22/4/20

Stability Analysis in S-Domain

* Examine the stability of the following systems described by the characteristic eq^n

① $s^4 + 2s^3 + 3s^2 + 8s + 2 = 0$

We use Routh's - Stability criteria

s^4	1	3	2
s^3	2	8	-
s^2	-1	2	-
s^1	12	-	-
s^0	2	-	-

Since two times the changes in sign appear in the first column, we find that two roots of the characteristic eq^n lie in the RHSP. Hence the s/m is unstable.

② $2s^4 + s^3 + 3s^2 + 5s + 10 = 0$

Using RH stability criteria

s^4	2	3	10
s^3	1	5	-
s^2	-7	10	-
s^1	45/7	-	-
s^0	10	-	-

The system is unstable

(3) $s^6+3s^5+4s^4+6s^3+5s^2+3s+2=0$

Using R-H criteria

s^6	1	4	5	2
s^5	3	6	3	0
s^4	2	4	2	0
s^3	0	0	0	0

Now a full row appears to be '0' ∴ We form a auxillary equation using 3rd row.

$$A(s)=2s^4+4s^2+2$$

$$\cancel{\frac{dA(s)}{ds}=0}$$

$$\frac{dA(s)}{ds}=8s^3+8s$$

s^6	1	4	5	2
s^5	3	6	3	0
s^4	2	4	2	0
s^3	8	8	0	0
s^2	2	2	0	0
s^1	0	0	0	0

Again find a auxillary eqn

$$A(s)=2s^2+2$$

$$A'(s)=4s$$

s^6	1	4	5	2
s^5	3	6	3	0
s^4	2	4	2	0
s^3	8	8	0	0
s^2	2	2	0	0
s^1	4	0	0	0
s^0	2	0	0	0

WCPM

No sign change

Now $A(s) = 2s^4 + 4s^2 + 2 = 0$ & $A'(s) =$ [illegible] $= 0$

$s = \pm j$

$s^4 + 2s^2 + 1 = 0$

$s^2(s^2+1) + 1(s^2+1) = 0$

$(s^2+1)(s^2+1) = 0$

$s^2 = -1, -1$

$\therefore s = \pm j, \pm j$

roots are $j\omega$ on imaginary axis not on real axis $\therefore$ marginally stable

(4) $s^6 + 4s^5 + 3s^4 - 16s^3 - 64s - 48 = 0$

~~s^6 1 16 -648~~

s^6	1	3	-16	-48
s^5	4	0	-64	0
s^4	3	0	-48	0
s^3	0	0	0	0

Since a row of zero appears we form a auxillary frequency eqn $A(s) = 3s^4 - 48$

$$\frac{dA(s)}{ds} = 12s^3$$

s^6	1	3	-16	-48
s^5	4	0	-64	0
s^4	3	0	-48	0
s^3	12	0	0	0
s^2	$0 \to \epsilon$	-48	0	0
s^1	$-\infty$	0	0	0
s^0	-48			

Sign changed only once hence 1 root is in RHSP

$\therefore$ S/m is unstable

(5) $s^6+2s^5+8s^4+12s^3+20s^2+16s+16=0$

Using R-H criteria of stability

s^6	1	8	20	16
s^5	2	12	16	-
s^4	2	12	16	-
s^3	0	0	0	-

Using auxillary eqn $A(s) = 2s^4+12s^2+16$

$\frac{dA(s)}{ds} = 8s^3+24s$

s^6	1	8	20	16
s^5	2	12	16	-
s^4	2	12	16	-
s^3	8	24	-	-
s^2	6	16	-	-
s^1	8/3	-	-	-
s^0	16	-	-	-

Since there is no change in sign the s/m is stable

Now $A(s)=0$

$s^4+6s^2+8=0$

$s^4+2s^2+4s^2+8=0$

$s^2(s^2+2)+4(s^2+2)=0$

$(s^2+2)(s^2+4)=0$

$s^2=-2,-4$

$\therefore s = \pm\sqrt{-2}, \pm 2j$

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(6) $s^6+3s^5+5s^4+9s^3+8s^2+6s+4=0$

Using R-H stability criteria

s^6	1	5	8	4
s^5	3	9	6	-
s^4	2	6	4	-
s^3	0	0	-	-

$A(s) = 2s^4 \; 6s^2+4$

$\frac{dA(s)}{ds} = 8s^3+12s$

s^6	1	5	8	4
s^5	3	9	6	-
s^4	2	6	4	-
s^3	8	12	-	-
s^2	3	4	-	-
s^1	4/3	-	-	-
s^0	4	-	-	-

Since there is no sign change s/m is stable

$\frac{dA(s)}{\cancel{ds}} = 0$

$s^4+3s^2+2=0$

$(s^2+2)(s^2+1)=0$

$s^2=-1, -2$

$s = \pm j, \pm\sqrt{-2}$

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(7) $s^6+s^5+3s^4+3s^3+3s^2+2s+1=0$

