^+)$

- at $t=0^+ \Rightarrow$ pos 2.



- To find $\frac{di}{dt}$, $t=0^+$



$$\underbrace{\frac{1}{C}\int_0^t i(t)\,dt}_{V_C(0^+)} + Ri(t) + L\frac{di}{dt} = 0 \quad -(1)$$

$$R\,\cancel{i(0^+)}^{\,2A} + L\frac{di(0^+)}{dt} = 0$$

$$\frac{di(0^+)}{dt} = -20\ A/sec$$

To find $\frac{d^2 i(t)}{dt^2}$ at $t=0^+$.

dif eq (1)

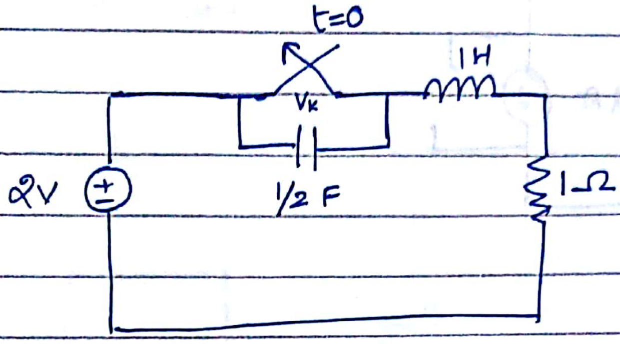
$$\frac{1}{C}i(t) + R\frac{di(t)}{dt} + L\frac{di^2(t)}{dt^2} = 0$$

$$\frac{2}{1\mu} + 10\times(-20) + 1\,\frac{di^2(t)}{dt^2} = 0$$

$$-200 + \frac{2}{1\mu} + \frac{di^2(t)}{dt} = 0$$

$$\frac{di^2(t)}{dt} = \cancel{198}\ 200 - \frac{2}{1\mu} = 200 - 2\times10^6 = -1.9\times10^6$$

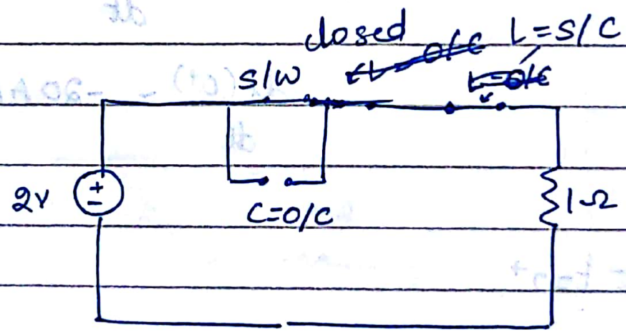
4. The circuit is in steady state with switch K closed. At t=0, switch is open, determine the voltage across the switch V_K, $\frac{dV_K}{dt}$ at $t=0^+$



$t=0^- \rightarrow$ switch was closed

$t=0^+ \rightarrow$ switch is opened

• at $t=0^-$:



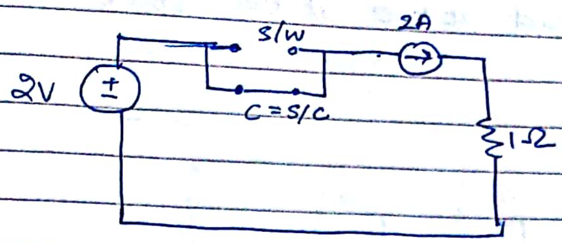
$$i_L(0^-) = 2A = i_L(0^+)$$

$$V_C(0^-) = 0 = V_C(0^+)$$

$$V_C = V_K$$

$$\Rightarrow V_K(0^+) = 0.$$

- at $t=0^+$



$$i_c(t) = C\frac{dV_c}{dt}$$

$$i_c(t) = C\frac{dV_K}{dt}$$

at $t=0^+$

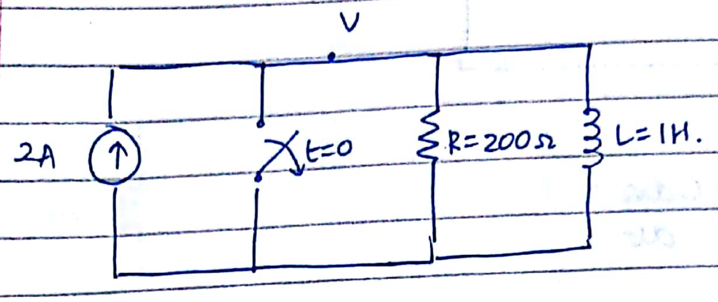
$$i_c(t) = C\frac{dV_K}{dt}$$

$$2 = \frac{1}{2}\frac{dV_K}{dt}$$

$$4 = \frac{dV_K}{dt}$$

5. switch K is opened at $t=0$ at $t=0^+$ solve $V, \frac{dV}{dt}, \frac{d^2V}{dt^2}$.

→

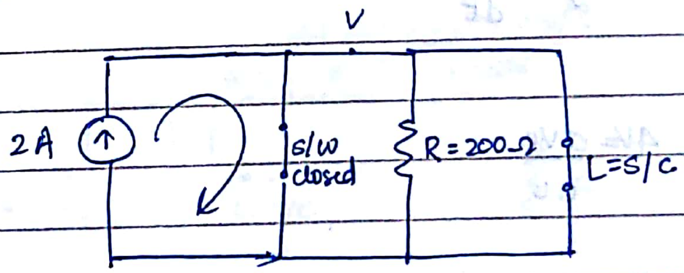


⇒ $t=0^-$ = switch was closed

$t=0$ = switch open

$t=0^+$ = switch open

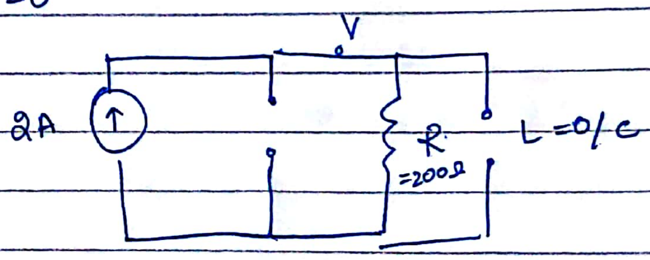
at $t=0^-$



$i_L(0^-) = i_L(0^+) = 0$

$V(0^-) = 0$

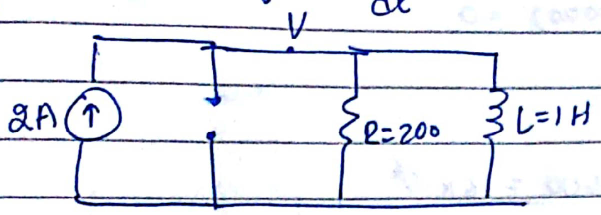
at $t=0^+$



store 67

$i_L(0^+) = 2\times 200 = 400\,V.$

At $t=0^+$ find $\frac{dv}{dt}$



$\frac{dv}{dt} \to$ write KCL eq.

$$-2+\frac{V}{200} + \frac{1}{L}\int_0^t V_C(t)dt = 0 \quad -①$$

dif this eq

$$0+\frac{1}{200}\frac{dV}{dt} + \frac{1}{L}V(t) = 0 \quad -②$$

at $t=0^+$

$$\frac{dv}{dt} = \cancel{200(}$$

$$\frac{1}{200}\frac{dv}{dt} + \frac{1}{L}V(0^+) = 0$$

$$\frac{dv}{dt} = -400\times 200$$
$$= -80000$$

dif eq②

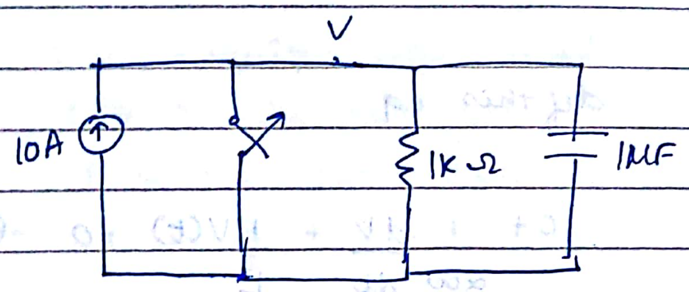
$$\frac{1}{200}\frac{d^2V}{dt^2} + \frac{1}{L}\frac{dv}{dt} = 0$$

$$\frac{1}{200}\frac{d^2v}{dt^2} + \frac{1}{L}\frac{dv}{dt} = 0$$

$$\frac{1}{200}\frac{d^2v}{dt^2} + (-80000) = 0$$

$$\frac{d^2v}{dt^2} = 16000000 = 16\times10^6$$

H/W

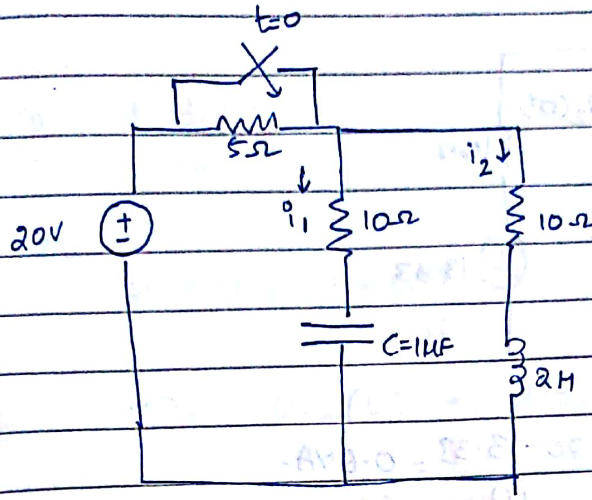


S/W K is open at $t=0$

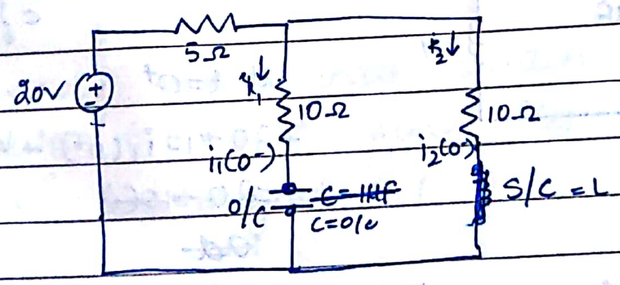
At $t=0^+$ find $v, \frac{dv}{dt}, \frac{d^2v}{dt^2}$

store
67

6. steady state is reached with switch K open, the switch is closed at t=0. Determine $I_1, I_2, \frac{di_1}{dt}, \frac{di_2}{dt}$ at $t=0^+$



t=0⁻ (previous stage)
open switch



$i_1(0^-)=0$.

