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Network Theorems

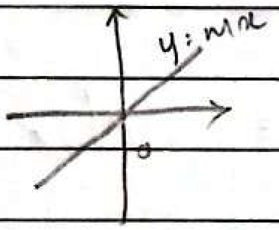
Applicable to linear systems: obeys

① principle of homogenity (Scaling)

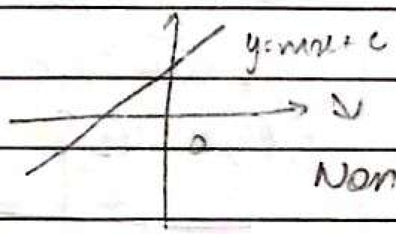
② principle of Superposition.

If $k x(t) \Rightarrow k y(t)$

$\alpha x_1(t) + \beta x_2(t) \Rightarrow \alpha y_1(t) + \beta y_2(t)$



→ linear systems



→ Nonlinear

Network Theorem:

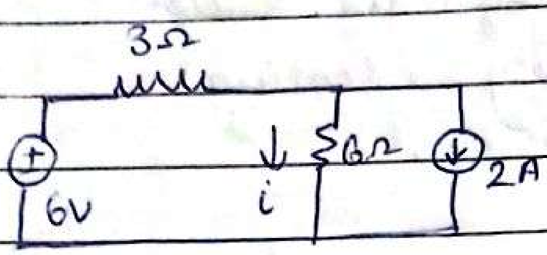
1) Law of Superposition
2) Thevenin's Theorem.
3) Norton's Theorem
4) Maximum power transfer Thm.
5) Reciprocity Thm.

① ✓ Superposition Theorem

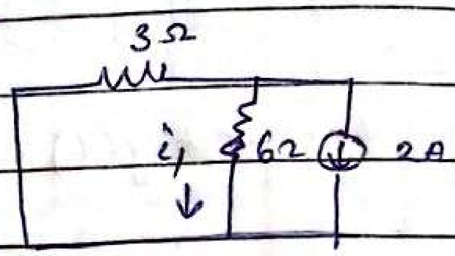
Deactivate independent voltage source → S/C

- Determine the current in 6Ω resistor in foll network using the principle of superposition.

Soln

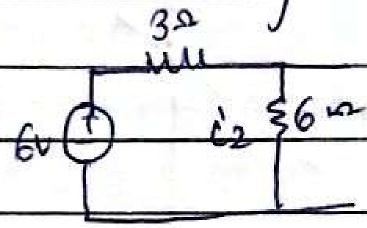


(a) Deactivating 6V source. (s/c)



$$i_1 = \frac{2\times 3}{6+3} = \frac{6}{9} = \frac{2}{3}A$$

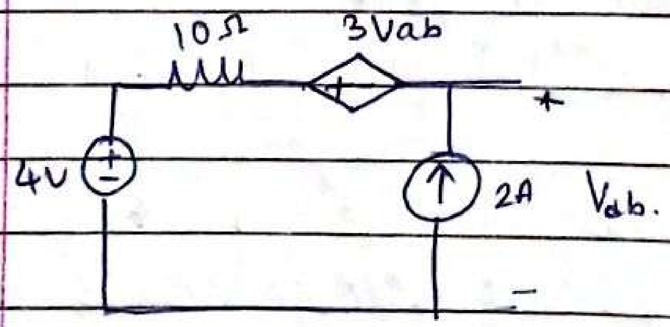
(b) Deactivating 2A current source. (o/c)

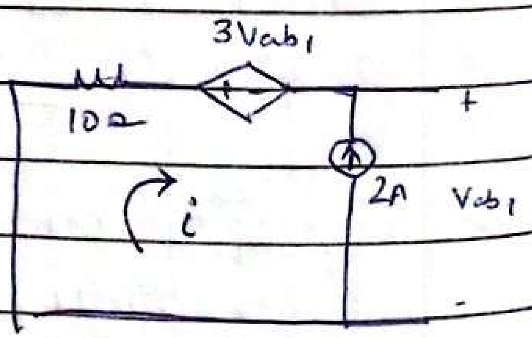


$$\rightarrow \quad i_2 = \frac{6}{3+6} = 2/3\ A$$

$$i = i_1 + i_2 = \frac{2}{3} + \frac{2}{3} = 4/3\ A$$

- Use superposition principle to find terminal voltage V_B in the foll ckt

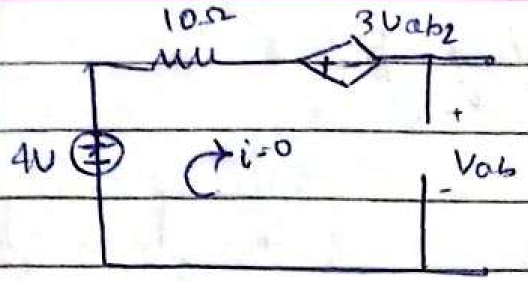




$$-i\times 10 - 3V_{ab_1} - V_{ab_1} = 0$$

$$-i\times 10 - 4V_{ab_1} = 0 \quad \Rightarrow \quad -(2)\times 10 - 4V_{ab_1} = 0$$

$$V_{ab_1} = 5V.$$



$$4 - i\times 10 - 3V_{ab_2} - V_{ab_2} = 0$$

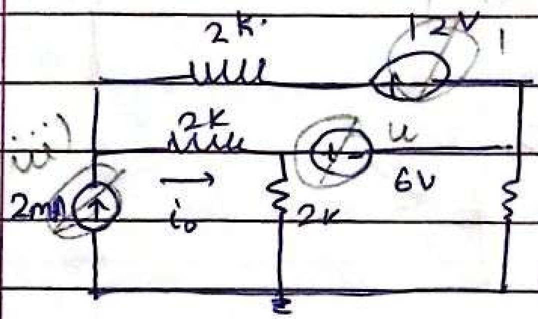
$$4 - 4V_{ab_2} = 0$$

$$V_{ab_2} = \underline{1V}$$

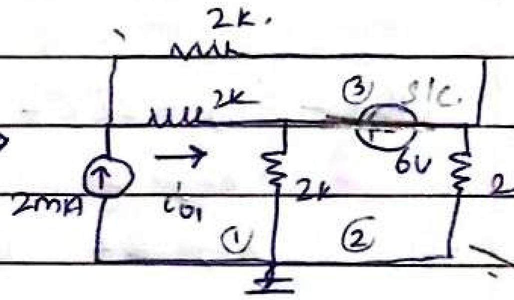
$$V_{ab} = V_{ab_1} + V_{ab_2} = 5+1 = 6V.$$

- Use superposition principle to find the current i_o in the following circuit.

→



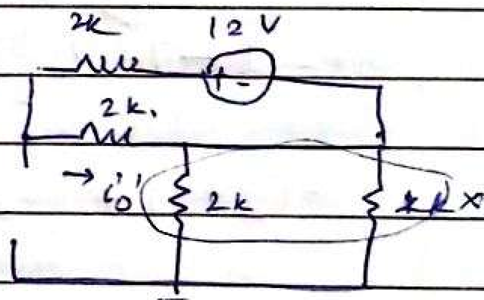
⇒

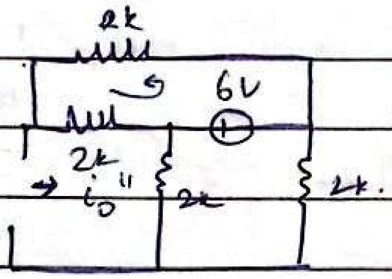


In loop 1, $i_{o_1} = 2$ mA.

$$i_o' = \frac{12}{4} = -3 \text{ mA}.$$

→





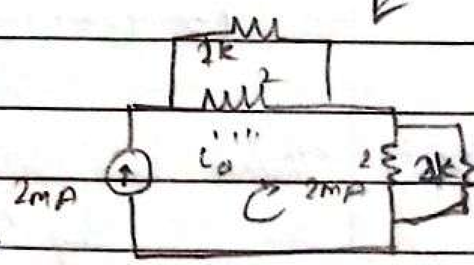
$$i_o'' = +\frac{6}{4} = +\frac{3}{2} \text{ mA}$$