Using RH stability criteria

s^6	1	3	3	1
s^5	1	3	2	–
s^4	$0 \to \epsilon$	1	1	–
s^3	$\frac{3\epsilon-1}{\epsilon}$	$\frac{2\epsilon-1}{\epsilon}$	–	–
s^2	$\dfrac{\frac{3\epsilon-1}{\epsilon}-2\epsilon}{\frac{3\epsilon-1}{\epsilon}}$		–	–
s^1	2	–	–	
s^0	0	0	0	

$$\lim_{\epsilon\to 0}\frac{3\epsilon-1}{\epsilon}=-\infty$$

$$\lim_{\epsilon\to 0}\frac{2\epsilon-1}{\epsilon}=-\infty$$

$$\lim_{\epsilon\to 0}\frac{3\epsilon-1-2\epsilon^2}{3\epsilon-1}=\frac{-1}{-1}=1$$

s^6	1	3	3	1
s^5	1	3	2	–
s^4	ϵ	1	1	–
s^3	$-\infty$	$-\infty$	–	–
s^2	1	–	–	–
s^1	2	–	–	–
s^0	$0 \to 2$ –		–	–

Sign changes hence sfm is unstable

$A(s)=2s$

$\frac{dA(s)}{ds}=2$

* find the range of values of K for which the system whose characteristics eqn is given by
$F(s) = s^3 + (K+0.5)s^2 + 4Ks + 50$ is stable.

Using R-H Stability criteria.

s^3	1	4K	-
s^2	K+0.5	50	-
s^1	$\frac{4k^2+2k-50}{k+0.5}$	-	-
s^0	50	-	-

for the s/m to be stable. there should not be any sign change

$\therefore K+0.5>0$ $\quad$ $\frac{4K^2+2k-50}{k+0.5}>0$

$k>-0.5$

$\cancel{K \Rightarrow (-\infty, -3.79) \& (-0.5, \infty)}$ $\quad$ $2K^2+k-25>0$

$k>3.29$

$3.29 < K < \infty$

$\cancel{K<-3.79}$

$K>-3.79$

* A open-loop transfer function of a positive feedback control system is given by

$$G(s)H(s) = \frac{(s+a)}{(s+1)} \times \frac{K}{s(s+2)(s+5)}$$

where 'K' and 'a' are the parameters of the system. Determine the range of values of 'K' & 'a' for which the system is stable

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Characteristic eq of a open loop transfer function is

$\Rightarrow 1 \mp G(s)H(s)$ with +ve fb.

$$\Rightarrow 1 - \frac{(s+a)k}{s(s+1)(s+2)(s+5)} = 0$$

$\Rightarrow s(s+1)(s+2)(s+5) - (s+a)k = 0$

$\Rightarrow (s^2+s)(s^2+7s+10) - ks - ak = 0$

$\Rightarrow s^4 + 7s^3 + 10s^2 + s^3 + 7s^2 + 10s - ks - ak = 0$

$\Rightarrow s^4 + 8s^3 + 17s^2 + (10-k)s - ak = 0$

Using RH stability criteria.

s^4	1	17	$-ak$
s^3	8	$10-k$	$-$
s^2	$\frac{126+k}{8}$	$-ak$	$-$
s^1	$\frac{(126+k)(10-k)+8a}{8}$	$-$	$-$
s^0	$-ak$	$-$	$-$

Since the s/m is stable there shd be no sign change

$\boxed{-126} \leftarrow \frac{126+k}{8} > 0$ $-a > 0$ $\therefore a < 0$ $-ak > 0$ $\boxed{\therefore ak < 0}$

$$\frac{(126+k)(10-k)+8a}{8} > 0$$

$1260 + k10 - 126k - k^2 + 64a > 0$

$k^2 + 116k - 1260 - 64ak < 0$

$k^2 + (116 - 64a)k - 1260 < 0$

WCPM

* The characteristic eq^n of a mechanical s/m consisting of mass, spring, damper elements in differential form is

~~$dx^2 - (k+2)x$~~

$$x'' - (k+2)x' + (2k+3)x = 0$$

(a) find the value of k for which the s/m is (i) Stable (ii) Marginally stable (iii) Unstable

→ $S^2 - (k+2)s + (2k+3) = 0$

S^2	1	$2k+3$
S^1	$-(k+2)$	–
S^0	$2k+3$	–

(i) Stable : if $-(k+2) > 0$ $\quad 2k+3 > 0$

$-k-2 > 0$ $\quad$ ~~$-k+2<0$~~ $\quad$ ~~$2k<-3$~~

$-k < 2$ $\quad$ ~~$k<2$~~ $\quad$ ~~$k<-3/2$~~

$k < -2$ $\quad$ ~~$-3/2 \le k \le -2$~~ $\quad$ ~~$k<-1.5$~~

~~$(-\infty, <2)$~~ $\quad$ ~~$-1.5 \le k \le 2$~~ $\quad$ ~~$-2 > k > -1.5$~~

~~$-1.5 \le k \le -2$~~ $\quad$ $(-\infty, -2)$ $(-1.5, \infty)$ $\quad k > -1.5$

(ii) ~~Unstable : if~~ marginal stable :

if $-(k+2) = 0$ $\quad 2k+3 = 0$

$k = -2$ $\quad k = -1.5$

(iii) Unstable : if ~~$-2<k$~~

$k \not< -2$ or $k \not> -1.5$

or both

(b) for what value of (stable s/m) 'K' the s/m is
(i) Underdamped (ie if $0<\xi<1$)
(ii) Overdamped (ie if $1<\xi<\infty$)

$$\omega_n^2 = 2k+3 \qquad 2\xi\omega_n = -(k+2)$$
$$\omega_n = \sqrt{2k+3} \qquad 2\xi = \frac{-k-2}{\sqrt{2k+3}}$$
$$\xi = \frac{-(k+2)}{2\sqrt{2k+3}}$$