$i_L = i_2(0^-) = \frac{20}{5+10} = \frac{20}{15} = 1.33$

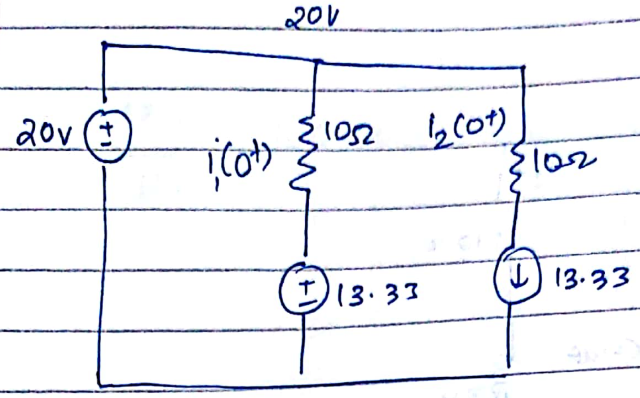
main V

opp branch R

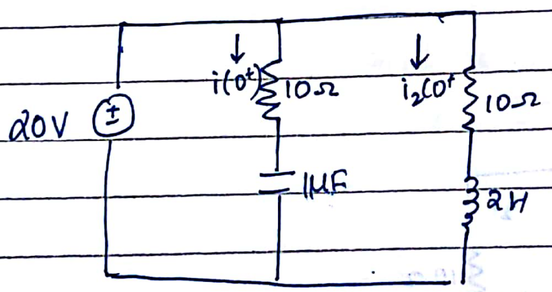
$V_c(0^-) = V_c(0^+) = \cancel{20}\ \cancel{20\times0} \frac{20\times10}{15} = 13.3\,V.$

Total R

- replace cap by 13.3V voltage source.
- replace inductor by current source ↓ 1.33



$$i_1(0^+) = \frac{20-13.33}{10} = 0.67A.$$



KVL

$$-20 + 10\,i_1(t) + \frac{1}{C}\int_0^t i_1(t)\,dt = 0 \quad \text{(1)}$$

at $t=0^+$

$$-20 + 10\,i_1(0^+) + V_c(0^+) = 0$$

~~$-20+10C$~~

~~$10d$~~

to find $\frac{di_1}{dt}$ at $t=0^+$.

dif eq (1)

$$0 + 10\frac{di_1(0^+)}{dt} + \frac{i_1(t)}{C} = 0$$

$$10\frac{di_1(0^+)}{dt} = -\frac{0.67}{1\mu F}$$

store
67

$$\frac{di(0^+)}{dt} = -0.67 \times 10^5 A/sec$$

$$\cancel{10\frac{di_1(0^+)}{dt} = -0.67}$$

To find $\frac{di_2}{dt}$

KVL :

$$-20 + 10 i_2(t) + L\frac{di_2}{dt} = 0$$

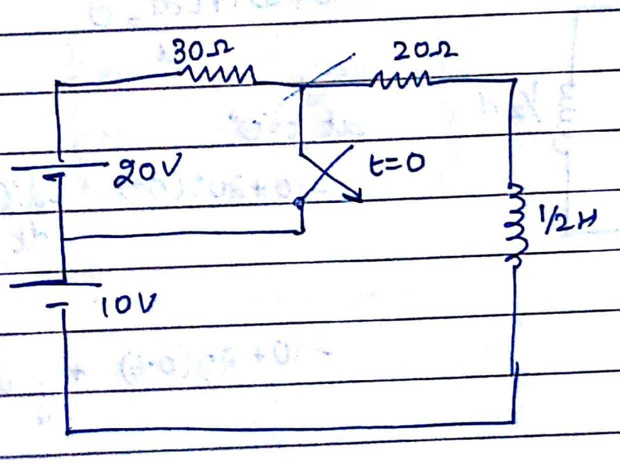
$$-20 + 10 i_2(0^+) + L\frac{di_2(0^+)}{dt} = 0$$

$\boxed{i_2(0^+) = 1.33}$

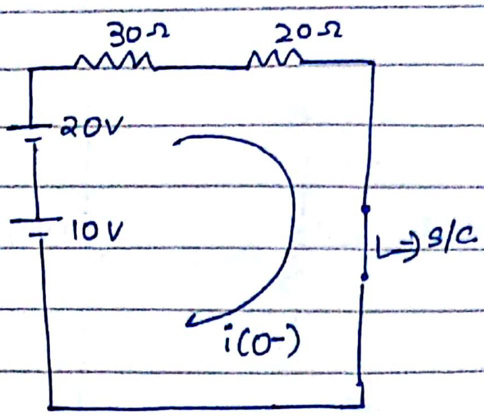
$$\frac{di_2(0^+)}{dt} = \frac{(-1.33)10 + 20.}{L}$$

$$= \frac{-13.3+20}{2.} = \frac{\cancel{7}6.7}{2} = 3.35$$

7) Steady state with switch K open at t=0, the switch is closed find i(t) at t=0+ and $\frac{di(t)}{dt}$ at $t = 0^+$ & also find $i(\infty)$.

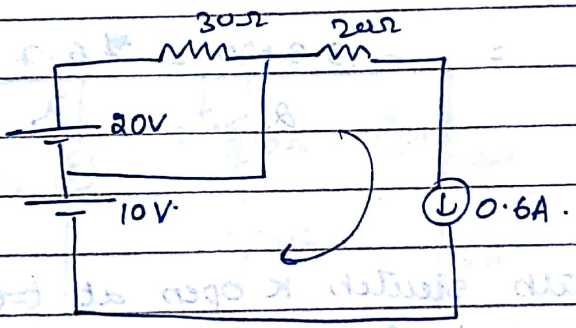


$t=0^- \Rightarrow$ s/w open



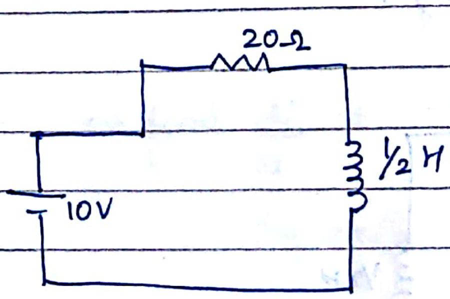
$$i(0^-) = \frac{30}{50} = 0.6A.$$

$$i_L = i(0^-) = i(0^+).$$



KVL

$$-10 + 20i + L\frac{di}{dt} = 0.$$



at $t=0^+$

$$-10 + 20\,i(0^+) + L\frac{di(0^+)}{dt} = 0$$

$$-10 + 20(0.6) + \frac{1}{2}\frac{di(0^+)}{dt} = 0$$

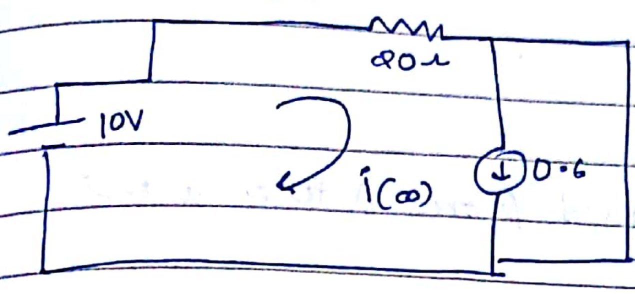
$-\cancel{10\times2} \rightarrow$

store 67

$$-10 + 1\cdot 2 + \frac{1}{2}\frac{di(0^+)}{dt} = 0$$

$$\frac{di(0^+)}{dt} = -2\times 2 = -4 \text{ A/sec}$$

$t=\infty$

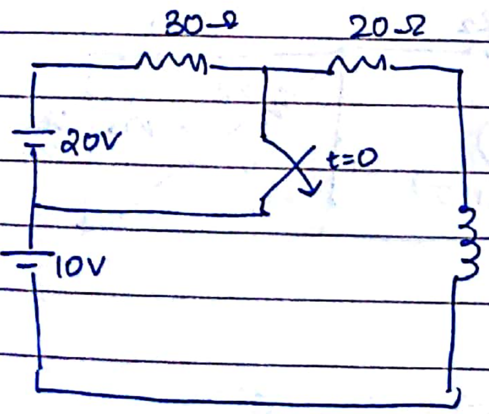


$i(\infty)$

$$-i(0) + 20i(\infty) = 0$$

$$i(\infty) = \frac{10}{25}.$$

8) The network reaches steady state with switch K open, at t=0, the switch is closed. Find i(t) for the numerical values given. Sketch the current waveform.



2 port
i/p o/p imp – x
G parameter – x

Fourier time.
concept of time const
system govern
charac
Initial cond
transfer func
CRC & RC
RL methods

$i_L = [\text{final value}] + [\text{initial} - \text{final value}]\, e^{-t/\tau}$.

$= 0.5 + [0.6 - 0.5]\, e^{-t/\tau}$.

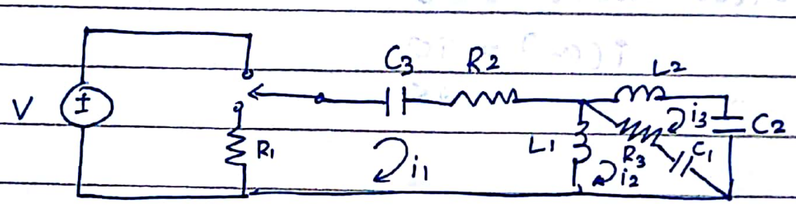
$\boxed{i_L(t) = 0.5 + 0.1\, e^{-t/\tau}}$.

$\tau = L/R = 0.5/20 = 25m$

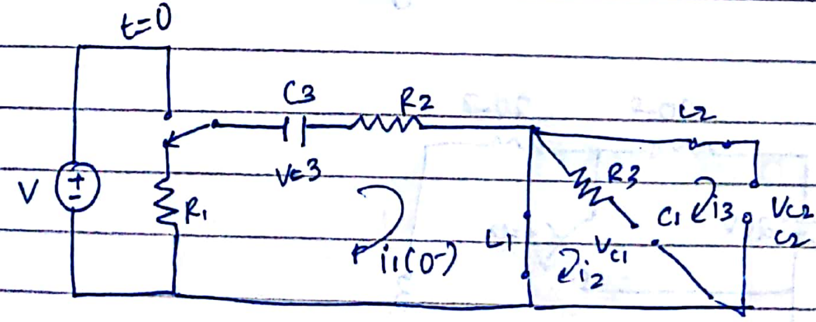
~~How much~~

Imp 9) The switch K is changed from A to B at $t = 0^-$. Show that at $t = 0^+$

$$i_1 = i_2 = \frac{-V}{R_1 + R_2 + R_3}, \quad i_3 = 0.$$



→ . switch at $t = 0^+$ switch will be at B.