⇓

~~$i_o''' = 2$mA~~

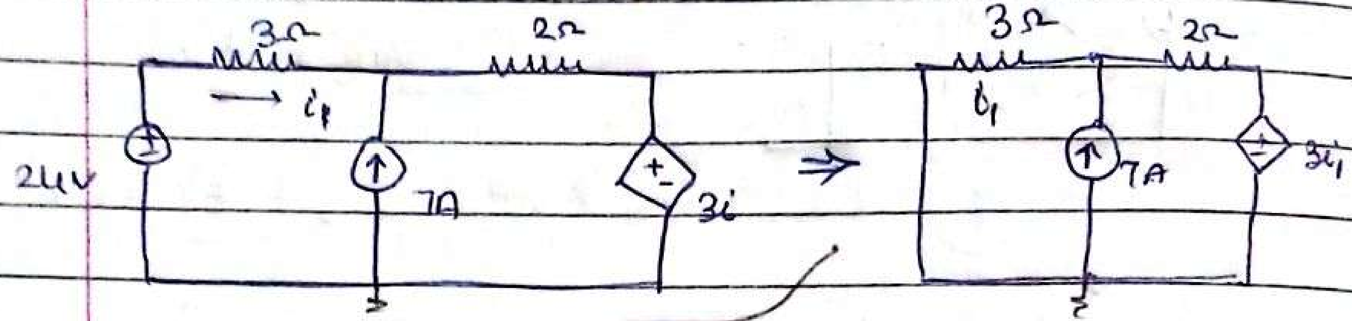
$i_o''' = 1$ mA at 2k resistor

$$= \frac{2\times 2}{2+2} = 1 \text{ mA}$$

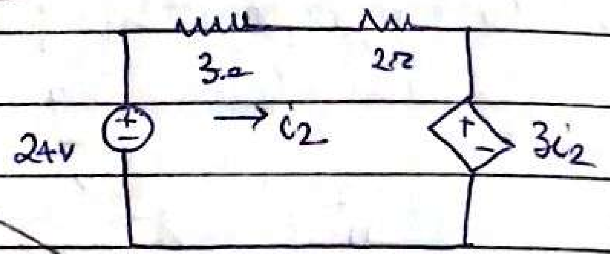


$$i_o = +3 \mp \frac{3}{2} + 1 = \cancel{-1/2}.\ +2.5 \text{ mA}$$

- Determine current i using p.s



$$\cancel{i_1 = \frac{2\times 7}{3+2} = \frac{-14}{5} A.}$$



$$24 - 3i_2 - 2i_2 - 3i_2 = 0$$
$$24 - 8i_2 = 0$$
$$8i_2 = 24 \quad i_2 = \underline{3A}$$

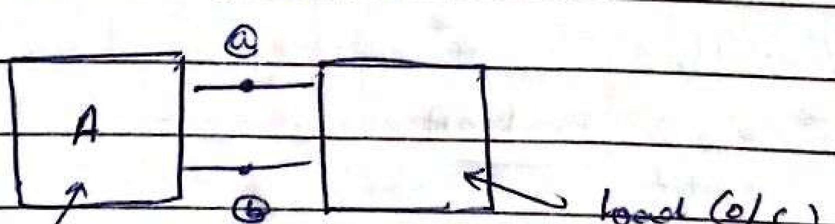
$$V = -3i_2.$$

$$-i_2 - 7 + \frac{V - 3i_2''}{2} = 0.$$

$$-i_2 - 7 + \frac{-3i_2 - 3i_2}{2} = 0 \Rightarrow -4i_2 - 7 = 0$$
$$-4i_2 = 7$$
$$i_2 = \underline{\underline{7/4\ A}}$$

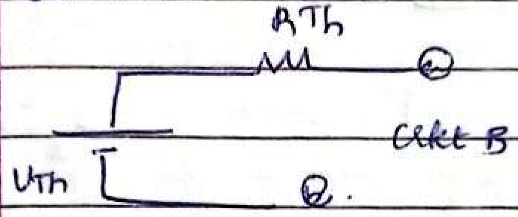
$$i = i_1 + i_2 = 3 - \frac{7}{4} = 5/4\ A$$

2) Thevenin's Theorem.



Thevenin's Eq ckt. V_{Th} & R_{Th}

1) when circuit has only resistors & independent sources.
2) when circuit has only resistors, dependent & independent.
3) when circuit has only resistors & only dependent source.

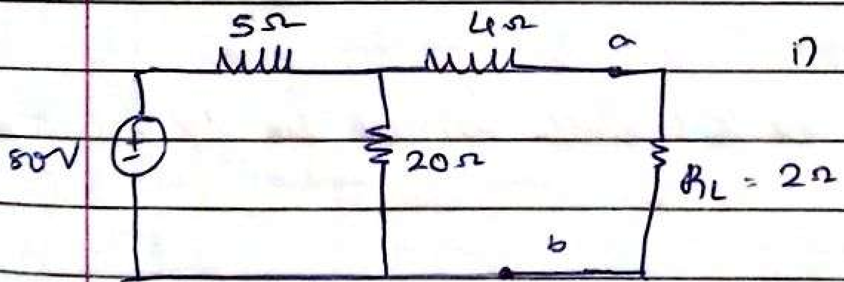
1* 

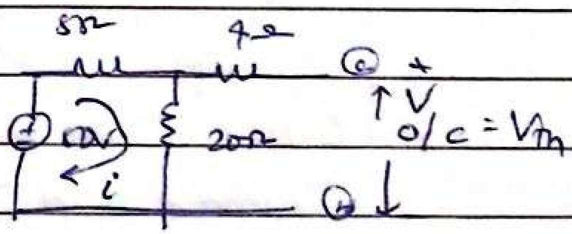
$V_{OC} = V_{Th}$ & R_{Th} deactivate source.

2* $V_{Th} = V_{OC}$; $I_{SC} = i_{sc}$; $R_{Th} = \frac{V_{OC}}{I_{SC}} = \frac{V_{Th}}{I_{SC}}$

3* $R_{Th} = \frac{V_{Test}}{1A}$

1) Using Thevenins Theorem, determine current i flowing through $R_L = 2\Omega$ in foll ckt

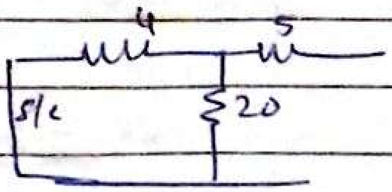


i) 

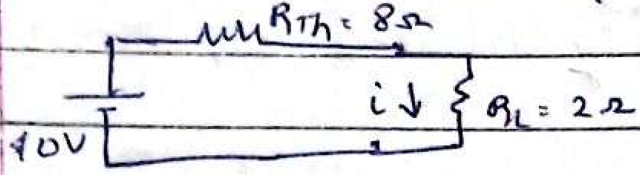
$I = \frac{50}{20+5} = 2A$; $V_{OC} = V_{20\Omega} = I \times 20 = 2 \times 20 = 40V$

2) Deactivating the 50V source

$R_{Th} = 20 \| 5 + 4 = 8\Omega$



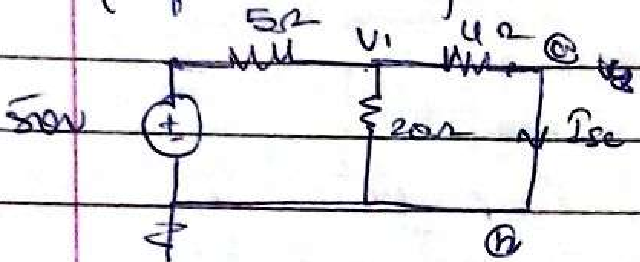
then the thevenin's Eq ckt is,



$$i = \frac{V_{Th}}{R_{Th}+R_L} = \frac{40}{8+2} = 4A$$

$$P_{RL} = i^2 R_L = 16 \times 2 = \underline{32W}$$

(If it is dependent source)