$\therefore$

(i) Underdamped:-

$$\xi>0 \Rightarrow \frac{-(k+2)}{2\sqrt{2k+3}}>0$$

$$k+2<0 \qquad \text{but } \sqrt{2k+3}>0$$
$$k<-2 \qquad 2k+3>0$$
$$k>-3/2$$

$$\xi<1 \qquad \frac{-(k+2)}{2\sqrt{2k+3}}<1$$

$$-k-2<2\sqrt{2k+3}$$
$$k^2+4k+4<4(2k+3)$$
$$k^2-4k-8<0$$
$$(k-5.46)(k+1.46)<0$$
$$k<5.46 \qquad k \geq -1.46$$

$-2 \quad -1.5 \quad -1.46 \quad 0 \quad 5.46$

$(-\infty,-2)$ $(-1.5, 5.46)$

(ii) Overdamped $\quad \cancel{2\xi\omega_n = -k-2} \quad \xi>1$

$k>5.46 \quad k \leq -1.46$ $\qquad (-1.46, 5.46)$

$$2\sqrt{2k+3} \neq 0$$
$$k \neq -1.5$$

WCPM

* Determine the range of values of k (k>0), such that the characteristic eqⁿ

$$s^3 + 3(k+1)s^2 + (7k+5)s + (4k+7) = 0$$ has

roots more negative than $s = -1$.

→ Using RH stability criteria

s^3	1	$7k+5$
s^2	$3k+3$	$4k+7$
s^1	$\frac{(3k+3)(7k+5)-(4k+7)}{3k+3}$	-
s^0	$4k+7$	-

Since given roots are more negative than $s = -1$ S/m is stable.

$\therefore 3k+3 > 0$ $\quad$ $21k^2 + 32k + 8 > 0$

$k > -1$ $\quad$ $\Rightarrow k > 0$

$4k+7 > 0$

$k > -7/4$

$\therefore$ Given $k > 0$

$\therefore$ Range of k $[0, \infty)$

$0 \leq k < \infty$

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Bode plot

(1) $G(s) = \frac{K}{s(s+2)(s+20)} = \frac{K/40}{s(1+s/2)(1+s/20)}$

Step 1: Corner frequency, Let assume $K/40 = 1$, $20\log\left(\frac{K}{40}\right) + 20dB = 20dB$

$T_1 = 0.5 \quad \omega_1 = 2\,rad/sec$
$T_2 = 0.05 \quad \omega_2 = 20\,rad/sec$

* Magnitude plot calculation

Sl.No.	factor	Corner frequency	Asymptotic log-Magnitude characteristic	Resultant slope
1	1	None	$20\log 1 + 20dB = 20dB$	None
2.	$1/s$	None	$\omega = 1\,rad/sec$	$-20dB/dec$
3.	$1/(1+0.5s)$	2	$\omega_1 = 1/0.5 = 2\,rad/sec$	$-40dB/dec$
4	$1/(1+0.05s)$	20	$\omega_2 = 1/0.05 = 20\,rad/sec$	$-60dB/dec$

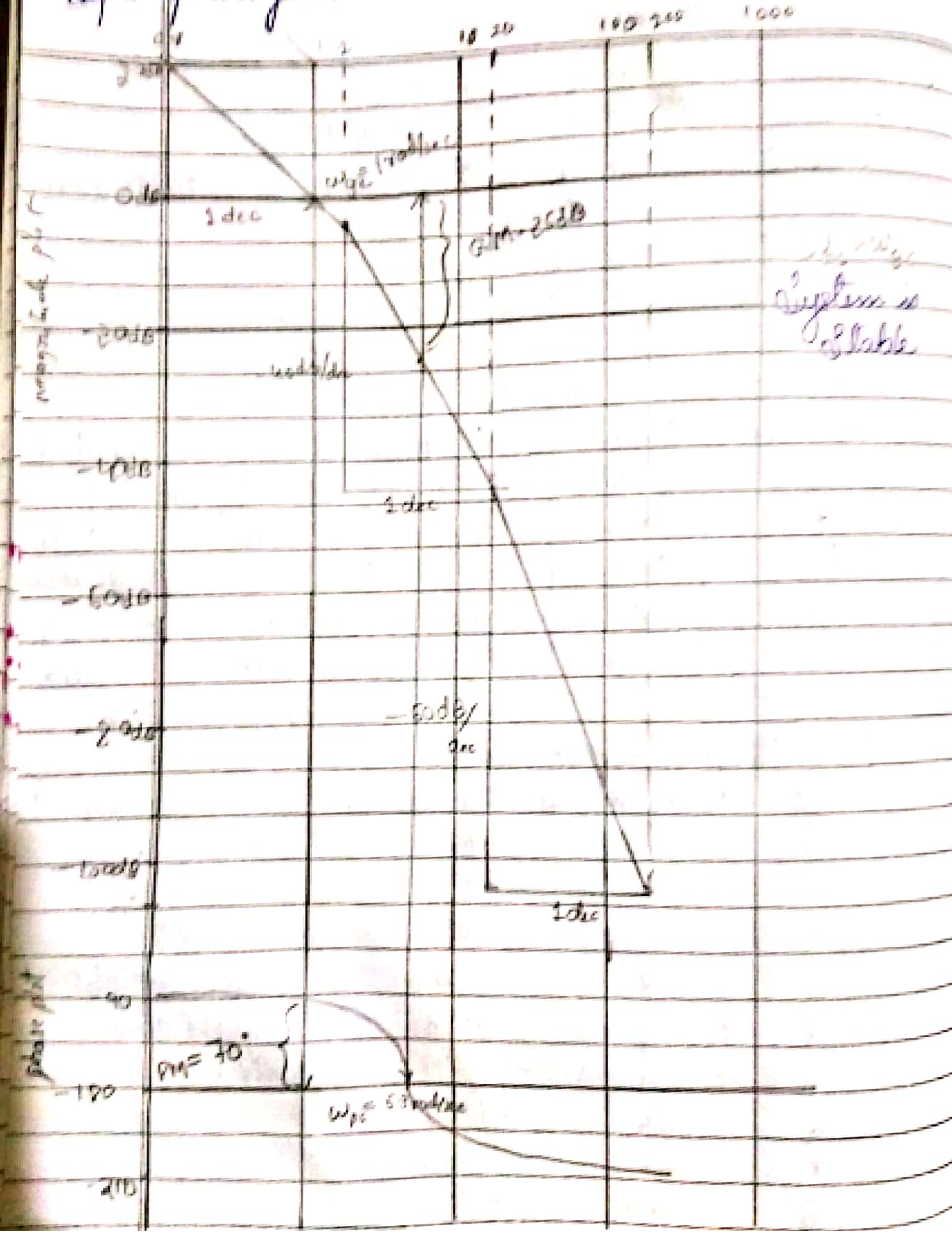
Step 2: Phase plot calculation.