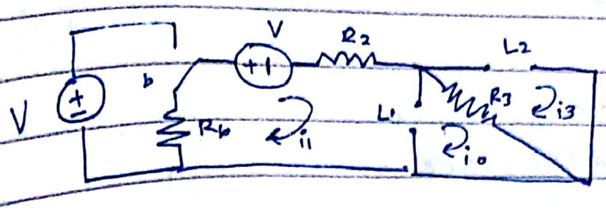
$i_1(0^-) = 0$

$V_{c3}(0^-) = V$ volts

$V_c(0^+) = V_c(0^-) = 0$

$V_{c2}(0^-) = 0 = V_{c2}(0^+)$

At $t=0^+$



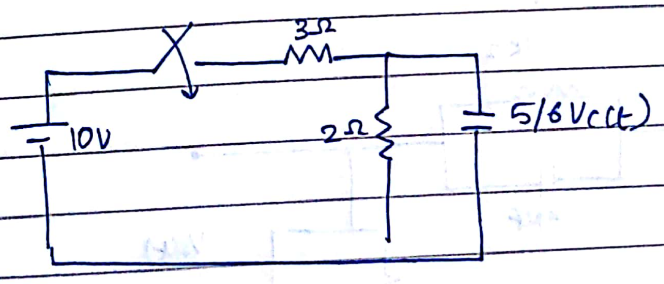
$R_1 i_1 + V + R_2 i_7 + R_3 i_1 = 0$

$i_1(R_1+R_2+R_3) = -V$

$\therefore \; i_1 = \dfrac{-V}{R_1+R_2+R_3}$

10 In the circuit shown the switch is closed at $t=0$. Assuming zero initial condition, find $V_C(t)$ at $t=1$ sec.

~~find the value of Vc(t)~~



$V_C(t) = [\text{final}] + [\text{initial} - \text{final}]\, e^{-t/\tau}.$

$\rightarrow V_C(\infty) = \dfrac{10\times 2}{5}$

$V_C(\infty) = 4V$

$V_C(t) = 4 + [0-4]\, e^{-t/\tau}.$

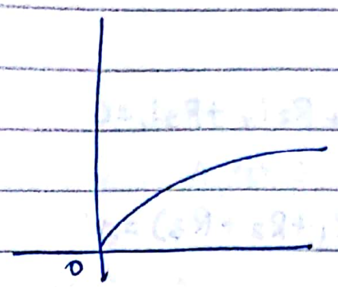
$V_c(t) = 4(1-e^{-t/\tau})V$

τ value is calculated.

$\tau = R_{eq} \times C$

$\tau = 1.2 \times \frac{5}{6} = 1 sec$

$\rightarrow$ $\boxed{V_c(t)\big|_{t=1sec} = 2.52 Volts}$



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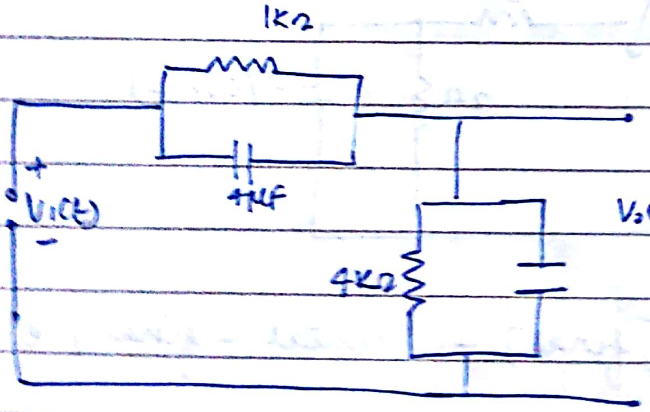
11) In the figure shown below assume that all the caps are initially uncharged. If $V_i(t) = 10u(t)$ volts find $V_o(t)$.

$V_i(t) = 10u(t)$

$V_o(t) = ?$

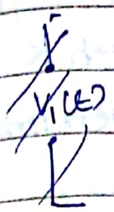
$\rightarrow$



$V_o(t) = [\text{final value}] + [\text{Initial value}]e^{-t/\tau}$

$V_c(0^-) = V_c(0^+) = 0V.$

final value = the capacitor behaves as



Final value = 8

Initial = 0.

$$V_o(t) = 8 + [0 - 8]\, e^{-t/\tau}$$

$1 \| 4$

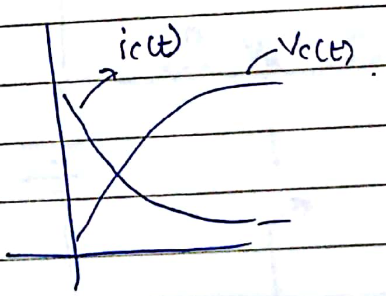
$$\tau = R_{eq} C_{eq} = 0.8k \times 8\mu.$$

$$\tau = 6.4ms$$

- Time Response

$$i_c(t) = \frac{V}{R} e^{-t/\tau}$$

$$V_c(t) = V(1 - e^{-t/\tau})$$



- Frequency response

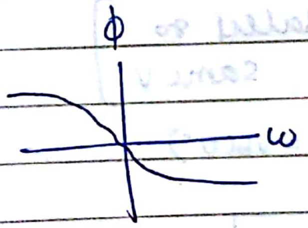
$$H(s) = \frac{1}{RCs + 1}$$

$$s = \cancel{\sigma}^{\,0} + j\omega$$

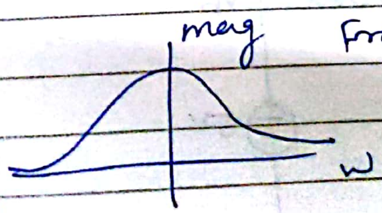
$$\boxed{H(s) = \frac{1}{RC(j\omega)}}$$

$$|H(j\omega)| = \frac{1}{\sqrt{1 + R^2C^2\omega^2}}$$

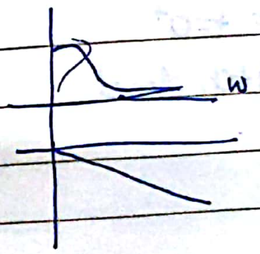
$$\phi = -\tan^{-1} \frac{\omega RC}{1}$$



Time response ←

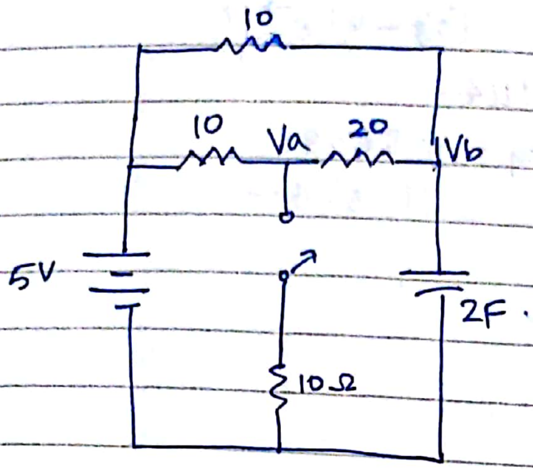


Freq Response ←

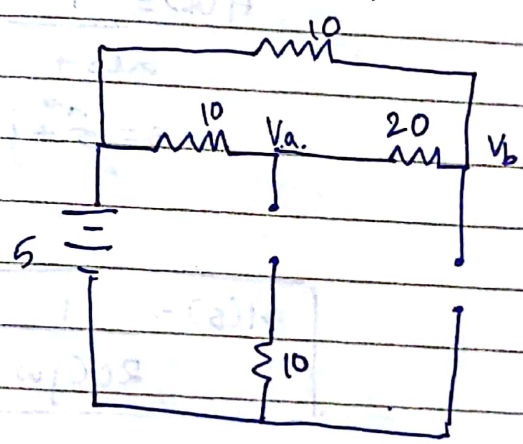


13 NOV 2018

12. The steady state is reached with switch K open. at t=0, the switch K is closed, Determine the values of Va(0⁻) and Va(0+)



→ At $t=0^-$ switch open.



* doubt

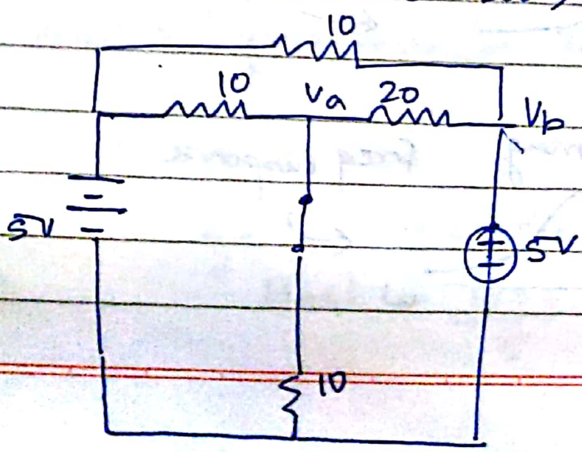
$Va(0^-) = 5V$ [parallel so same V]

also $Vb(0^-) = 5V$

$\Rightarrow$ ~~Vc~~ $Vb(0^-) = 5V = V_C = Vb(0^+)$

At $t=0^+$

switch closed



applying KCL.

$$\frac{Va-5}{10} + \frac{Va}{10} + \frac{Va-5}{10} = 0$$

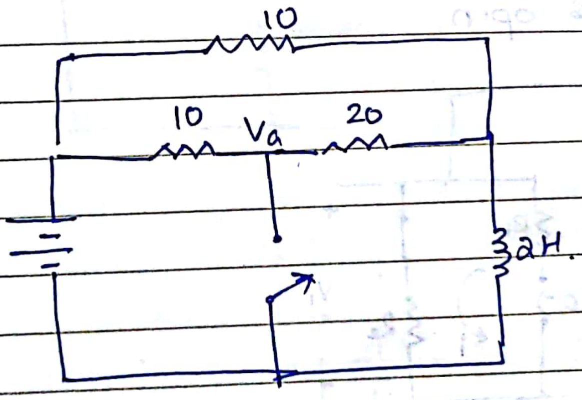
$$\frac{Va(0^+)-5}{10} + \frac{Va(0^+)}{10} + \frac{Va(0^+)-5}{20} = 0$$

$$Va(0^+)\left[\frac{1}{10}+\frac{1}{10}+\frac{1}{20}\right] = \frac{5}{20}+\frac{5}{10}$$

$$Va(0^+) = \frac{0.75}{0.\cancel{2}25}$$

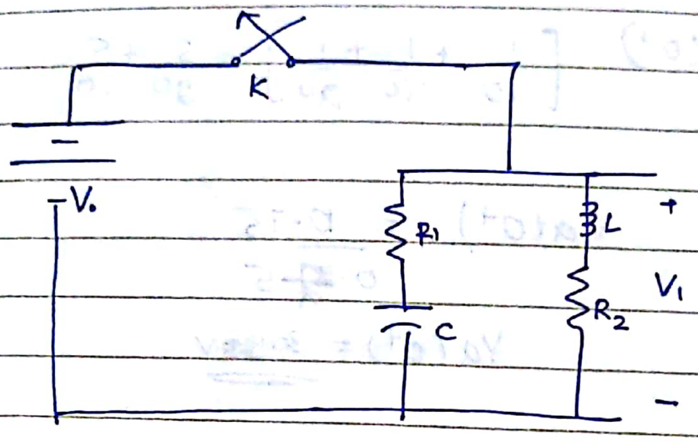
$$Va(0^+) = 3.\cancel{33}V.$$

H.W.