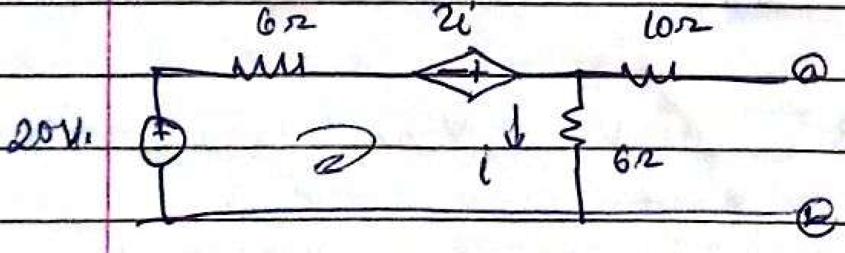
$$\frac{V_1-50}{5} + \frac{V_1}{20} + \frac{V_1}{4} = 0.$$

$$\frac{V_1}{2} = 10$$

$$V_1 = 20V. \qquad I_{sc} = \frac{V_1}{4} = \frac{20}{4} = 5A$$

$$R_{Th} = \frac{V_{oc}}{I_{sc}} = \frac{40}{5} = 8\Omega$$

2) Draw Thevenin's Eq ckt with respect to o/p circuited terminals a & b



$$20 - 6i + 2i - 6i = 0.$$

$$20 - 10i = 0 \rightarrow i = 2A$$

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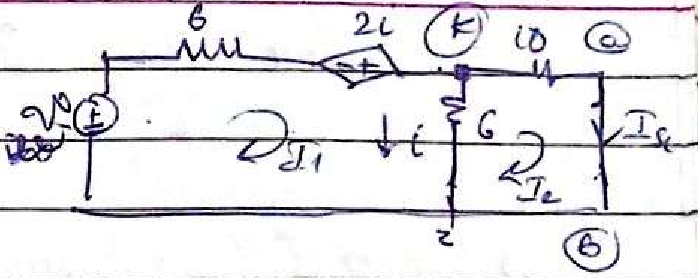
$V_{oc} = 2\times 6 = \underline{12V} \therefore V_{Th}$



KCL at node K

$-I_1 + i + I_2 = 0$

$I_2 = I_{SC}$

$-I_1 + i + I_s = 0. \qquad i = I_1 - I_2 \; - (2)$

loop1 $\cancel{20 - 12I_1 - 12I_2 + 2i + 6I_2 = 0}$

$20 - 6I_1 - 6(I_1 - I_2) + 2i = 0$

$20 - 10I_1 + 4I_2 = 0.$

loop 1 $-6(I_2 - I_1) - 10I_2 = 0.$

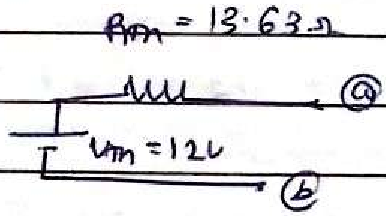
$6I_1 - 16I_2 = 0.$

$I_1 = \frac{40}{17} = \cancel{1.139A} \; 2.35A$

$I_2 = I_{sc} = \underline{0.8823A}$

$R_{th} = \frac{V_{oc}}{I_{sc}} = \frac{12}{0.88} = \underline{13.63\Omega}$

Thevenin's eq ckt is;



Verification

Using Nodal

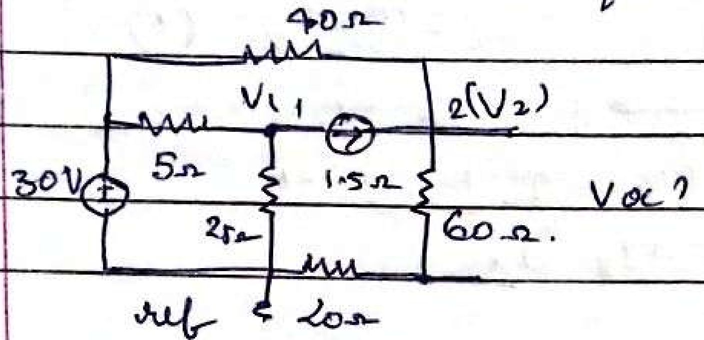
$\frac{V_1}{6} + \frac{V_1}{10} + \frac{V_1 - 2i - 20}{6} = 0. \qquad \frac{V_1}{6} = i.$

$V_1\left(\frac{1}{6} + \frac{1}{10} + \frac{1}{6} - \frac{2}{36}\right) = \frac{20}{6}. \Rightarrow V_1 = \frac{200}{8}$

$V_1 = 8.82V.$

$$I_{sc} = \frac{V_1}{10} = \frac{8.895}{10} = \cancel{8.8}\ 0.88$$

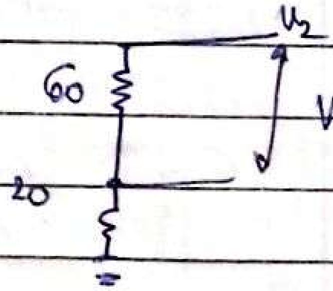
3) Determine Thevenins Eq circuit across o/p circuited terminals a & b in foll ckt



At node 1;

$$\frac{V_1 - 30}{5} + \frac{V_1}{25} + 1.5 = 0. \Rightarrow V_1 = 18.75$$

At node 2;

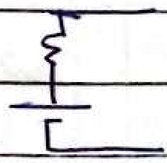


$$\frac{V_2}{60+20} + \frac{V_2 - 30}{40} - 1.5 = 0$$

$$V_2 = \underline{60V}$$

$$i = \frac{V_2}{80} = \frac{60}{80} = 0.75.$$

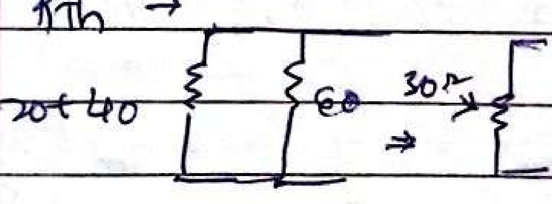
Eq ckt

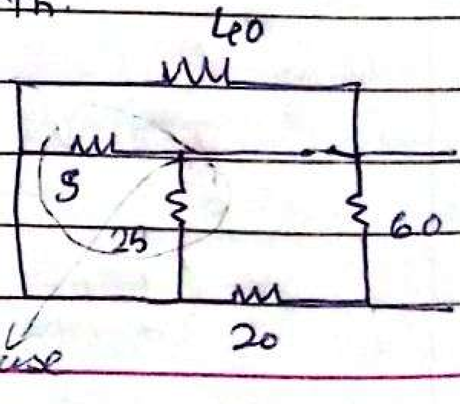


$$V_{oc} = i \times 60. = \underline{45V} = V_{Th}.$$

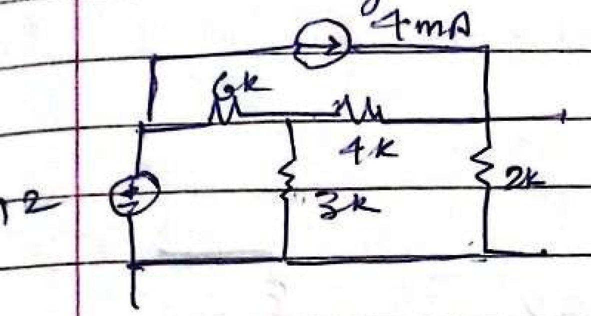
verify by I_{sc}

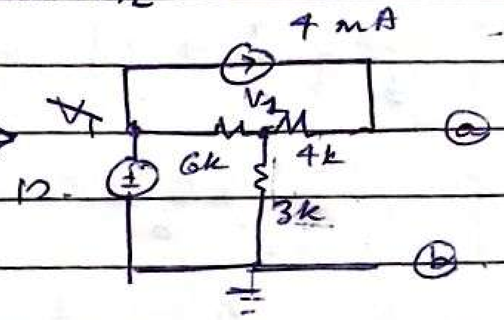
$R_{Th} \rightarrow$





Use T.T to determine T.E.C parameters for the following ckt with R_L of $2k\Omega$, also determine the voltage V_0 across this $R_L = 2k\Omega$