$\phi = -90° - \tan^{-1}0.5\omega - \tan^{-1}0.05\omega$

Sl.No	ω	ϕ
1	0	$-90°$
2	1	$-119°$
3	2	$-140°$
4	3	$-154.8°$
5	4	-164.7
6	5	$-172.2°$
7	6	-178.2
8	6.5	-181.2

Starting of the Magnitude plot
$= 20\log(1) + 20dB$
$= 20\,dB.$

Step 3: plotting



System is Stable

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Step 4: Calculation of K

The gain margin is 26dB

$20\log(0.025K) = 26dB$

$\therefore K = 798$

(2) $G(s) = \dfrac{10}{s(1+s)(1+0.2s)}$

Step 1: Corner frequency :- $k = 10$, $20\log(10) + 20dB = 40dB$

Magnitude plot calculation.

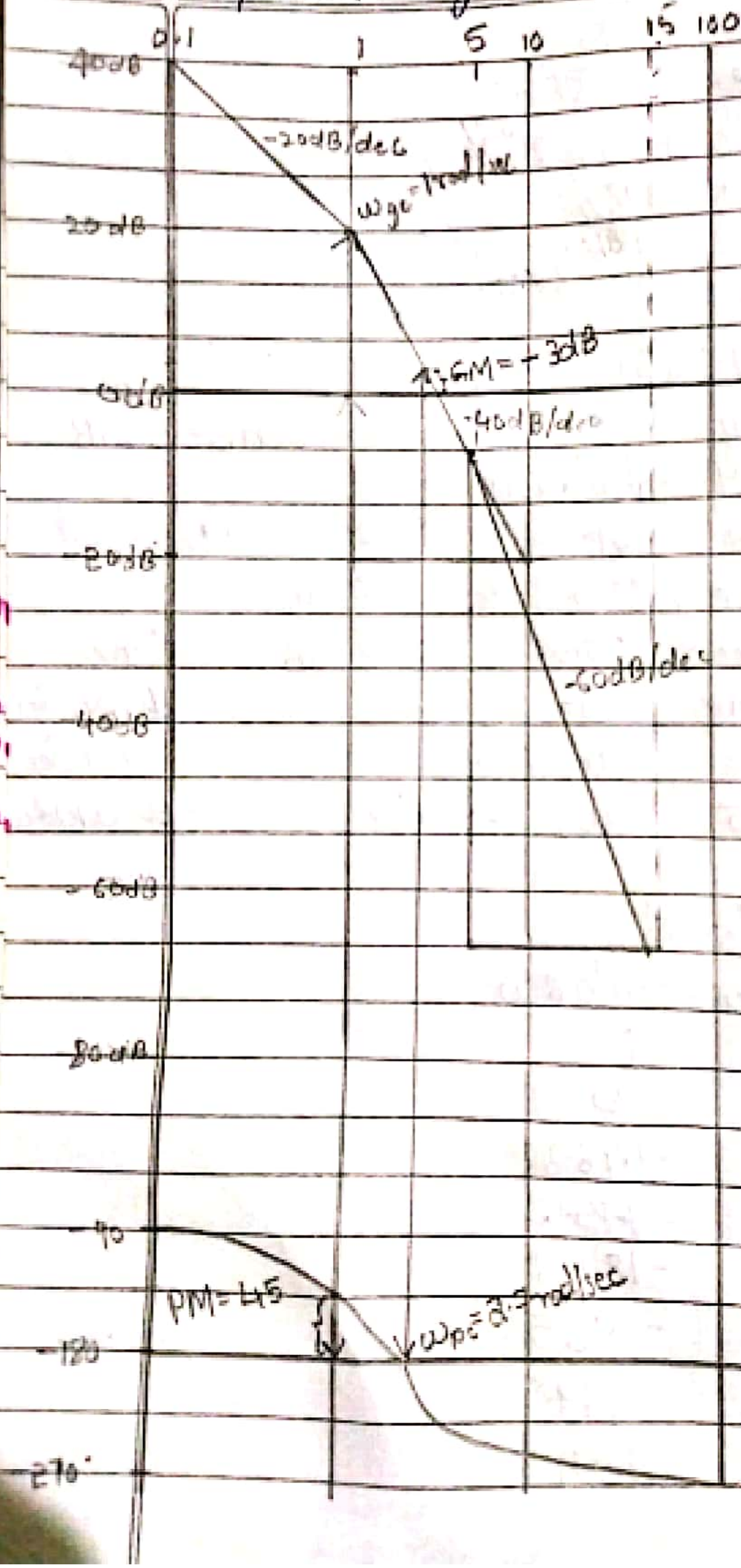
Sl.No.	factor	Corner frequency	Asymptotic log-Magnitude characteristic	Resultant slope
1	10	None	$20\log(10) + 20dB = 40dB$	None
2	$1/s$	None	$\omega = 1$ rad/sec	-20 dB/dec
3	$1/(1+s)$	1	$\omega_1 = 1$ rad/sec	-40 dB/dec
4	$1/(1+0.2s)$	~~10~~ 5	$\omega_2 =$ ~~10~~ 5 rad/sec	-60 dB/dec