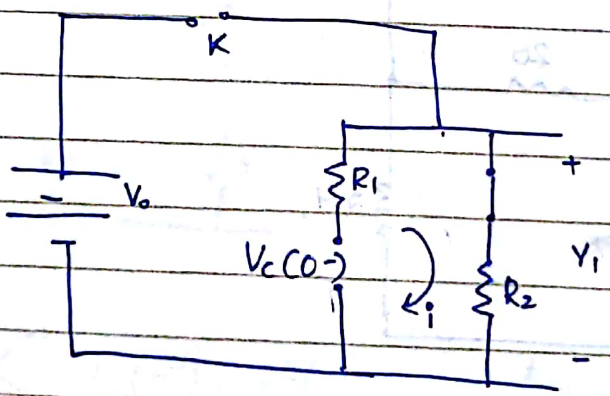


13. The switch K is the network is closed at $t=0$. Connecting the battery to an unenergised network, determine $i, \frac{di}{dt}$ and $\frac{d^2i}{dt^2}$ at $t=0^+$.

Determine $V_1, \frac{dV_1}{dt}$ and $\frac{d^2V_1}{dt^2}$ at $t=0^+$.



→ At $t=0^-$ switch was open.



$V_c(0^-)=0=V_c(0^+)$

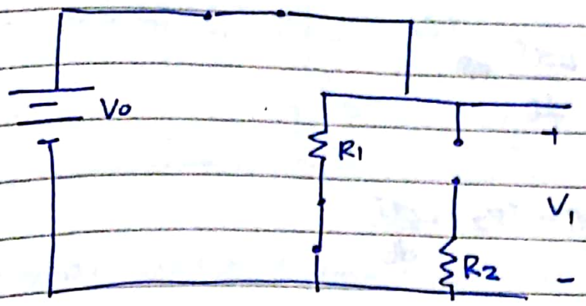
$V_1(0^-)= i/R_2\ 0.$

$i(0^-)=0$

$i(0^-)=i(0^+)=0.$

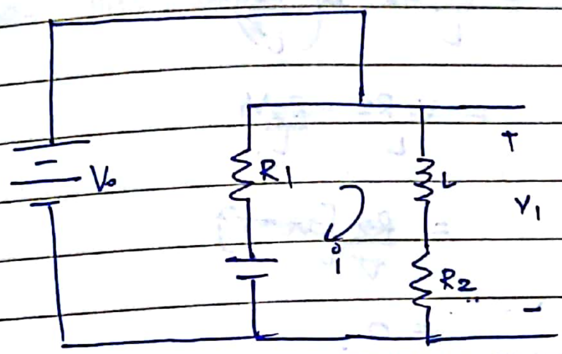
store
67

At $t=0^+$ switch was closed



$V_1(0^+) = V_0$

$\Rightarrow$



$$-V_0 + L\frac{di}{dt} + \cancel{R_2 i}^{0} = 0 \quad -①$$

$$\boxed{\frac{di}{dt} = \frac{V_0}{L} \quad A/sec} \rightarrow ④$$

dif eq ①

$$\cancel{\frac{di^2}{dt^2}} =$$

$$+L\frac{d^2i}{dt^2} + R_2\frac{di}{dt} = 0$$

$$L\frac{di^2}{dt^2} + R_2\frac{V_0}{L} = 0$$

$$\boxed{\frac{di^2}{dt^2} = -\frac{R_2 V_0}{L^2}} \quad A/sec^2$$

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$V_1(0^-) =$

$$V_1 \cancel{=} iR_2 + L\frac{di}{dt}$$

$$V_1 = iR_2 + L\frac{di}{dt}$$

dif this eq

$$\frac{dV_1}{dt} = \frac{di}{dt} R_2 + \frac{Ld^2i}{dt^2}$$

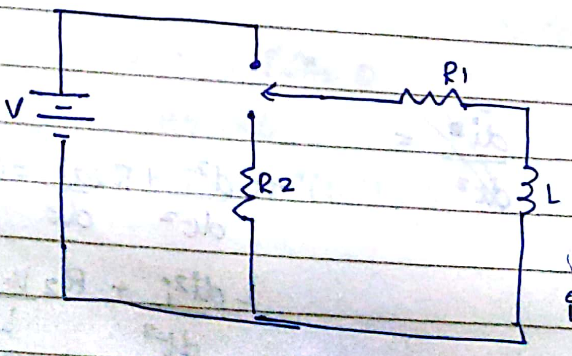
$$= \frac{V_0}{L} R_2 + \cancel{L}\left(-\frac{R_2}{L^{\cancel{2}}}\right)V_0$$

$$= \frac{V_0 R_2}{L} - \frac{R_2 V_0}{L}.$$

$$= \cancel{\frac{R_2}{V}\left[V_0 - 1\right]}$$

$$= 0.$$

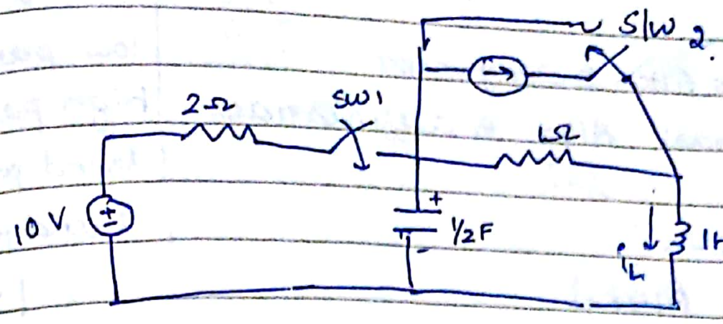
14) In the network of figure, switch K is moved from position 1 to position 2 at t=0. Steady state current having previously being reached in the RL circuit, find the particular solution for the current i(t).



Ans ↓

$$i(t) = \frac{V e^{-\frac{R_1+R_2}{L}t}}{R}$$

2)



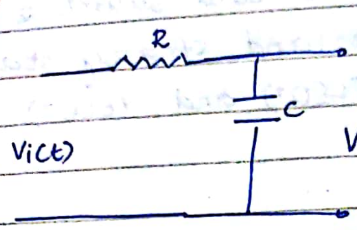
Assume that switch 1 has been opened and switch 2 has been closed for a long time and steady state condition is at $t=0^-$. Find $i_L(0^+)$ and $V_c(0^+)$ $\frac{dv(0^+)}{dt}$ and $\frac{di(0^+)}{dt}$.

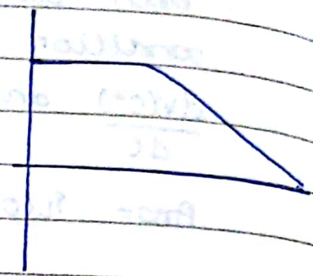
Ans= $i_L(0^+) = 0$ $\quad V_c(0^+) = -2$ $\quad \frac{di(0^+)}{dt} = -2\text{A/sec}$ $\quad \frac{V_c(0^+)}{dt} = 12\text{V/s}$

* Application of RC.
  - ⇒ RC as low pass filter & integrator
  - ⇒ RC as high pass filter & differentiator.

low pass
high pass
band pass
band rejection

* RC as low pass filter.





$X_c = \frac{1}{2\pi f c}$ = Reactance

- $f=0$ $X_c=\infty$ cap = o/c Vout = Vin
- $f=\infty$ $X_c = 0$ cap = s/c Vout = 0.

$$H(s) = \frac{1}{RCs+1}$$ ← Transfer function.

sub $s=j\omega$

$$H(s) = \frac{1}{RCj\omega+1}$$

$$\omega = 2\pi f$$

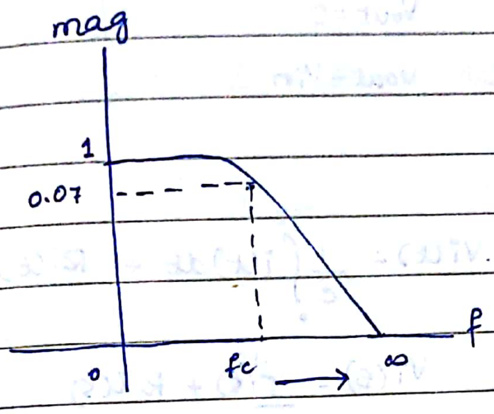
$$H(j\omega) = \frac{1}{RCj2\pi f+1}$$ f_c = cutoff freq = $\frac{1}{2\pi RC}$

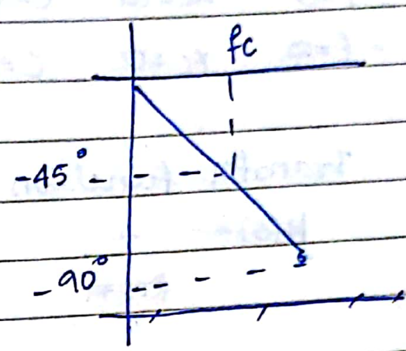
$$H(j\omega) = \frac{1}{j\, f/f_c + 1}$$

$$|H(j\omega)| = \frac{1}{\sqrt{1+\left(\frac{f}{f_c}\right)^2}}$$

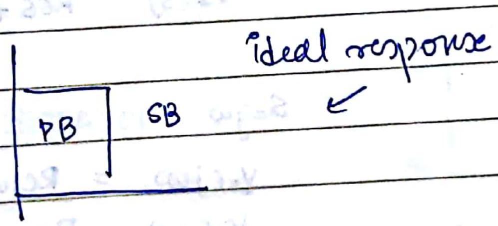
$\phi = -\tan^{-1}\left(\frac{f/fc}{1}\right)$

	mag	ϕ
$f=0$	1	0
$f=fc$	$1/\sqrt{2}$	$-\pi/4$; -45°
$f=\infty$	0	$-\pi/2$; -90°

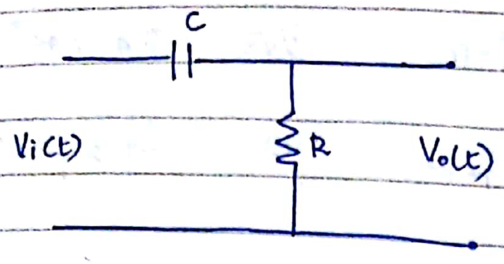


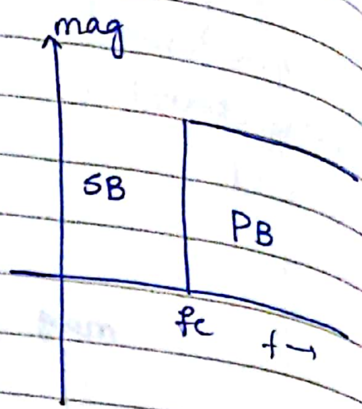


- fc is also called as −3db line



* RC as a HIGH PASS FILTER.