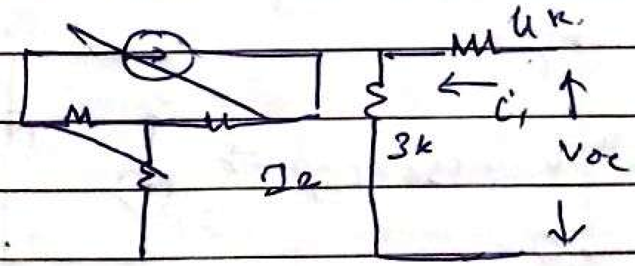


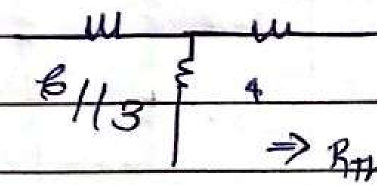
$$\frac{V_1}{3} + \frac{V_1 - 12}{6} + \cancel{\frac{V_1 - 14}{}} - 4$$

$$= V_1\left(\frac{1}{3} + \frac{1}{6}\right) = 6.$$

$\underline{V_1 = 12\,V}$. $\quad \underline{i_1} = \frac{12}{3} = 4mA$

$V_{oc} = i \times 3 + i \times 4 = 4 \times 3 + 4 \times 4 = 12 + 16 = 28V$

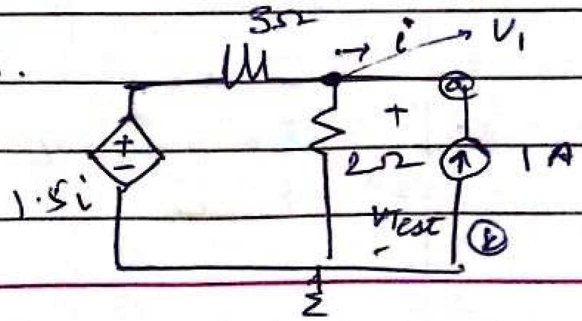




$\Rightarrow R_{Th} = 6$

$$V_0 = \frac{V_{Th}}{R_{Th} + R_L} R_L = \underline{\underline{7\,V}}$$

(5) Find T.E.C for foll ckt.

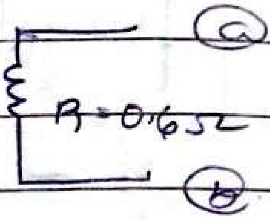


$\frac{V_1}{2} + \frac{V_1 \text{ test}}{6} \quad \frac{V_1 - 1.5i}{3} - 1$

$\frac{V_1}{2}\left(\frac{1}{2} + \frac{1}{3}\right) + \frac{1.5}{3} - 1 = 0.$

$V_1 \times \frac{5}{6} = \frac{1}{2}$

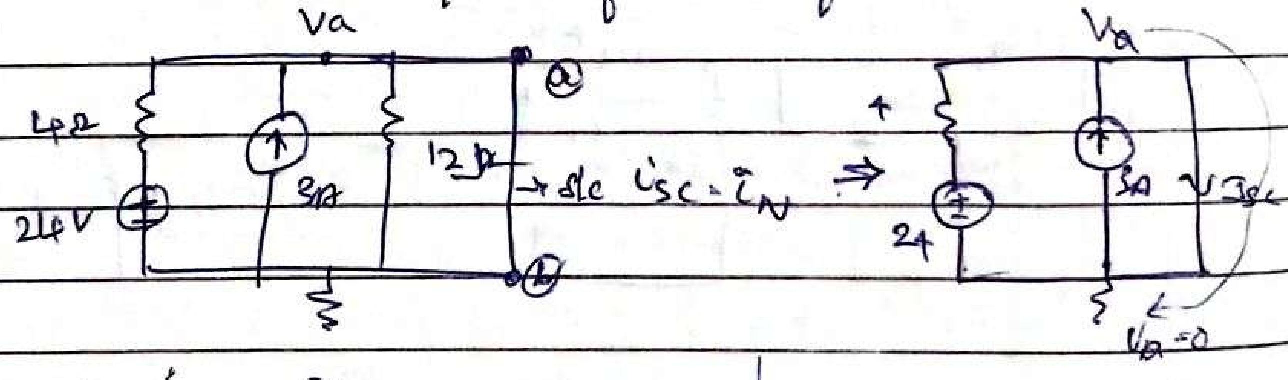
$V_1 = \frac{6}{10} = 0.6V$



$R_{Th} = \frac{V_{test}}{1A} = \frac{0.6V}{1A} = 0.6\Omega$

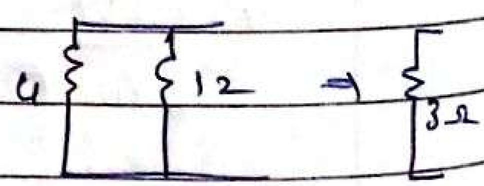
Norton's Theorem.

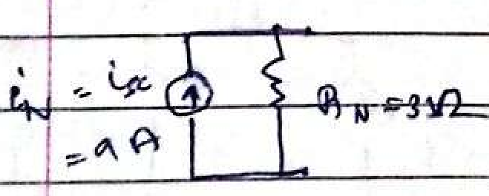
Draw Norton eq ckt for the foll ckt.



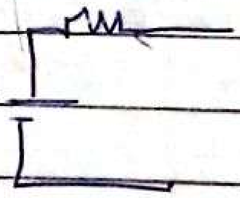
$\frac{Va - 24}{4} + Isc - 3 = 0.$

$Va = 0. \Rightarrow Isc = +9A$





→ Thevenins



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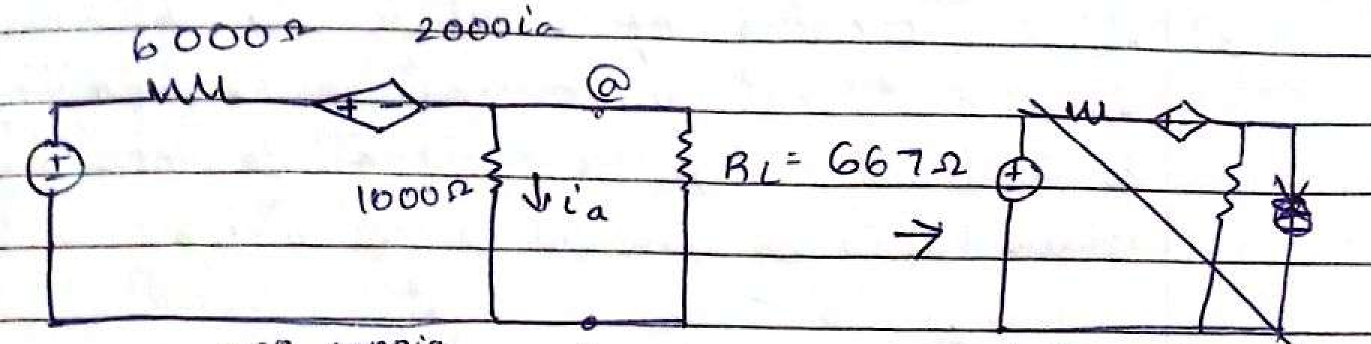
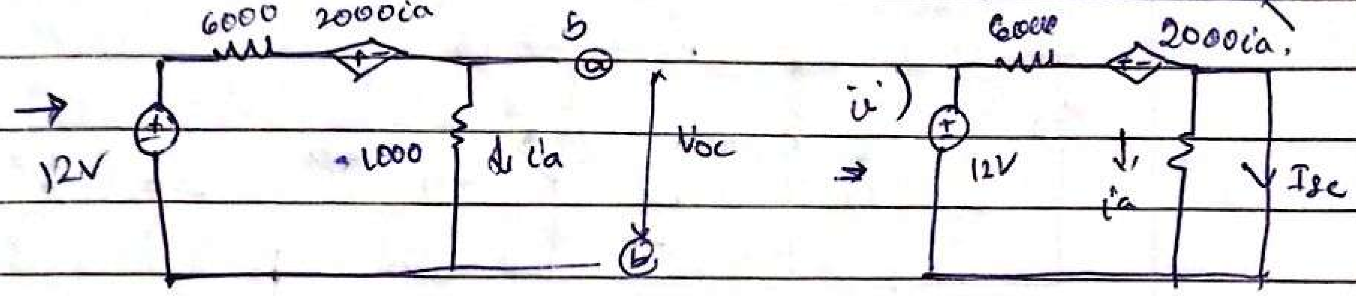