Step 2: Phase plot calculation

$\phi = -90° - \tan^{-1} 1\omega - \tan^{-1} 0.2\omega$

Sl.No	ω	ϕ
1	0	$-90°$
2	1	$-146.3°$
3	2	$-175.2°$
4	2.5	-184.7
5	3	-192.5
6	3.5	-199.0
7	4	$-204.62°$

Step 3 :- Plotting.



$\omega_{pc} > \omega_{gc}$
S/m is stable

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(3) $G(s) = \dfrac{10}{s(1+0.01s)(1+0.1s)}$

Step1:- Magnitude plot calculation:-

Sl.No.	Factor	Corner frequency	Asymptotic log. magnitude characteristic	Resultant slope
1	10	–	$20\log(10) = 20dB = 40dB$	–
2	$1/s$	–	$\omega = 1 rad/sec$	$-20 dB/dec$
3	$1/(1+0.01s)$	10	$\omega_1 = 10 rad/sec$	$-40 dB/dec$
4	$1/(1+0.01s)$	100	$\omega_2 = 100 rad/sec$	$-60 dB/dec$

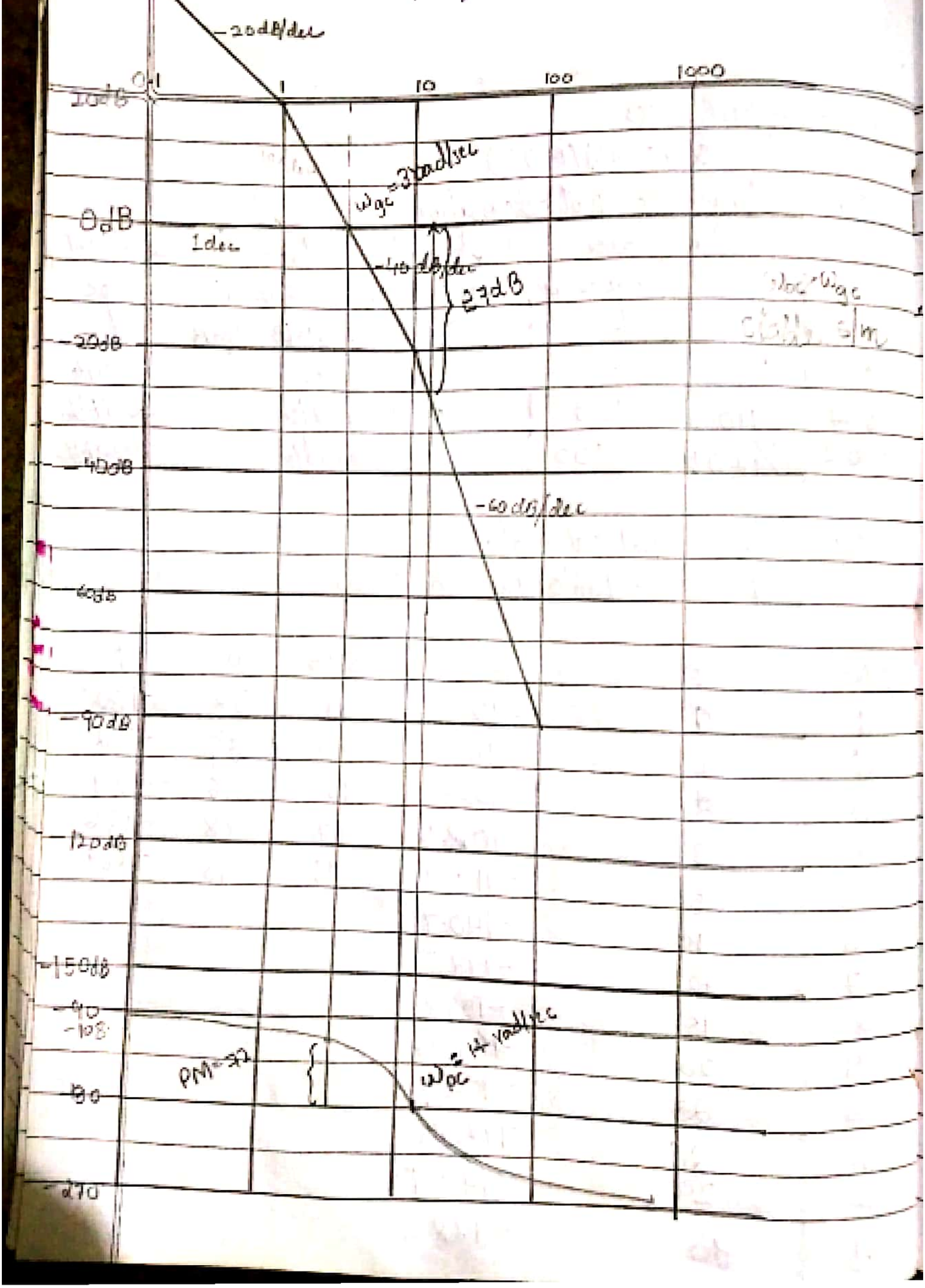
Step2: Phase plot calculation