- $f=0$ $\quad X_C=\infty$ $\quad C = O/C$ $\quad V_{out}=0$
- $f=\infty$ $\quad X_C=0$ $\quad C=S/C$ $\quad V_{out}=V_{in}$

Transfer function

~~$H(s) = \frac{1}{RCs+1}$~~

$$V_i(t) = \frac{1}{C}\int_0^t i(t)\,dt + R\,i(t)$$

~~sub s=jw~~.

$$V_i(s) = \frac{I(s)}{Cs} + R\,I(s)$$

$$V_o = R\,i(t)$$

$$V_o(s) = R\,I(s)$$

$$\frac{V_o(s)}{V_I(s)} = \frac{RCs}{RCs+1}$$

$$s = j\omega$$

$$\frac{V_o(j\omega)}{V_i(j\omega)} = \frac{RCj\omega}{RCj\omega+1}$$

$$\omega = 2\pi f$$

$$H(j\omega) = \frac{RCj2\pi f}{RCj2\pi f+1}$$

$$= \frac{j(f/f_c)}{j(f/f_c)+1}$$

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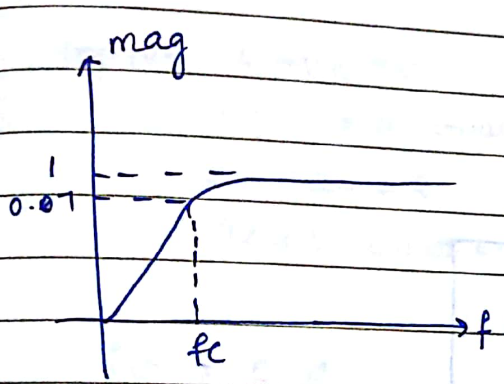
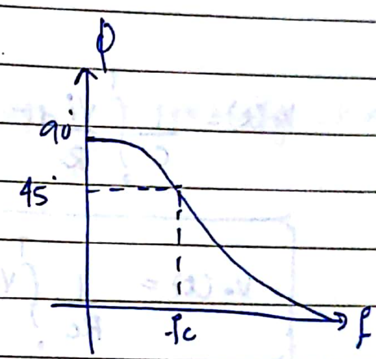
$$|H(j\omega)| = \frac{f/f_c}{\sqrt{(f/f_c)^2 + 1}}$$

$$\phi = \underbrace{\tan^{-1}\frac{f}{0}}_{90^\circ} - \tan^{-1}\frac{f}{f_c}$$

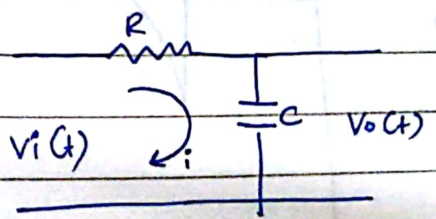
$$\phi = 90 - \tan^{-1}(f/f_c)$$

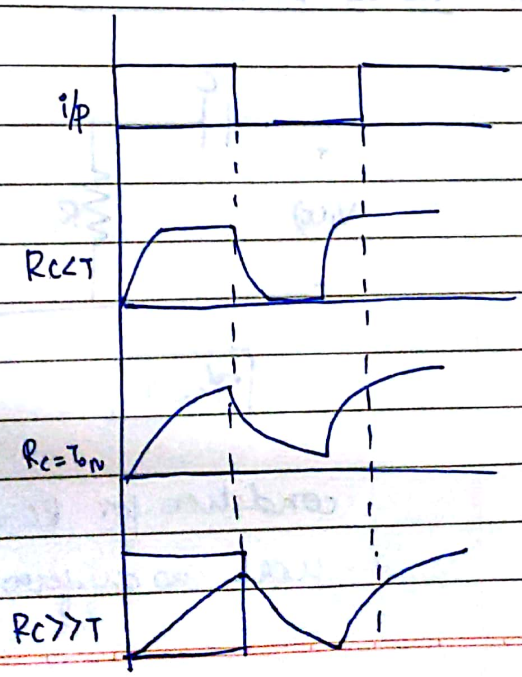
	mag	ϕ
$f=0$	0	90°
$f=f_c$	$1/\sqrt{2}$	45°
$f=\infty$	1	0°





* RC as INTEGRATOR.





To make RC work as integrator

⇒ ∴ RC>>T to get triangular wave from square wave

small τ - cap charges fast
large τ - cap charges slow

$$V_o(t) = \frac{1}{C}\int_0^T i(t)\,dt$$

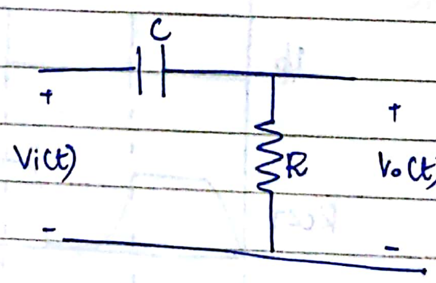
$$i(t) = \frac{V_R}{R}$$

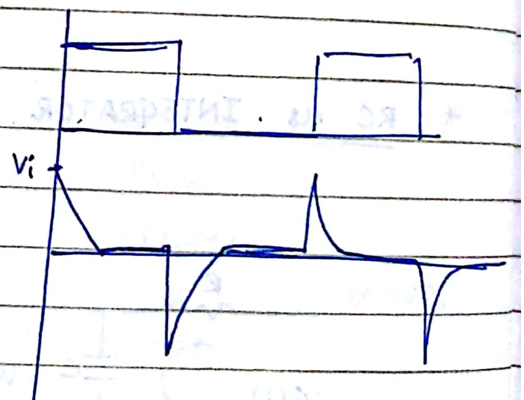
Approx $V_R \simeq V_{in}$ under the condn $RC \gg T$.
cuz cap charges very slowly cuz of less τ and takes less Vin $\therefore$ V_R takes more Vin $\therefore V_R \simeq V_{in}$.

$$V_o(t) = \frac{1}{C}\int_0^T \frac{V_i}{R}\,dt$$

$$\boxed{V_o(t) = \frac{1}{RC}\int_0^T V_i(t)\,dt}$$

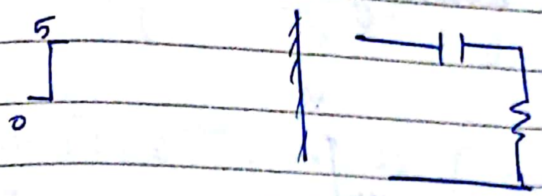
* RC as DIFFERENTIATOR

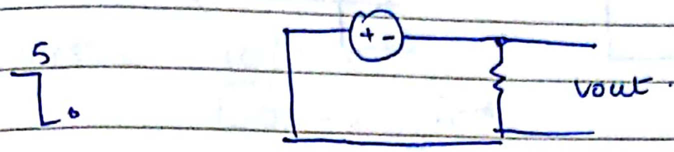




$\boxed{\tau \ll T}$

condition for RC to work as differentiator

for


for


$V_o(t) = R \cdot i(t)$

$i(t) = C\frac{dv}{dt}$

Approx $V_c \simeq V_{in} \simeq V$

cuz τ is << small. ∴ cap is compeetely charged

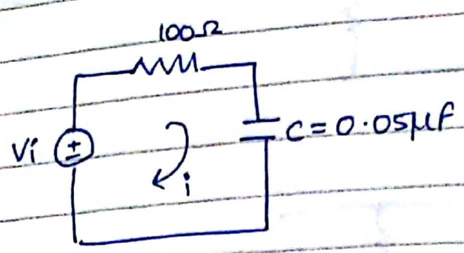
less V across R

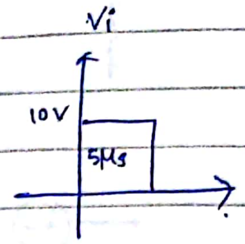
more V across C.

$\therefore\ i(t) = C \cdot \frac{dV_i}{dt}$

$$\boxed{V_o(t) = R \cdot C\frac{dV_i(t)}{dt}}$$

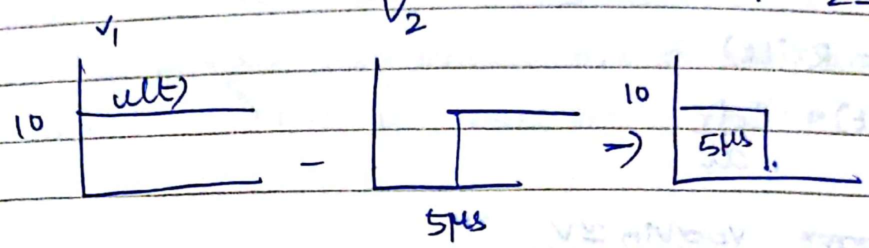
Q.1 Find the current I(t) assuming 0 initial condition





$\cancel{-V_i + 100 i + \frac{1}{C}\int i\,dt = 0}$

$V_1 - V_2 = V_i$ ✓



$V_i(t) = 10\,[u(t) - u(t-5\mu s)]$

$V_1(s) = 10\left[\frac{1}{s} - \frac{e^{-5\mu}}{s}\right]$

KVL

$-V_i(t) + R.i(t) + \frac{1}{C}\int_0^t i(t)\,dt = 0$

LT.

$-V_i(s) + 100 \cdot I(s) + \frac{I(s)}{Cs}$

$V_i(s) = I(s)\left[100 + \frac{1}{Cs}\right]$

$\cancel{V_I(s) = I(s)\left[\frac{100s + 1}{0.05}\right]}$

store
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$$V_i(s) = I(s)\left[\frac{100s + 20\times10^{-6}}{s}\right]$$

$$\frac{10}{s}\left[1 - e^{-5\mu}\right] = I_s\left[\frac{100s + 20\times10^{-6}}{s}\right]$$

$$I(s) = \frac{10(1-e^{-5\mu})}{100s + 20\times10^{-6}}$$

$$I(s) = \frac{10(1-e^{-5u})}{100(s+2\times10^{5})}$$

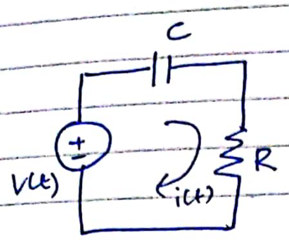
$$I(s) = 0.1\frac{(1-e^{-5\mu})}{s+2\times10^{5}} = \frac{0.1}{s+2\times10^{5}} - \frac{e^{-5u}\cdot 0.1}{s+2\times10^{5}}$$

$L^{-1}T$

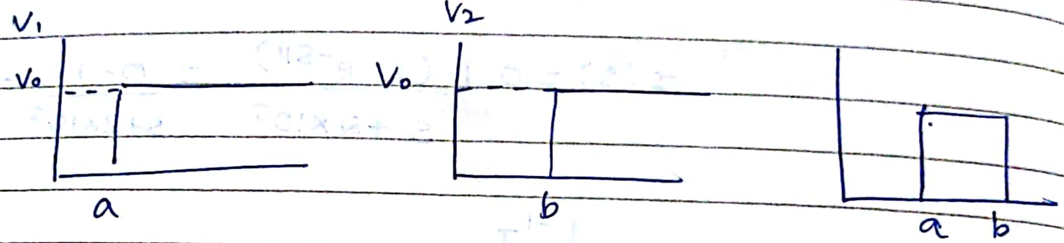
$$i(t) = 0.1e^{-2\times10^{5}t} \cancel{e^{-2\times10}\ 0.1}$$

$$- 0.1e^{-2\times10^{5}t(t-5\mu)} \cdot u(t-5\mu)A$$

Q.2 Consider the RC network shown below. Input is the self pulse as shown below, find $i(t)$ by assuming zero initial condition.