$$12 - i_a(6000 + 1000) - 2000\, i_a = 0$$

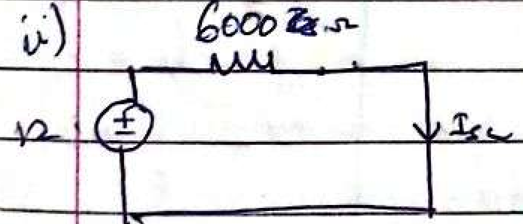
$$12 - 9000\, i_a = 0$$

$$12 = 9000\, i_a$$

$$i_a = 1.33\ mA$$

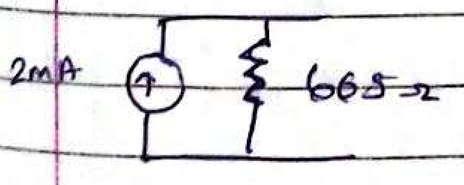
$$V_{oc} = V_{ab} = i_a \times 1000 = 1.33\ V.$$

ii)



$$I_{sc} = \frac{12}{6000} = 2 \times 10^{-3}\ A. = i_N.$$

$$R_N = R_{Th} = \frac{V_{oc}}{I_{sc}} = \frac{1.33}{2 \times 10^{-3}} = 665\ \Omega = R_{Th}.$$



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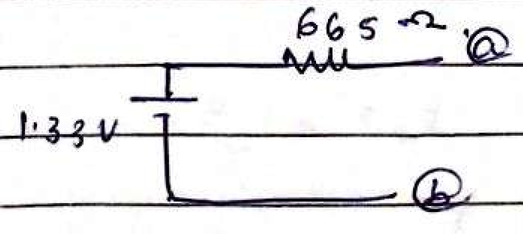

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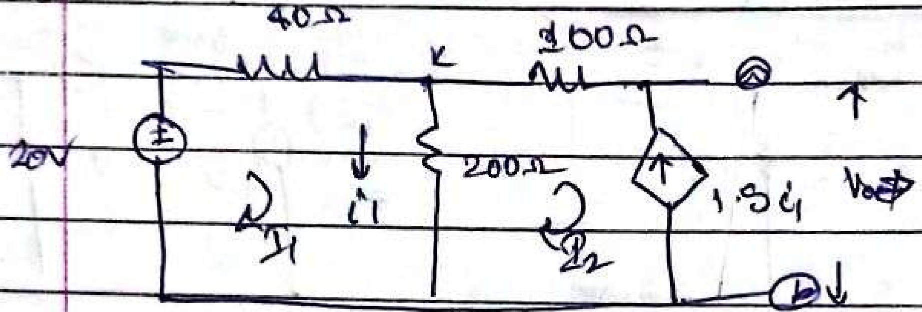
3) Find Thevenin's Eq Circuit for the following Ckt diagram also draw corresponding Norton Eq ckt & determine the power delivered to 100Ω load resistor which has to be connected across a & b



$I_b = -1.5 i_1$

Applying KVL

$20 - 40I_1 - 200I_1 + 200I_2 = 0$.

$200I_2 - 240I_1 = -20$

~~$-200 I_2 + 200 I_1 - 100 I_2 = 0$.~~

~~$I_2 = \frac{200 I_1}{300} = \frac{2}{3} I_1$~~

~~$200 \cdot \frac{2}{3} I_1 - 240 I_1 = -20$~~

~~$133.33 I_1 - 240 I_1 = -20$~~

~~$-106.6 I_1 = -20$~~

~~$I_1 = 0.187$.~~

$I_2 = \frac{2}{3} \times 0.187$

$= \cancel{0.1246}$

$I_2 = -1.5 i_1$

KCL at k.

$-I_1 + i_1 + I_2 = 0$

$i_1 = I_1 - I_2$

loop 2

$I_2 = -1.5 i_1$

$-0.5 I_2 = -1.5 I_1$

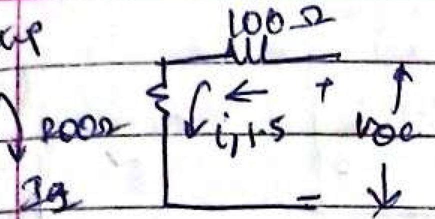
$I_2 = \frac{1.5}{0.5} I_1 = 3I_1$

$I_1 = -\frac{0.5}{9}$

$I_2 = 3I_1$

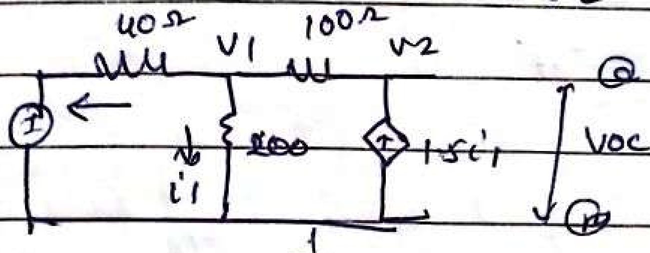
$I_2 = \frac{-1.5}{9}$ $\Rightarrow i_1 = I_a - I_b = I_1 - I_2 = \frac{1}{9}$ A

Imp step



$$V_{oc} = 1.5 i_1 \times 100 + 200(I_1 + 1.5 i_1)$$
$$= 1.5 \cdot \frac{1}{9} \times 100 + 200\left(\frac{-0.5}{9} + 1.5 \times \frac{1}{9}\right)$$

$$V_{oc} = 38.889\,V = V_{th}$$

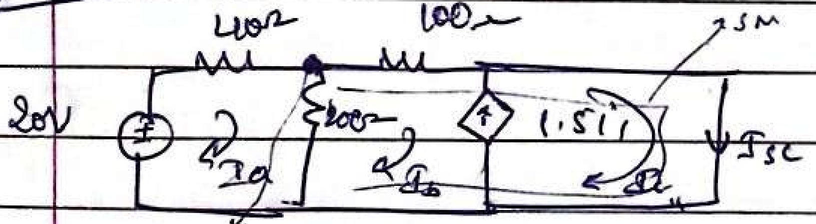


At node V1.