$\phi = -90 - \tan^{-1}0.01\omega - \tan^{-1}0.1\omega$

Sl.No.	ω	φ	Sl.No	ω	φ
1	0	-90°	14	32	-180°
2	1	-96.3°	15	34	-182°
3	2	-102.4	16	36	-184.
4	3	-108.4	17	38	-186.
5	5	-119.42	18	40	-187
6	10	-140.7			
7	12	-147°			
8	15	-154.8			
9	20	-167.74			
10	25	-172.23			
11	27	-174.78			
12	29	-177.14			
13	30	-178.26			

WCPM

Step 3: plotting



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(1) $G(s) = \dfrac{10}{s(1+0.2s)(1+0.01s)}$

Step 1:- Magnitude plot calculation

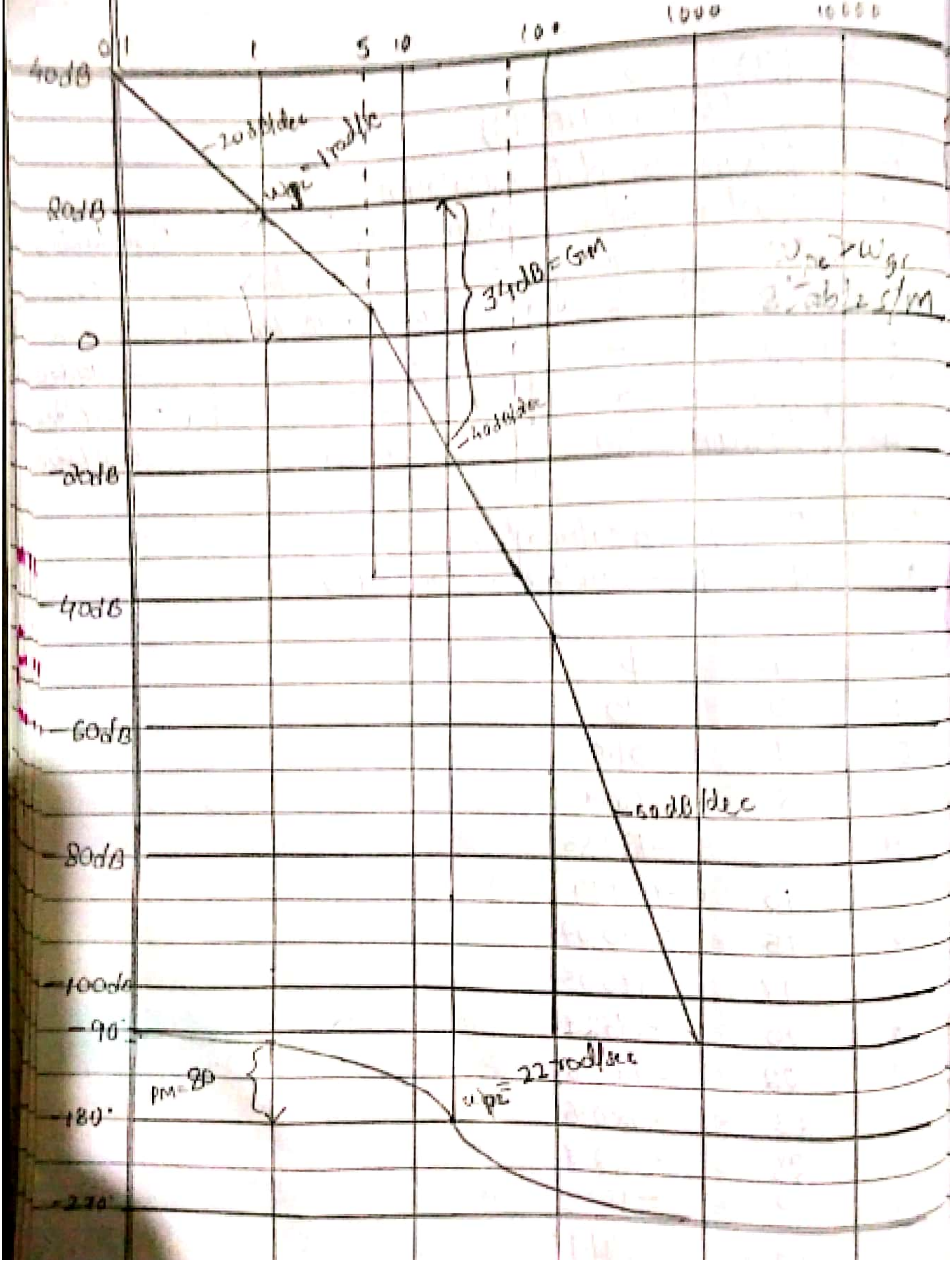
Sl.No.	factors	Corner frequency	Asymptotic log-magnitude characteristic	Resultant slope
1	10	-	$20\log(10)+20dB = 40dB$	-
2	$1/s$	-	$\omega = 1$ rad/sec	-20dB/dec
3	$1/(1+0.2s)$	5	$\omega_1 = 5$ rad/sec	-40dB/dec
4	$1/(1+0.01s)$	100	$\omega_2 = 100$ rad/sec	-60dB/dec