→ 

V_1



$$V(t) = V_0 \left[u(t-a) - u(t-b) \right]$$

KVL eq

$$V_i(t) = \frac{1}{C}\int_0^t i(t)\,dt + R \cdot i(t)$$

$$V_0\left[u(t-a) - u(t-b)\right] = \frac{1}{C}\int_0^t i(t)\,dt + R \cdot i(t)$$

taking Laplace

$$\frac{V_0}{s}\left[e^{-as} - e^{-bs}\right] = \left[R + \frac{1}{Cs}\right] I(s)$$

$$\frac{V_0}{s}\left[e^{-as} - e^{-bs}\right] = I(s)\left[\frac{RCs+1}{Cs}\right]$$

$$\frac{V_0}{s}\left[e^{-as}-e^{-bs}\right] = \frac{I(s)}{Cs}\left[RC\left(s+\frac{1}{RC}\right)\right]$$

$$V_0\left[e^{-as}-e^{-bs}\right] = I(s)\cdot R\left(s+\frac{1}{RC}\right)$$

$$I(s) = \frac{V_0\left(e^{-as}-e^{-bs}\right)}{R\left(s+\frac{1}{RC}\right)}$$

$$I(s) = \frac{V_0}{R}\left[\frac{e^{-as}}{s+\frac{1}{RC}} - \frac{e^{-bs}}{s+\frac{1}{RC}}\right]$$

$L^{-1}T$

inverse laplace

$$i(t) = \frac{V_0}{R}\left[e^{-\frac{(t-a)}{RC}}u(t-a) - e^{-\frac{(t-b)}{RC}}\cdot u(t-b)\right]$$

PHASORS

Rotating vector which helps to represent phase shift

Representation of phasor 1.

polar form $V_m \angle \theta$

rectangular form $A + jB$

Exp form $V_m e^{j\phi}$

Rec $\longrightarrow$ Polar

$V_m = \sqrt{A^2 + B^2}$

$\phi = \tan^{-1}\left(\frac{B}{A}\right)$

polar $\rightarrow$ rect

$A = V_m \cos\theta$

$B = V_m \sin\theta$

For $R \Rightarrow$ V & I are in phase

For cap, C

$X_C = \frac{1}{j\omega C}$

$\frac{V}{I} = \frac{1}{j\omega C}$

$V j\omega C = I$

$I = j\omega C \cdot V$

$\therefore$ I leads V

For inductor

$X_L = j\omega L$

$\frac{V}{I} = j\omega L$

$\frac{V}{j\omega L} = I$

store
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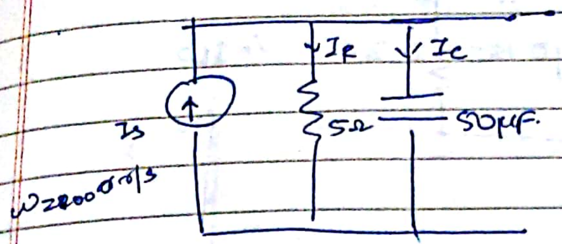
1) Determine I(S) using phasor diagram.



$$I(s) = I_R + I_c$$

$$V = 1\angle 0^\circ \quad = 1 + j0.$$

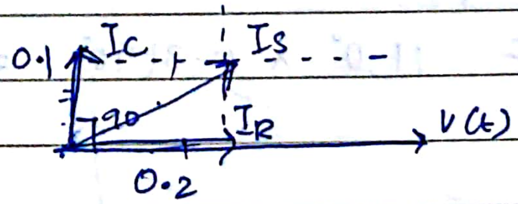
$$I_R = \frac{V}{R} = \frac{1\angle 0^\circ}{5} = 0.2\angle 0^\circ = 0.2 + j0 \text{ Amp}.$$

$j = 90^\circ$ always

$$X_c = \frac{1}{j\omega C} = \frac{1}{j \times 2000 \times 50\mu} = \frac{1}{10j} = -j10 = 10\angle -90^\circ.$$

$$I_c = \frac{V}{X_c} = \frac{1\angle 0^\circ}{10\angle -90^\circ} = 0.1\angle 90^\circ = j0.1A$$

$$I(s) = I_R + I_c$$



llgm law of addition

$$I_s = I_R + I_c$$

$$I_s = 0.2 + j0.1 \rightarrow 0.22\angle 26.56^\circ$$

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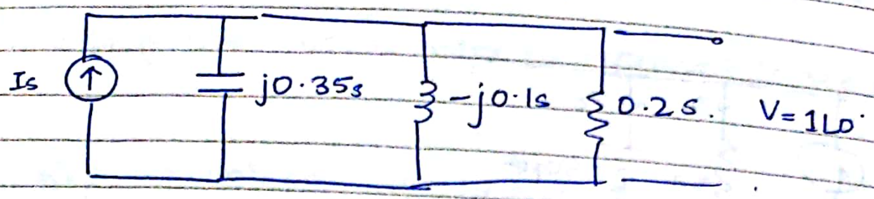
2)



⇒ $I_x = I_L + I_R$

$I_s = I_c + I_x.$

→ $I_R = \frac{V}{R} = V\times 0.2 = 0.2\angle 0^\circ$

$I_L = \frac{V}{X_L} \Rightarrow \frac{\cancel{1\angle 0^\circ \times 0.1\angle 90^\circ = 1.66}}{\cancel{62.8\angle 90^\circ}}\ 0.1\angle -90^\circ =$ 3)

they are in siemens so multiply

$\cancel{X_L = \omega L = 2\pi f = 2\pi \times 0.1 = 0.6\angle 90^\circ} = \cancel{62.8\angle +90^\circ}$

$\cancel{-0.1j}$ $X_L = -j0.1s = 0.1\angle -90^\circ$

$I_c = \frac{V}{X_c}$

in siemens so multiply.

$X_c = V \times j \times 0.35 = 0.3\angle 90^\circ$

$= \frac{\cancel{1\angle 0^\circ}}{\cancel{0.3\angle 90^\circ}} = \cancel{3.33\angle -90^\circ}$

$I_c = 1\angle 0^\circ \times 0.3\angle 90^\circ = 0.3\angle 90^\circ$

$= 0.3j$

$I_x = I_L + I_R$

$= \cancel{0} - 0.1j + 0.2$

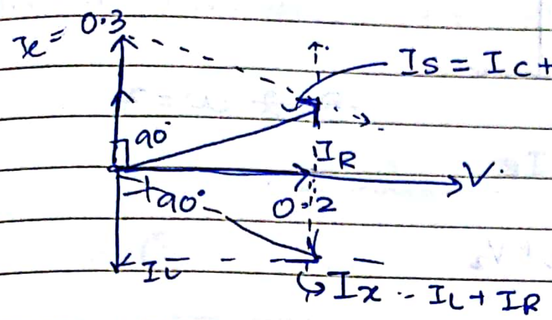
$= 0.2 - 0.1j = 0.22\angle -26.5$

$I_S = I_C + I_x$

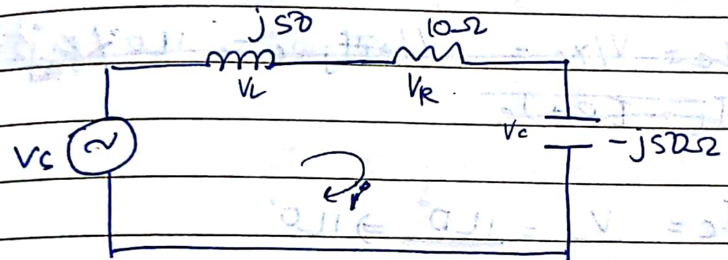
$= 0.3j + 0.2 - 0.1j$

$= 0.2 + 0.2j$

$= 0.28\angle 45^\circ$

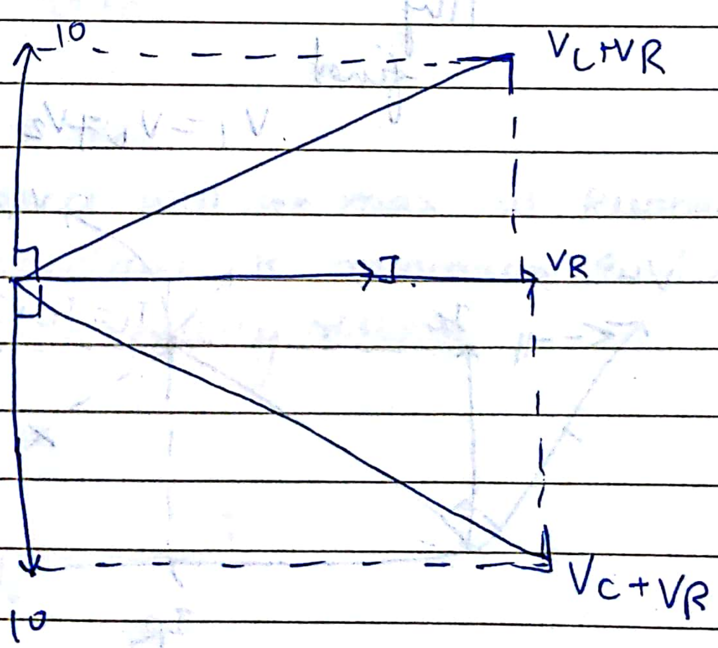


3)

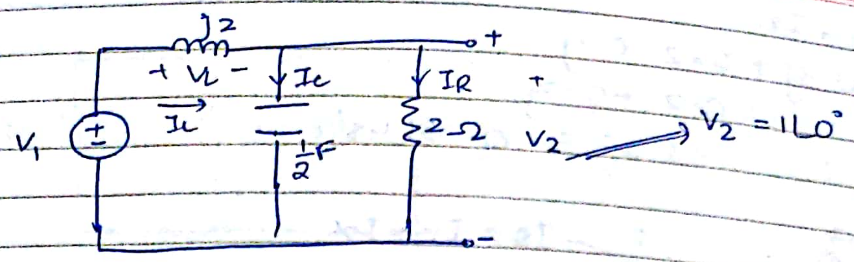


Let $I = 0.2\angle 0^\circ$ Amp

Ans =) $V_R = 2\angle 0^\circ$ $V_C = 10\angle -90^\circ$ $V_L = 10\angle 90^\circ$



4)