$$\frac{V_1 - 20}{40} + \frac{V_1}{200} + \frac{V_1 - V_2}{100} = 0 \quad (1)$$

At node V_2 $\quad \frac{V_2 - V_1}{100} - 1.5 i_1 = 0.$

$\frac{V_1}{200} = i_1 \quad \& \quad V_2 = V_{oc} = ?.$



$I_c = I_{sc}$

$-I_a + i_1 + I_b = 0.$

$I_b - I_a = -i_1. \quad — (A)$

$-I_b - 1.5 i_1 + I_c = 0$

$I_c - I_b = 1.5 i_1. \quad — (B)$

unify using nodal analysis

$20 - 40 I_a - 200 I_a + 200 I_b = 0.$

$-240 I_a + 200 I_b = -20. \quad — (1)$

$I_a = \frac{3}{16}\,A$

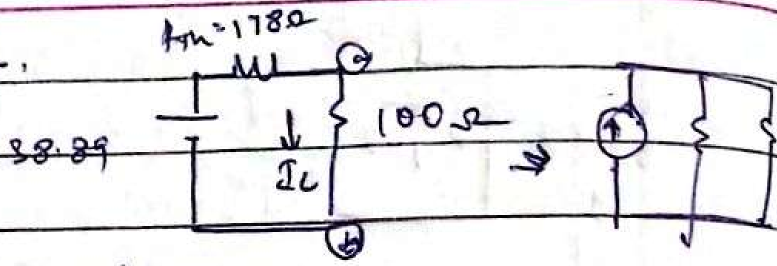
$-200 I_b + 200 I_a - 100 I_b = 0.$

$-300 I_b + 200 I_a = 0.$

$I_b = 1/8\,A$

$i_1 = \frac{1}{16}$; $I_c = 1.5 i_1 + I_b = \frac{7}{32} = 0.218A$

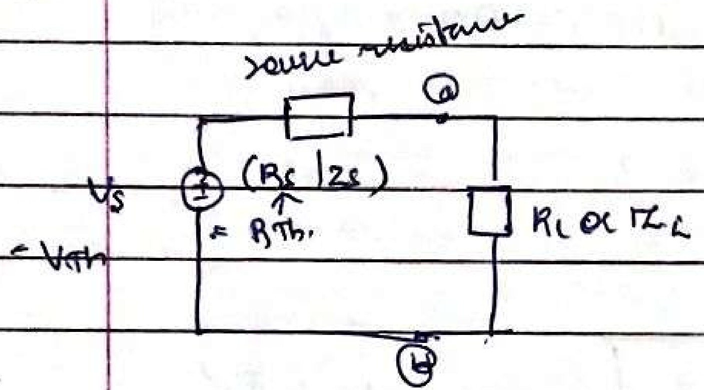
$R_{Th} = \dfrac{V_{oc}}{I_{sc}} = 178\Omega.$



$P_L = R_L I_L^2 = \left(\dfrac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L = 1.96W.$

$= \cancel{13.89W}$

(4) Maximum Power Transmission Theorem



(A) When source & ~~Stite~~ load are purely resistive (R_S & R_L)

(B) When source ~~& load are~~ is an impedance (Z_S) & the load is purely resistive (R_L)

(C) When the source & load are impedance (Z_S & Z_L)

$Z_S = (R_S + j X_S)$

$Z_S = R_S + j X_S$

$Z_{load} = R_{load} + j X_{load}$

(A) Maximum power is transferred to load when $R_L = R_S = R_{Th}$.

(B) Maximum power will be transferred to load

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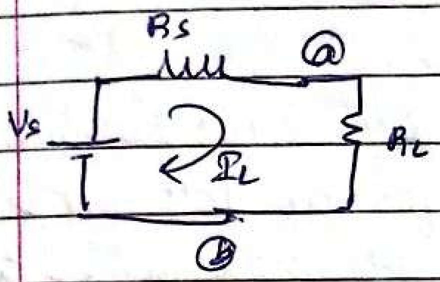
when $R_L = R_s + jX_s = \sqrt{R_s^2 + X_s^2}$

c) Max power will be transferred to Z_L when
$Z_L = Z_s^* = (R_s - jX_s)$

<u>when source & load are purely resistive</u>



$$P_L = I_L^2 \cdot R_L = \left(\frac{V_s}{R_s + R_L}\right)^2 R_L \quad -(1)$$

$$= \frac{V_s}{(R_s + R_L)^2} \cdot R_L$$

To determine the condition for M.P. transferred to load
Let us diff with respect to R_L & equate to zero

$$\frac{dP_L}{dR_L} = 0 ; \quad \frac{(R_s + R_L)^2 \cdot V_s^2 \cdot 1 - V_s^2 \cdot R_L \cdot 2(R_s + R_L)(1)}{(R_s + R_L)^4} = 0$$

$$V_s^2 \left[R_s^2 + R_L^2 + 2R_sR_L - 2R_LR_s - 2R_L^2\right] = 0$$

$$V^2 \left[R_s^2 - R_L^2\right] = 0.$$

As $V_s^2 \neq 0$; there $R_s^2 - R_L^2 = 0$

$$\underline{\underline{R_s = R_L}} \quad -(2)$$

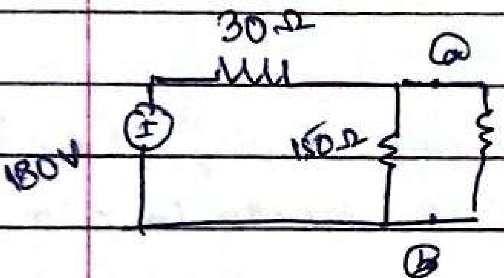
Condition for maximum power transfer
Substitute in (1)

$$P_{Lmax} = \frac{V_s^2}{(2R_s)^2} \cdot R_s = \frac{V_s^2}{4R_s} = \frac{V_s^2}{4R_L} \quad -(3)$$

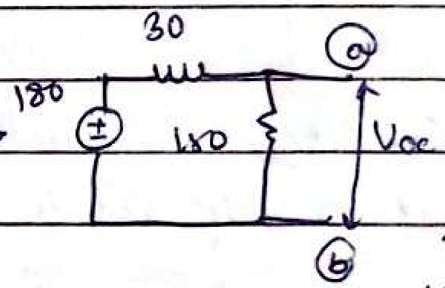
usually given circuit will be reduced to Thevenin ckt, $V_s = V_{Th}$, $R_s = R_{Th}$. Then condition for max power transfer in the load can be written as $R_L = R_{Th}$.

$$P_{Lmax} = \frac{V_{Th}^2}{4 R_{Th}}$$

- Find the value of R_L that will result in max power transfer in the load for the foll network & also determine the value of max power to load



$\Rightarrow$

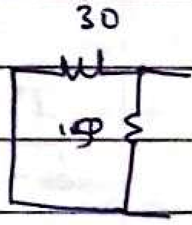


$$180 - 30i - 180i = 0.$$
$$180 = 180i$$
$$i = 1A$$
$$V_{Th} = 150V.$$

$R_{Th} = 25\,\Omega$

$30 \| 150$

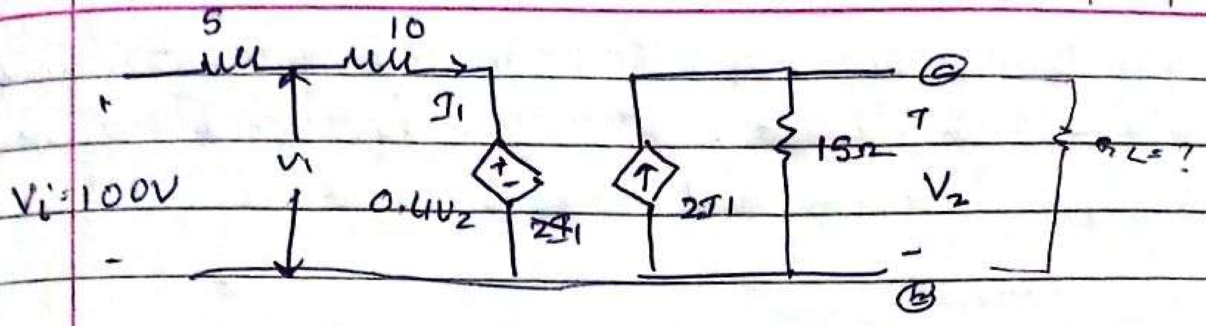
$= 25\,\Omega$



$R_{Th} = R_L = 25\,\Omega.$

$$P_L = \frac{(150)^2}{4 \times 25} = \cancel{324}\ 225\,W$$

- Find the value of R_L which is to be connected b/w A & B terminals to absorb max power in the foll ckt.