Step 2:- Phase plot calculation

$\phi = -90° - \tan^{-1}0.2\omega - \tan^{-1}0.01\omega$

Sl.No	ω	ϕ
1	0	-90°
2	1	-101.9°
3	2	-112.9
4	5	-137.86
5	10	-159.14
6	15	-170.09
7	17	-173.25
8	20	-177.27
9	22	-179.6
10	23	-180.6°
11	25	-182.7°
12	30	-187.2°
13	35	-191.1°

WCPM



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(5) $G(s) = \dfrac{50}{s(s+10)(s+5)(s+1)} = \dfrac{1}{s(1+0.1s)(1+0.2s)(1+s)}$

$\Rightarrow \dfrac{1}{s(1+s)(1+0.2s)(1+0.1s)}$

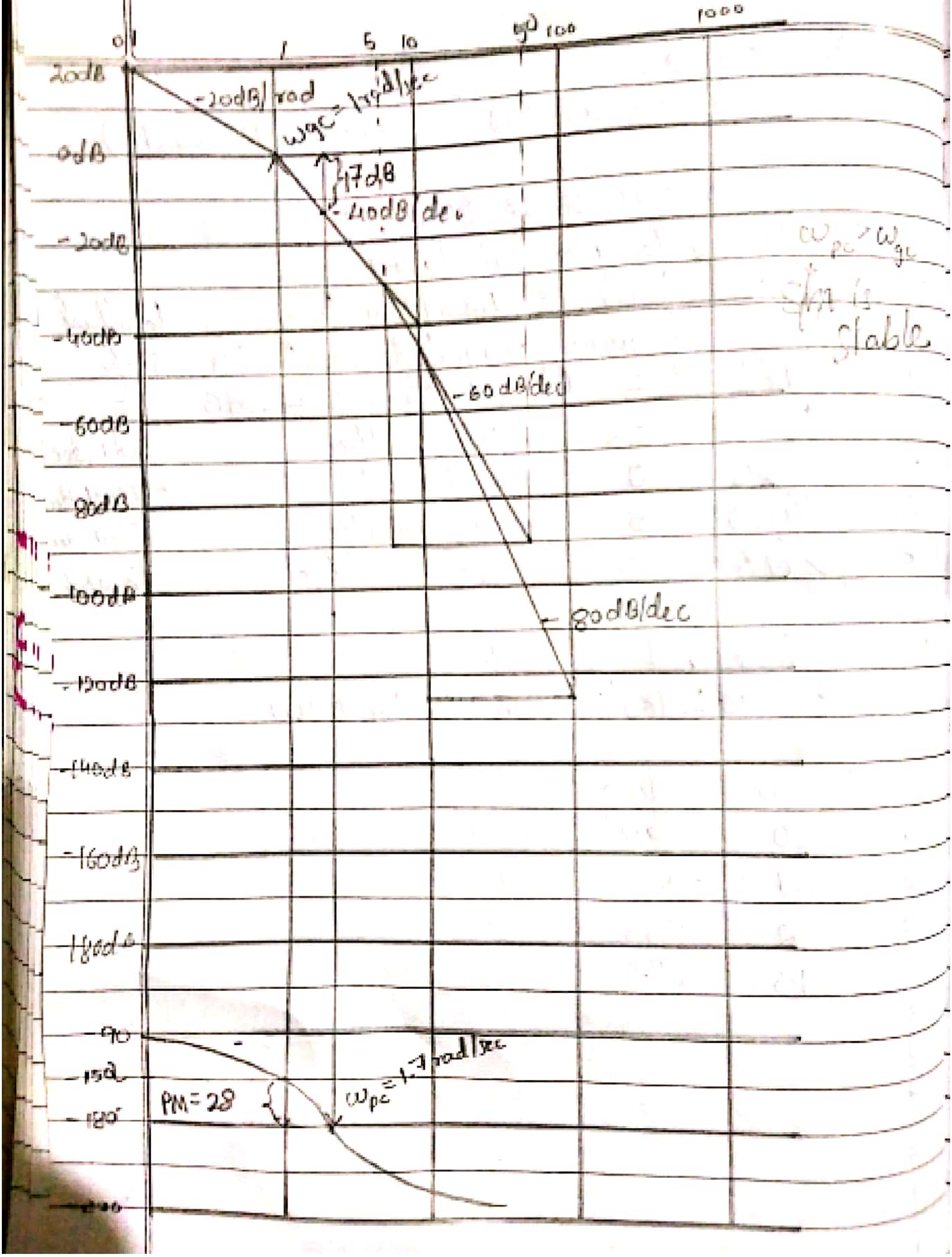
Step 1:- Magnitude plot calculation

S.No	Factor	Corner frequency	Aysmptotic-log-magnitude characteristics	Resultant slope
1	1	–	$20\log(1) + 0\,dB \Rightarrow 20\,dB$	–
2	$1/s$	–	$\omega = 1$ rad/sec	-20 dB/dec
3	$1/(1+0.1s)$	10	$\omega_1 = 1$ rad/sec	-40 dB/dec
4	$1/(1+0.2s)$	5	$\omega_2 = 5$ rad/sec	-60 dB/dec
5	$1/(1+0.1s)$	10	$\omega_3 = 10$ rad/sec	-80 dB/dec

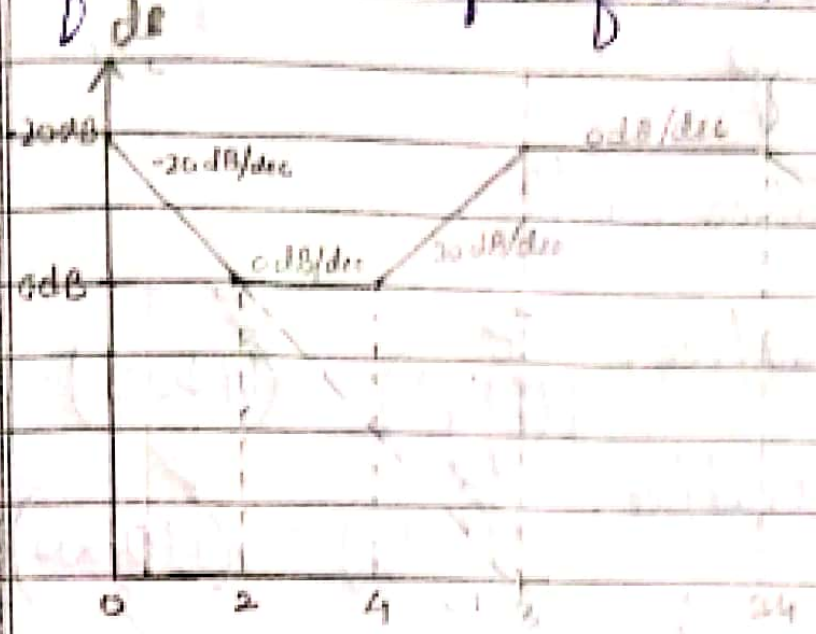
Step 2:- Phase plot calculation

$\phi = -90^\circ - \tan^{-1}\omega - \tan^{-1}0.2\omega - \tan^{-1}0.1\omega$

S.No.	ω	ϕ
1	0	-90°
2	1	-152.0
3	2	-186.5
4	1.5	-171.53
5		
6		
7		


0.1
1
5
10
50
100
1000
20dB
0dB
-20dB
-40dB
-60dB
-80dB
-100dB
-120dB
-140dB
-160dB
-180dB
-20dB/rad