~~Ω = 2~~ $\omega = 2$

$I_L = I_c + I_R$

$V_1 = V_L + V_2$

$I_R = \frac{V}{R} = 0.5\angle 0°$

$\cancel{I_c = V/X_c = V/2\pi f j\omega c = 1\angle 0° \times j\frac{1}{2}}$

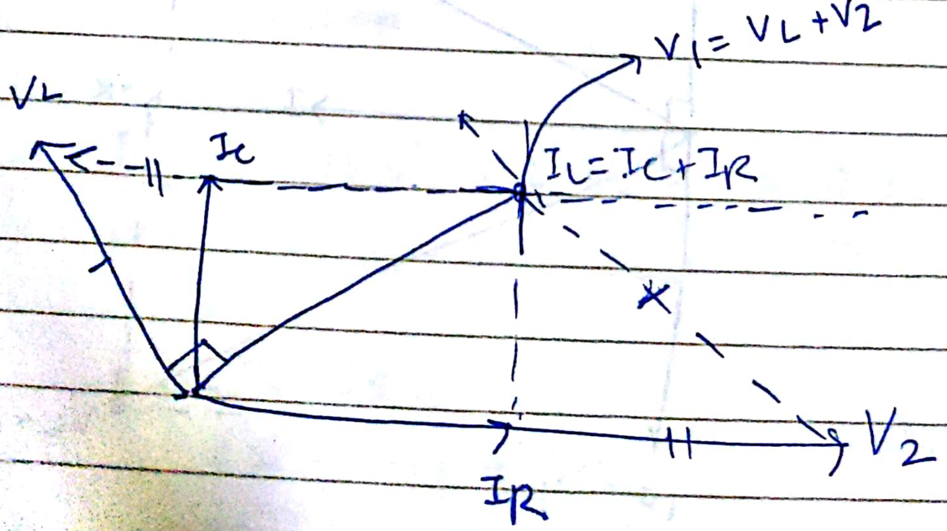
$\cancel{I_L = I_R + I_c}$

$I_c = \frac{V}{X_c} = \frac{1\angle 0°}{1} \Rightarrow 1\angle 0°$

$I_L = I_R + I_c$

IIly find

$V_1 = V_L + V_2$



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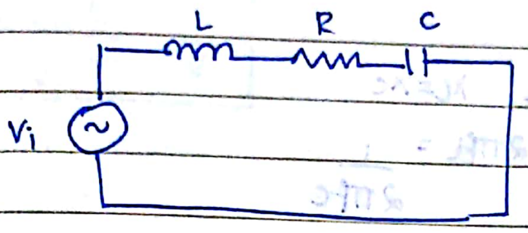
RESONANCE

→ Series Resonance - acceptor circuit
parallel Resonance - rejector circuit



$$Z = R + X_L + X_c$$

$$Z = R + j\omega L + \frac{1}{j\omega C}$$

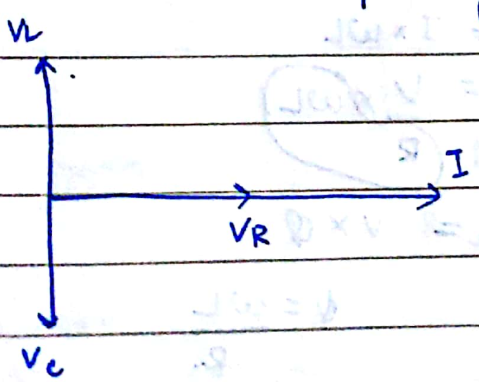
$$Z = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

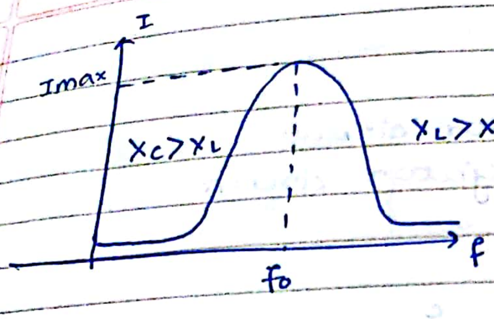
$$|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

At resonance $\boxed{X_L = X_C}$
then

$$|Z| = R.$$

the current will be max at Resonance & impedence is minimum & the circuit is purely resistive





At resonance $X_L = X_C$

$$2\pi f L = \frac{1}{2\pi f C}$$

$$f^2 = \frac{1}{4\pi^2 LC}$$

$$f = \sqrt{\frac{1}{4\pi^2 LC}}$$

$$f = \frac{1}{2\pi\sqrt{LC}}$$

$$\boxed{\omega_0 = \frac{1}{\sqrt{LC}}.}$$

Series Resonance — voltage magnifying device

$$V_{XL} = I \times X_L$$

$$= I \times \omega L$$

$$= \frac{V}{R} \times \omega L$$

$$V_{XL} = V \times Q$$

$$Q = \frac{\omega L}{R}.$$

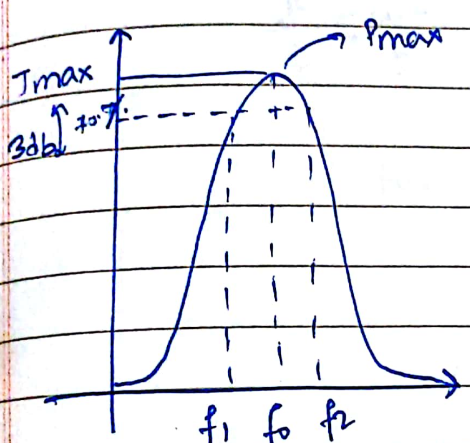
$Q = \frac{\omega_0 L}{R}$

quality factor

$\omega_0 = \frac{1}{\sqrt{LC}}$ $Q = \frac{\omega_0 L}{R}$

$$\boxed{Q = \sqrt{\frac{L}{C}} \times \frac{1}{R}}$$

Bandwidth

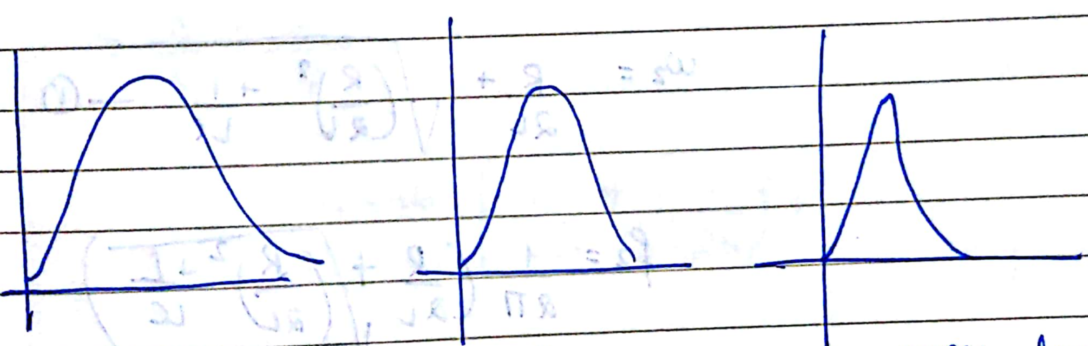


$P_{max} = I_{max}^2 \cdot R$

$I = \frac{I_{max}}{\sqrt{2}}$

$P_{f_1 f_2} = \left(\frac{I_{max}}{\sqrt{2}}\right)^2 \times R$

half power frequency



BW more — less — very less

R ↑ — R ↓ — R very less

Q = less
less selectivity

high selectivity
↑ Q high

Halfpath frequency

Der

ω_1 & ω_2

$$I = \frac{I_{max}}{\sqrt{2}}$$

$$\frac{V}{Z} = \frac{I_{max}}{\sqrt{2}}$$

$$\frac{V}{Z} = \frac{V}{R\sqrt{2}}$$

$$Z = \sqrt{2}\, R$$

$$\sqrt{R^2+(X_L-X_C)^2} = \sqrt{2}\, R$$

$$R = X_L - X_C$$

$$R = \omega_2 L - \frac{1}{\omega_2 C}$$

$$\omega_2^2 LC - \omega_2 CR - 1 = 0$$

$$\omega_2 = \frac{(RC) \pm \sqrt{(RC)^2 + 4LC}}{2LC}$$

$$\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \quad \text{—(1)}$$

$$f_2 = \frac{1}{2\pi}\left(\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right)$$

store
67

$X_C - X_L = R$

$\omega_1^2 LC + \omega_1 CR - 1 = 0$

$$\omega_1 = \frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \quad -② $$

$$f_1 = \frac{1}{2\pi}(\omega_1)$$

Bandwidth $= \omega_2 - \omega_1$

$$Q = \frac{\omega_0 L}{R}$$

$$\frac{R}{L} = \frac{\omega_0}{Q}$$

$$BW = \omega_2 - \omega_1$$

$$= \frac{R}{L} \text{ rad/sec}$$

$$= \frac{f_0}{Q} \text{ freq}$$

$$= \frac{\omega_0}{Q} \text{ rad/sec.}$$

$$\Rightarrow \frac{R}{L} = \frac{\omega_0}{Q}$$

$$\omega_2 = \omega_0 \left[\frac{-1}{2Q_0} + \sqrt{\left(\frac{1}{2Q_0}\right)^2 + 1} \right.$$

2) Resonant freq is the GM of the 2 half path frequencies

→ $R = \omega L - \frac{1}{\omega C}$

at ω_1

$R = \frac{1}{\omega_1 C} - \omega_1 L$

at ω_2

$R = \omega_2 L - \frac{1}{\omega_2 C}$.

equating the above eq:

$$\frac{1}{\omega_1 C} - \omega_1 \cancel{L} = \omega_2 L - \frac{1}{\omega_2 C}.$$

$$\frac{1}{\omega_1 C} + \frac{1}{\omega_2 C} = \omega_2 L + \omega_1 L$$

$$\frac{1}{C}\left[\frac{\omega_2 + \omega_1}{\omega_1 \omega_2}\right] = L(\omega_1 + \omega_2)$$

$$\omega_1 . \omega_2 = \frac{1}{LC}.$$

WKT,

$$\omega_0^2 = \frac{1}{LC}.$$

$$\omega_0^2 = \omega_1 \omega_2$$

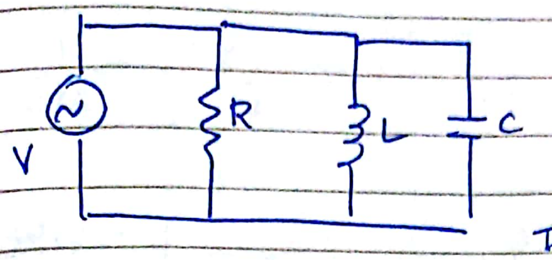
$$\omega_0 = \sqrt{\omega_1 \omega_2}$$

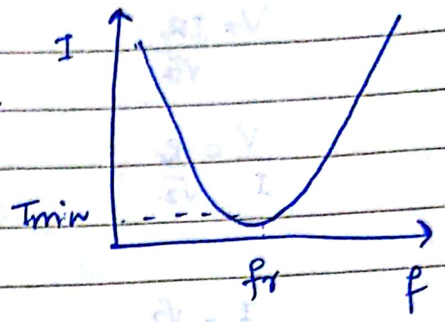
or

$$f_0 = \sqrt{f_1 f_2}.$$

Scanned by CamScanner

⇒ Parallel Resonance (Rejector circuits).