$V_2 = V_{oc} = 2I_1 \times 15 = 30I_1.$

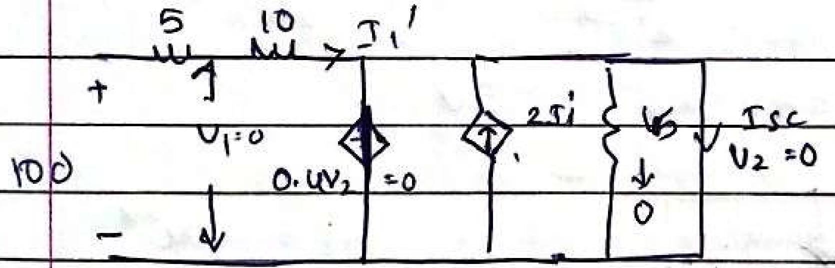
$100 - I_1 \times 15 - 0.4V_2 = 0.$

$100 - 15I_1 - 0.4 \times 30\, I_1 = 0$

$27I_1 = 100$

$I_1 = \frac{100}{27} = \cancel{4.76A}\ \underline{3.704A}$

$V_{oc} = V_2 = V_{Th} = 30I_1 = 111.12V.$



$I_{sc} = 2I_1'.$

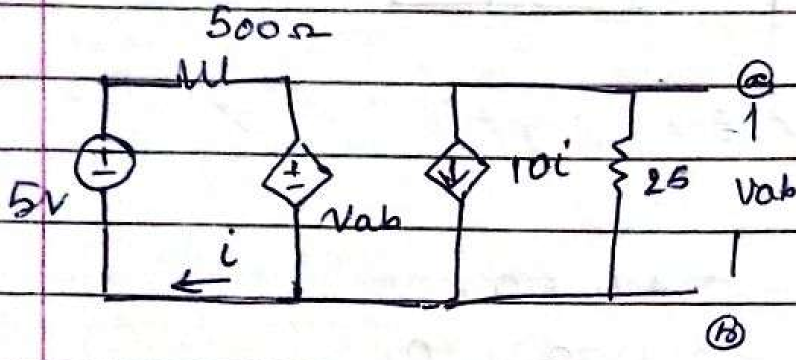
$I_1' = \frac{100}{15} = 6.67A$

$I_{sc} = 2 \times 6.67 = 13.334A$

$R_{Th} = \frac{V_{oc}}{I_{sc}} = \frac{V_{Th}}{I_{sc}} = \frac{111.12}{13.34} = 8.33\Omega = R_L$ for max power transfer. $P_{Lmax} = \frac{(V_{Th})^2}{4R_{Th}} = \frac{111.1^2}{4 \times 8.33}$

$= \underline{370.02W}$

6 Find nortons eq ckt on left of the terminals a & b. Also draw thevenin's eq ckt & det max ~~pto~~ power transferred across load connected at a & b



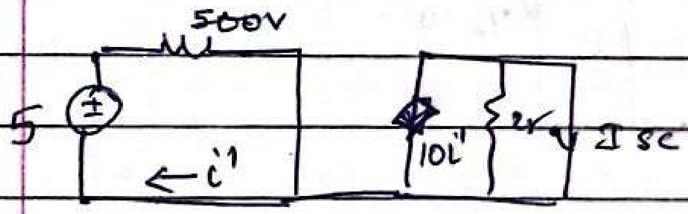
$V_{ab} = V_{Th} = -25\times 10i = -250i$.

$5\ \cancel{100} - 500i - V_{ab} = 0$.

$5\ \cancel{100} - 500i + 250i = 0$

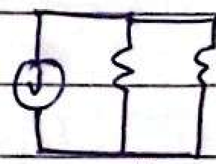
$i = \frac{\cancel{100}}{\cancel{250}} = \cancel{0.4A}\ \frac{5}{250} = 0.02A$

$V_{ab} = -250\times 0.02 = -5V$

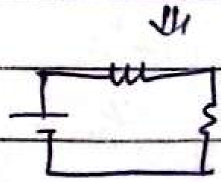


$I_{sc} = 10i'$

$I_{sc} = 10\times\frac{5}{500} = \cancel{0.05}\ -0.01\ A \times 10 = \cancel{0.5}\ -0.1\ A.$

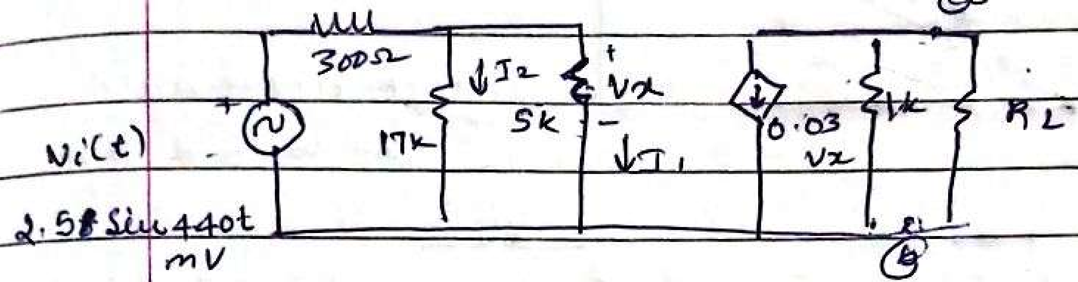


$R_{Th} = \frac{V_{oc}}{I_{sc}} = \frac{-5}{0.1} = \cancel{100}\ \cancel{-0.25}\ 50$

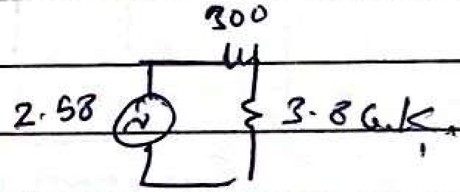


$P_{max} = \frac{(5)^2}{4\times 50} = \frac{25}{200} = \cancel{0.625W}\ \underline{0.125\,\omega}$

- Foll ckt is BJT model in CE mode. Determine the load resistance value through which BJT amplifier can supply max power & also determine the value of the max power



~~Vab = 0.03 Vx~~ K.



$\cancel{2.58 - 300i} - 17i = 0.$

$\frac{V_1 - 2.58}{300} + \cancel{\frac{V_1}{17}} + \cancel{\frac{V_1 - V_x}{5}} = 0.$

$\cancel{2.58 - 300i - 3.86i} = 0.$ 17

$\cancel{296.14i = 2.58}$

$\cancel{i = 8.7 mA}.$

$V = \cancel{8.5 \times 3.86}$

$\cancel{V = 3.86 \times 10^3 \times 0.6 \times 10^{-6}}$

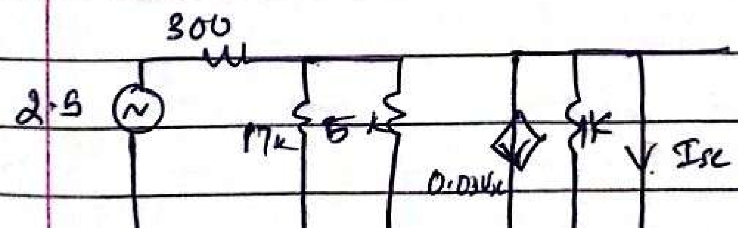
$i = \frac{2.58 \times 10^{-3}}{4.16 \times 10^{3}}$