-40dB/dec
-60dB/dec
-80dB/dec
17dB
-90
-150
-180
PM = 28
stable

* find the transfer function of bode plot



Solⁿ :- Step1: find value of k
Since graph starts at -20dB
k = 1

Step 2 - Corner frequencies & slope
Change in slope is -20 dB a simple zero pole is added at $\omega_p = 0.1$ rad/sec
change in slope is 20dB a simple zero is added at $\omega_1 = 2$ rad/sec
Change in slope is 20 dB a simple zero is added at $\omega_2 = 4$ rad/sec
Change in slope is -20dB a simple pole is added at $\omega_3 = 8$ rad/sec
Change in slope is -20 dB a simple pole is added at $\omega_4 = 24$ rad/sec
Change in slope is -40dB a pole is added at $\omega_5 = 36$ rad/sec

Step 3 – Standard form at each of corner frequencys

At $\omega = 1$ rad/sec, ; $1/S$

At $\omega_1 = 2$ rad/sec, ; $\frac{1}{(1+T_1S)} = \frac{1}{(1+0.5S)}$

At $\omega_2 = 4$ rad/sec, ; $\frac{1}{(1+T_2S)} = \frac{1}{(1+0.25S)}$

At $\omega_3 = 8$ rad/sec, ; $\frac{1}{1+T_3S} = \frac{1}{1+0.125S}$

At $\omega_4 = 24$ rad/sec, ; $\frac{1}{1+T_4S} = \frac{1}{1+0.042S}$

At $\omega_5 = 36$ rad/sec, ; $\frac{1}{1+T_5S} = \frac{1}{1+0.027S}$

Transfer function:

$$H(S) = \frac{1(1+0.5S)(1+0.25S)}{S(1+0.125S)(1+0.042S)(1+0.027S)}$$