At resonance

$R = max$

$I = min$

$$\omega_0 = \frac{1}{\sqrt{LC}}.$$

admittance: $Y = Y_R + Y_C + Y_L$

$$Y = \frac{1}{R} + j\omega C + \frac{1}{j\omega L}.$$

$$Y = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right).$$

$$|Y| = \sqrt{\left(\frac{1}{R}\right)^2 + \left(\omega C - \frac{1}{\omega L}\right)^2} \quad -(1)$$

Derv of ω_1 & ω_2.

$$V = \frac{V_0}{\sqrt{2}}$$

$$V = \frac{IR}{\sqrt{2}}$$

$$\frac{V}{I} = \frac{R}{\sqrt{2}}$$

$$\frac{I}{V} = \frac{\sqrt{2}}{R}.$$

$$|Y| = \frac{\sqrt{2}}{R}$$

$$|Y|^2 = \frac{2}{R}$$

After equating both (this & ⊛)

$$\left(\frac{2}{R^2}\right) = \left(\frac{1}{R^2}\right) + \left(\omega C - \frac{1}{\omega L}\right)^2$$

$$\frac{1}{R^2} = \left(\omega C - \frac{1}{\omega L}\right)^2$$

$$\omega C - \frac{1}{\omega L} = \frac{1}{R}$$

$$\omega_2^2 LC - \omega_2 L - R = 0.$$

$$\omega_2 = \left(\frac{L + \sqrt{L^2 - 4LCR^2}}{2LCR} \right)$$

$$\omega_2 = \frac{1}{2CR} + \sqrt{\left(\frac{1}{2CR}\right)^2 + \frac{1}{LC}}$$

$$\omega_1 = -\left(\frac{1}{2CR}\right) + \sqrt{\left(\frac{1}{2CR}\right)^2 + \frac{1}{LC}}.$$

$$BW = \omega_2 - \omega_1 = \frac{1}{CR}.$$

$$\frac{BW}{2} = \frac{1}{2CR}.$$

$$\omega_2 = \frac{BW}{2} + \sqrt{\left(\frac{BW}{2}\right)^2 + \omega_0^2}.$$

$$\left(\frac{2}{R^2}\right) = \left(\frac{1}{R^2}\right) + \left(\omega C - \frac{1}{\omega L}\right)^2$$

$$\left(\omega C - \frac{1}{\omega L}\right)^2 = \frac{1}{R^2}$$

$$\omega C - \frac{1}{\omega L} = \frac{1}{R}$$

$$\omega_2^2 LC - \omega_2 L - R = 0.$$

$$w_2 = \left(\frac{L + \sqrt{L^2 \pm 4LCR^2}}{2LCR}\right)$$

$$w_2 = \frac{1}{2CR} + \sqrt{\left(\frac{1}{2CR}\right)^2 + \frac{1}{LC}}$$

$$w_1 = -\left(\frac{1}{2CR}\right) + \sqrt{\left(\frac{1}{2CR}\right)^2 + \frac{1}{LC}}$$

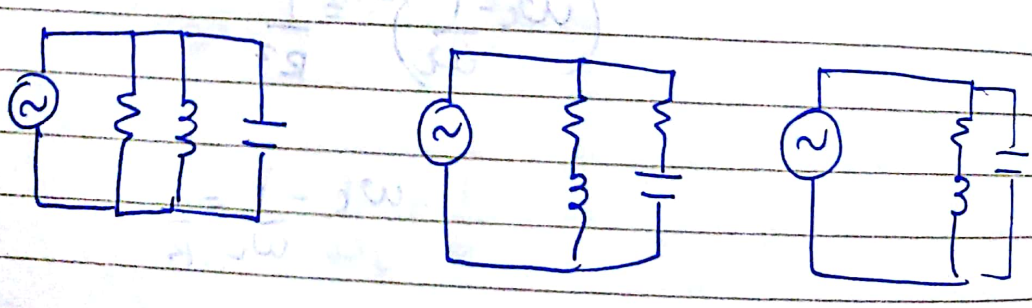
$BW = w_2$

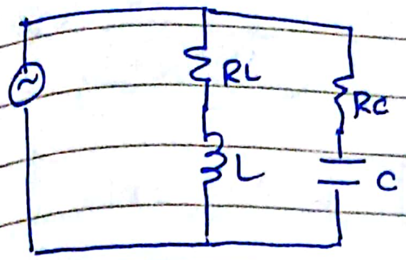
Quality factor

- $Q_0 = \omega_0 CR$
- $Q = \frac{R}{\omega_0 L}$

$\cancel{Q = \frac{R}{\omega_0 L}}$

- $Q = \frac{R}{L} \times \sqrt{LC}$





$$Y_L = \frac{1}{R_L + j\omega L}$$

$$Y_C = \frac{1}{R_C - j\frac{1}{\omega C}} = \frac{R_C + j\,{}^{1}/_{\omega C}}{R_C^2 + \frac{1}{\omega^2 C^2}}$$

Total admittance

$$Y = Y_L + Y_C$$

$$Y = \left[\frac{R_L}{R_L^2 + \omega^2 L^2} + \frac{R_C}{R_C^2 + \frac{1}{\omega^2 C^2}}\right] - j\left[\frac{\omega L}{R_L^2 + \omega^2 L^2} - \frac{{}^{1}/_{\omega C}}{R_C^2 + \frac{1}{\omega^2 C^2}}\right]$$

@ Resonance

$$\frac{\omega L}{R_L^2 + \omega^2 L^2} = \frac{{}^{1}/_{\omega C}}{R_C^2 + \frac{1}{\omega^2 C^2}}$$

$$\frac{\omega_r L}{R_L^2 + \omega_r^2 L^2} = \frac{\omega_r C}{1 + \omega_r^2 R_C^2 C^2}$$

$$L\left[1 + \omega_r^2 R_C^2 C^2\right] = C\left[R_L^2 + \omega_r^2 L^2\right]$$

$$\omega_r^2 \left[L R_C^2 C^2 - L^2 C\right] = C R_L^2 - L$$

$$\omega_r^2 LC\left[R_C^2 C - L\right] = R_L^2 C - L$$

$$\omega_r^2 = \frac{1}{LC}\frac{(CR_L^2 - L)}{(R_c^2 C - L)}$$

$$\omega_r = \sqrt{\frac{1}{LC}\left(\frac{L - CR_L^2}{L - CR_c^2}\right)}$$

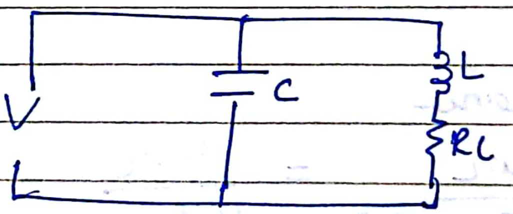
$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC}\left(\frac{L - CR_L^2}{L - CR_c^2}\right)}$$

If $R_c = 0$ $\quad f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R_L^2}{L^2}}$

if $R_L = R_c$, $f_r = \frac{1}{2\pi\sqrt{LC}}$.

$\cancel{Y}$

- <u>Impedance at Resonance</u>



$Y = Y_L + Y_C$

$$Y = \frac{R_L}{R_L^2 + \omega^2 L^2} - j\left(\frac{\omega L}{R_L^2 + \omega^2 L^2} - \omega C\right)$$

@ Resonance

$$\frac{\omega L}{R_L^2 + \omega^2 L^2} = \omega C$$

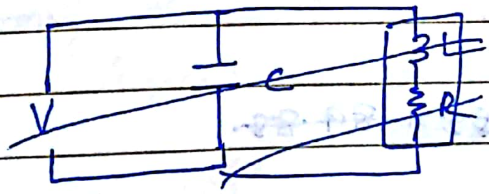
$$R_L^2 + \omega^2 L^2 = \frac{L}{C}$$

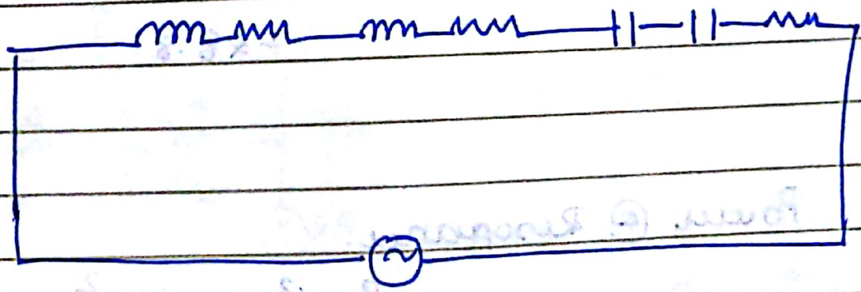
$$Y_r = \frac{R_L}{R_L^2+\omega^2L^2} \rightarrow L/C$$

$$Y_r = \frac{RLC}{L} \Rightarrow \boxed{Z_r = \frac{L}{CR_L}}$$ imp @ Resonance

Q. 2 coils of one of 0.5Ω & $L_1 = 32mH$ & other 1.3Ω & $L_2 = 18mH$ and 2 capacitors $C_1 = 20\mu F$ $C_2 = 60\mu F$ are all connected in series with 0.2Ω resistance. Determine resonance frequency Q of the coils Q of entire circuit, power dissipated at resonance if the voltage applied is 10V.

→





$$R_T = 0.5 + 1.3 + 0.2$$
$$R_T = 2\Omega$$
$$L_T = 50mH$$
$$C_T = 15\mu F.$$

$$\omega_0 = \frac{1}{\sqrt{LC_T}}.$$

$$\omega_0 = \frac{1}{\sqrt{50\times10^{-3}\times 15\times10^{-6}}}$$

$$\omega_0 = \cancel{7.5\times10^{-7}}\ 1154.700 \text{ rad/sec.}$$

$$Q_1 = \frac{\omega_0 L_1}{R} \qquad Q_2 = \frac{\omega_0 L_2}{R}$$

$$= \frac{1154.7\times32\times10^{-3}}{0.5} \qquad = \frac{1154.7\times80\times10^{-3}}{1.3}$$

$$= 73.9 \qquad = 15.98$$

$$Q_T = \cancel{Q_1+Q_2 = 89.88}.$$

$$Q_T = \frac{\omega_0 L_T}{R_T} = \frac{1154.7\times50\times10^{-3}}{2}$$

$$= 28.8$$

Power @ Resonance:

$$P = \frac{V^2}{R_T} = \frac{10^2}{2} = 50\text{ W}$$

Q) Det the values of RLC ckt parameters whose plot is shown in fig below. what are the new values of ω_0 & BW, if C is increased in 4 times

given data is