$= 0.6 \mu A$

$I_1 = \frac{I \times 17k}{17k + 5k} = 0.464 \mu A$

$V_x = I_1 \times 5k = 2.32 = 2.32 \sin 440t$ mV

$V_m = V_x \times 0.03 = 0.696.$



$I_{sc} = -0.03 V_x$

$I' = \frac{2.5}{4.16 \times 10^3} = 0.6 \times 10^{-3}$

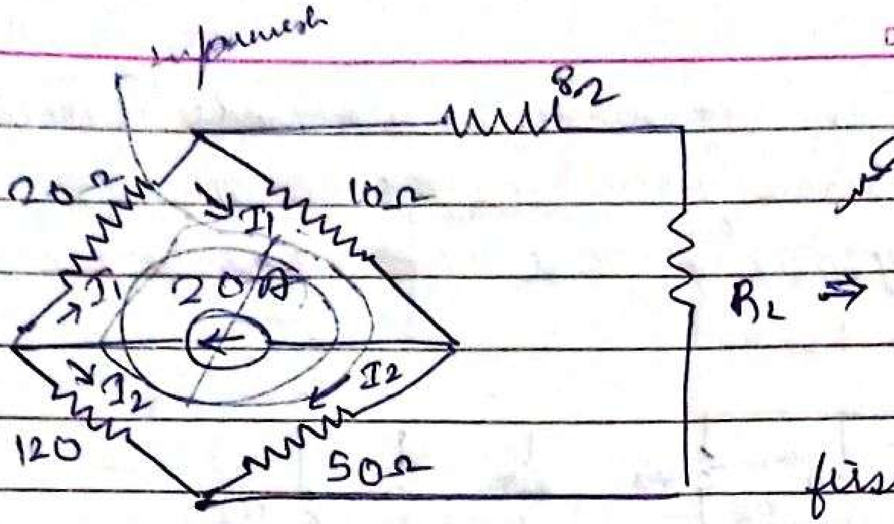
$\cancel{I_{sc} = 0.0}$

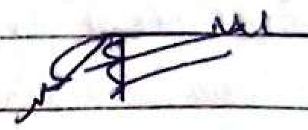
$R_{th} = 0.696 / \cancel{0.69 \times 10^{-3}}$

$= 1k.$

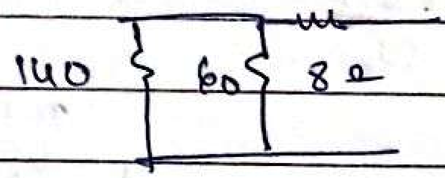
$P_{max} = 1.211 \sin^2(440t) \mu W.$

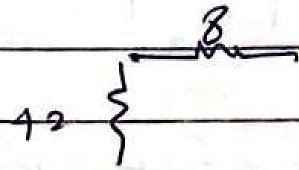
5]





first deactivate
2A to find R_{Th}.





$R_{Th} = 42 + 8 = 50\Omega$

Supermesh,

$I_1 - I_2 - 20 = 0.$

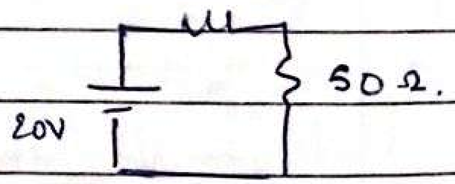
$I_1 - I_2 = 20.$ → Constraint equation.

$I_1 = 17A \quad I_2 = -3A$

$20 I_1 + 10 I_1 + 50 I_2 + 120 I_2 = 0.$

$30 I_1 + 170 I_2 = 0.$

$I_1 = +17 \quad \underline{I_2 = -3 A}$

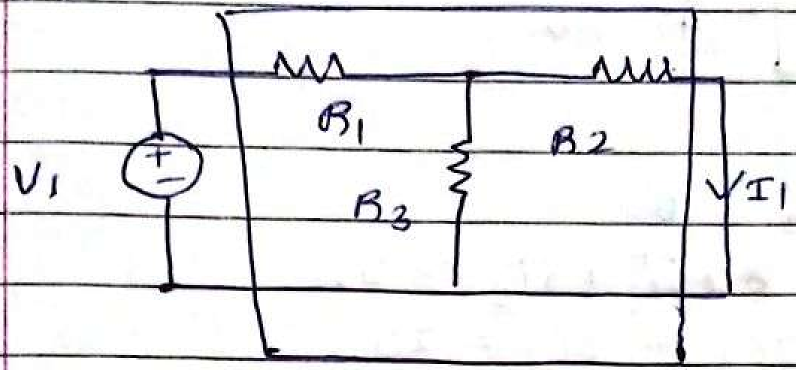


$V_{ab} = 10 \times 17 - 3 \times 50$

$= \underline{\underline{20V}}$

$P_{Lmax} = \frac{400}{4 \times 50} = \underline{2W}$

Reciprocity Theorem (or) Principle.



linear, bilateral

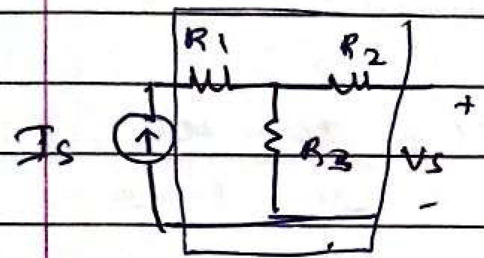
& vice versa.

$$\frac{V_1}{I_1} = \frac{V_1'}{I_1'}$$

or $I_1 = I_1'$

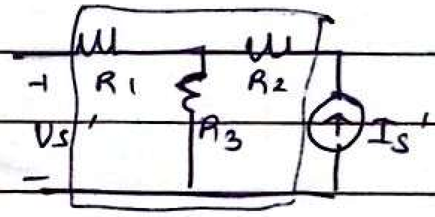
V → Excitation

I → Response.



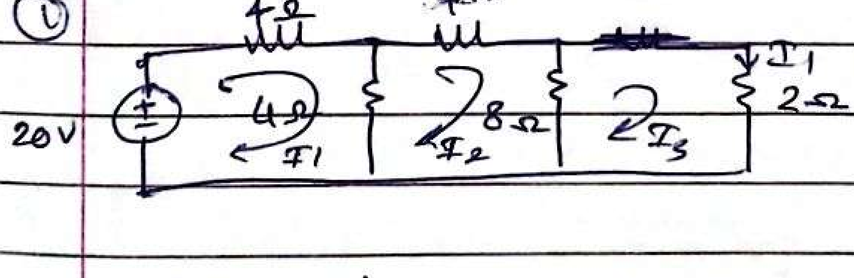
I → excitation

V → Response



• Find the current in 2Ω resistor & verify reciprocity Theorem

①



$20 - 4i_1 - 4(i_1 - i_2) = 0.$

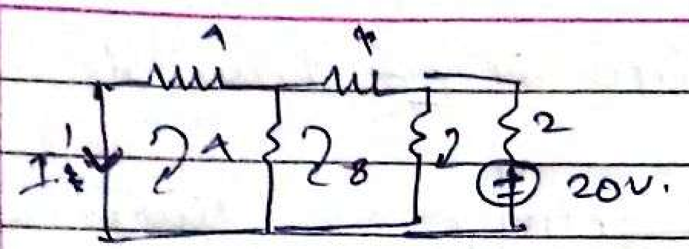
$20 - 4i_1 - 4i_1 + 4i_2 = 0 \Rightarrow 20 = 8i_1 - 4i_2$

$-4i_2 + 4i_1 - 4i_2 - 8i_2 + 8i_3 = 0.$

$-16i_2 + 4i_1 + 8i_3 = 0.$

$-8i_3 + 8i_2 - 2i_3 = 0 \Rightarrow -10i_3 + 8i_2 = 0.$

$i_1 = 3.19 \quad i_2 = 1.31 \quad i_3 = 1.052\,A.$



$$-8i_1' + 4i_2' = 0.$$
$$-16i_2' + 84i_1' + 8i_3' = 0.$$
$$-8i_3 - 2i_3 + 8i_2 = 20.$$

$$i_1' = \underline{1.052A}$$

Reciprocity principle is verified.

$$\frac{20}{I} = \frac{20}{1.052} = I\,1.052 \,; \quad \frac{20}{I_1} = \frac{20V}{1.052} = I' = 0.92$$